International Research Journal of Natural and Applied Sciences
ISSN: (2349-4077)
Impact Factor- 5.46, Volume 4, Issue 12, December 2017
Website- www.aarf.asia, **Email** : editor@aarf.asia , editoraarf@gmail.com

MHD BOUNDARY LAYER FLOW OVER A STRETCHING SHEET IN A NANOFLUID WITH CONVECTIVE BOUNDARYCONDITION AND VISCOUS DISSIPATION

Pamita

Dept of Mathematics Gulbarga University Kalaburagi

ABSTRACT

This paper analyses the MHD boundary layer nano fluid flow over a stretching sheet with convective heat transport and frictional heating.The governing boundary value problem is transformed into a set of nonlinear ordinary differential equations,using similarity transformation and are solved using Runge-Kutta 4th order method with shooting technique. Portrayals of flow,heat and nano particle concentration profiles, equivalent to abundant somatic parameters are graphically as well as in tabularization form are scrutinized.

KEY WORDS: Nanofluids;Boundary layer flow;Similarity transformation;Convective heat transport;Runge-Kutta shooting method.

1.Introduction

A Nanofluid is a fluid containing nanometer sized particles, called Nanoparticles. These fluids are engineered colloidal suspension of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene Glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engine, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler gas temperature reduction. They demonstrate enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behavior of nanofluids is found to be very vital deciding their suitability for convective heat transfer applications.

The fluid flow over a stretching surface is important in applications such as extrusion, wire drawing, metal spinning, hot rolling, etc[1-3]. A wide variety of problems dealing with heat and fluid flow over a stretching sheet have been studied with both Newtonian and non-Newtonian fluids and with the inclusion of imposed electric and magnetic fields, different thermal boundary conditions, and power law variation of the stretching velocity. A representative sample of the recent literature on the topic is provided by references [4-12]. After the poineering work by Sakiadis [13], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surface. The problem of natural convection in a regular fluid past a vertical plate is a classical problem first studied theoretically by E. Pohlhausen in contribution to an experimental study by Schmidt and Beckmann[14].

In the past few years, convective heat transfer in nanofluids has become a topic of major current interest. Recently Khan and Pop [15] used the model of Kuznetsov and Nield [16] to study the boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature. Makinde, and Aziz [17] considered to study the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta Fourth order method.

The solution of boundary layer equation for a power law fluid in MHD was obtained by Helmy[18]. Chiam[19] investigated hydromagnetic flow over a surface stretching with power law velocity using shooting method. Ishak etal[20] investigated MHD flow and heat transfer adjacent to a stretching vertical sheet. Nourazar etal[21] investigated MHD forced convective flow of nanofluid over a horizontal stretching sheet with variable magnetic field with the effect of viscous dissipation. the numerical solution of unsteady MHD flow of nanofluid on the rotating stretching sheet. Hamad [22] obtained ananalytical solution by considering the effect of magnetic field for electrical conducting nanofluid flowover a linearly stretching sheet. Rana et al.[2011] investigated the numerical solution of unsteady MHD flow of nanofluid on the rotating stretching sheet. Wang and Mujumdar [23], Kakaç and Pramuanjaroenkij [24],Chandrasekar et al. [25] and Wu and Zhao [26].The effects of nanofluids could be considering in different ways such as dynamic effects which include the effects of Brownian motion and thermophoresis diffusion, see[27-29], and the static part of Maxwell’s theory,see [30-33].Recently, many researchers, using similarity solution, have examined the boundary layer flow, heat and mass transfer of nanofluidsover stretching sheets. Khan and Pop [34] have analyzed theboundary-layer flow of a nanofluid past a stretching sheet using amodel in which the Brownian motion and thermophoresis effects were taken into account. They reduced the whole governing partialdifferential equations into a set of nonlinear ordinary differential equations and solved them numerically. In addition, the set ofordinary differential equations which was obtained by Khan andPop [35] has been solved by Hassani et al. [36] using HAM. After that, many researchers, using similarity solution approach, have extended the heat transfer of nanofluids over stretching sheets and examined the other effects such as the chemical reaction and heat radiation [37], convective boundary condition [38], nonlinear stretching velocity [39], partial slipboundary condition [40],magnetic nanofluid [41], partial slip and convective boundary condition [42], heat generation/absorption[43], thermal and solutal slip [44], nano non-Newtonian fluid[45], and Oldroyd-B Nanofluid [46]. As mentioned, the enhancement of the thermal conductivity of nanofluids is the most outstanding thermo-physical properties of nanofluids. In all of the previous studies [37–50], the effect of local volume fraction of nano particles on the thermal conductivity of the nanofluid was neglected . However, in the work of Buongiorno[51], it has been reported that the local concentration of nanoparticles may significantly affect the local thermal conductivity of the nanofluids.

In this present work, the objective is to investigate the combined effect of magnetic field and viscous dissipation on the boundary layer flow over a stretching sheet in a nanofluid, with convective boundary condition. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables, and these have been solved numerically. The effects of embedded parameters on fluid velocity, temperature and particle concentration have been shown graphically. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2.Convective transport equations

consider steady two-dimensional (x, y) boundary layer flow of a nanofluid past a stretching sheet with a linear velocity variation with the distance x i.e. $u_w = cx$ where c is a real positive number, is stretching rate, and x is the coordinate measured from the location, where the sheet velocity is zero

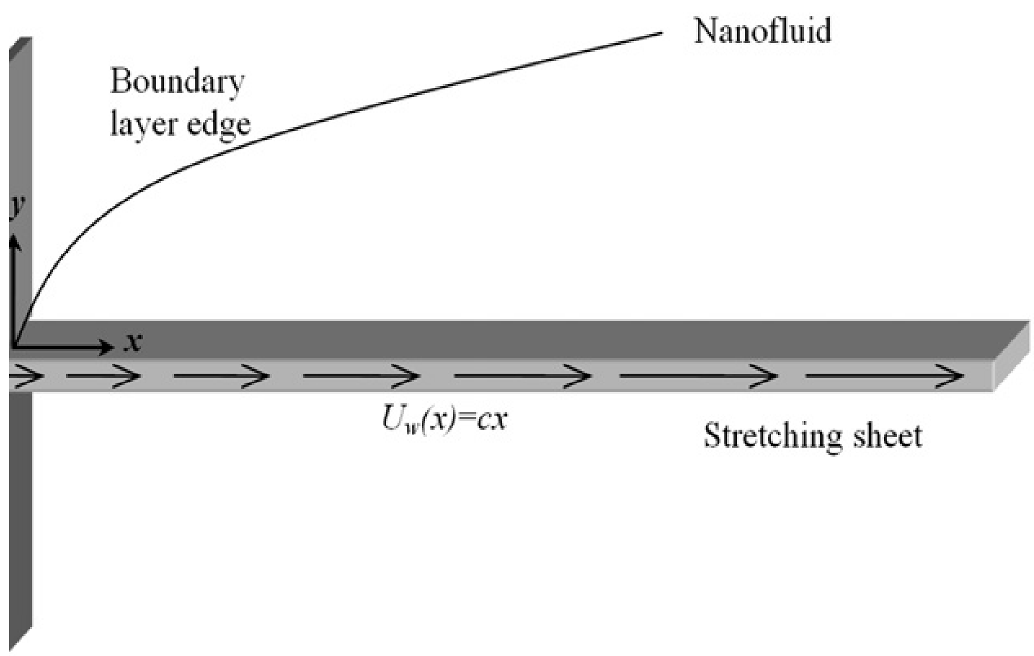

Fig A: boundary layer flow over a stretching sheet.

The sheet surface temperature T_w, to be determined later, is the result of a convective heating process which is characterized by temperature T_f and a heat transfer coefficient h . The nanoparticle volume fraction C at the wall is C_w, while at large values of y ,the value is C_∞. The Boungiorno model may be modified for this problem to give the following continuity, momentum, energy and volume fraction equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0{}^2 u}{\rho},$$

(2)

$$\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where u and v are the velocity components along the x and y directions, respectively, p is the fluid pressure, ρ_f is the density of base fluid, ν is the kinematic viscosity of the base fluid, α is the thermal diffusivity of the base fluid, $\tau=(\rho c)_p/(\rho c)_f$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and T is the local temperature. The subscript ∞ denotes the values of at large values at large values of y where the fluid is quiescent. The boundary condition may be written as

$$y=0, u=cx, v=0, -k\frac{\partial T}{\partial y}=h\left(T_f-T\right), C=C_{w,} \tag{5}$$

$$y\to\infty, u=0, v=0, T=T_\infty, C=C_\infty, \tag{6}$$

We introduce the following dimensionless quantities

$$\eta=(c/\upsilon)^{1/2}y, \psi=(c\upsilon)^{1/2}xf(\eta), \theta=\frac{T-T_\infty}{T_f-T_\infty},$$
$$\phi=\frac{C-C_\infty}{C_w-C_\infty}, \tag{7}$$

where ψ is the stream function with $u = \partial\psi/\partial y, v = -\partial\psi/\partial x$.

By using eq.(7) into eqs. (2) to (6), we obtain the following set of nonlinear ordinary differential equations,

$$f''' + ff'' - f'^2 - Mf' = 0, \quad (8)$$

$$\theta'' + Prf\theta' + PrNb\phi'\theta' + PrNt\theta'^2 + \Pr Ecf''^2 = 0, \quad (9)$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0, \quad (10)$$

subject to the following boundary conditions.

$$f(0) = 0, f'(0) = 1, \theta'(0) = -Bi[1-\theta(0)], \phi(0) = 1, \quad (11)$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \quad (12)$$

where primes denote differentiation with respect to η and the five parameters appearing in Eqs. (9) –(12) are as follows,

$$Pr = \frac{\nu}{\alpha}, Le = \frac{\nu}{D_B}, Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_{f^\nu}},$$
$$Nt = \frac{(\rho c)_p D_T (T_f - T_\infty)}{(\rho c)_{f^\nu} T_\infty}, Bi = \frac{h(\nu/a)^{1/2}}{k} \quad (13)$$
$$EC = \frac{a^2 x^2}{c_p (T_w - T_\infty)}$$

With $Nb = 0$ there is no thermal transport due to buoyancy effects created as a result of nanoparticle concentration gradients.

Here, we note that Eq. (8) with the corresponding boundary conditions on f provided by Eq. (11) has a closed form solution which is given by

$f(\eta) = 1 - e^{-\eta}$, (14)

In Eq. (14), Pr, Le, Nb, Nt and Bi denote the Prandtl number, the Lewis number, the Brownian motion parameter, the thermophoresis parameter and the Biot number respectively. The reduced Nusselt number Nur and the reduced Sherwood number Shr may found in terms of the dimensionless temperature at the surface, $\theta'(0)$ and the dimensionless concentration at the sheet surface , $\phi'(0)$, respectively i.e.

$$Nur = \mathrm{Re}_x^{-1/2}, Nu = -\theta'(0), \tag{15}$$

$$Shr = \mathrm{Re}_x^{-1/2}, Nu = -\phi'(0), \tag{16}$$

where

$$Nu = \frac{q_w x}{k(T_w - T_\infty)}, Sh = \frac{q_m x}{D_B(\phi_w - \phi_\infty)}, \mathrm{Re}_x = \frac{u_w(x)x}{\nu}, \tag{17}$$

and q_w is the surface (wall) heat flux and q_m is the surface (wall) mass flux.

3. Result and discussion

Eqs. (8-10) subject to the boundary conditions, Eqs.(11) and (12), were solved numerically using Runge- kutta fourth order method. As a further check on the accuracy of our numerical computations, Table1 contains a comparison of our results for the reduced Nusselt number and the reduced Sherwood number with those reported by Khan and Pop[15] for $Le = 10, Pr = 10, Bi = \infty, M = 10$. The infinitely large Biot number simulates the isothermal stretching model used in[15] as noted earlier. The results for all combination values of Brownian motion parameter Nb and the thermophoresis parameter Nt used in our computations, showed an exact match between our results and the results reported in [15]. The first five entries show that for a fixed thermophoresis parameter, $Nt = 0.1$,the reduced Nusselt number decreases sharply with the increasing in Brownian motion, that as Nb is increased from 0.1 to 0.5. However, the reduced Sherwood number increases substantially as Nb is increased from 0.1 to 0.2 but tends to plateau beyond $Nb = 0.2$. These observations are consistent with the initial slopes of the temperature and concentration profiles to be discussed later. As the Brownian motion intensifies, it impacts a larger extent of the fluid, causing the thermal boundary layer to thicken, which in turn decreases the reduced Nusselt number. The thickening of the boundary layer due to stronger Brownian motion will be high-lighted again when the temperature profiles are discussed. It will be seen from the concentration profiles appearing later in the discussion that the initial slope of the curve and the extend of the concentration boundary layer are not affected significantly beyond $Nb = 0.2$ and hence the plateau in the sherwood number behavior. The last four entries in Table2 show that the reduced Nusselt number decreases as the thermophoresis diffusion penetrates deeper into the fluid and causes the thermal boundary layer to thicken. However, the increase in the thermophoresis parameter enhance the Sherwood number, a conclusion that is consistent with the results of Khan and Pop[15].

We now turn our attention to the discussion of graphical results that provide additional insights into the problem under investigation.

Temperature profiles

Fig. 1 shows the temperature distribution in the thermal boundary layer for different values of Brownian motion and the thermophoresis parameters. As both Nb and Nt increase, the boundary layer thickens, as noted earlier in discussing the tabular data, the surface temperature increases , and the curves become less step indicating a diminution of the reduced Nusselt number. As seen in Fig. 2, the effect of Lewis number on the temperature profiles is noticeable only in a region close to the sheet as the curves tend to merge at larger distances from the sheet.

The Lewis number expresses the relative contribution of thermal diffusion rate to species diffusion rate in the boundary layer regime.An increase of Lewis number will reduce thermal boundary layer thickness and will be accompanied with a decrease in temperature. Larger Le will suppress concentration values. I.e inhibit nanoparticle species diffusion.There will be much greater reduction in concentration bopundary layer thickness than thermal boundary layer thickness over an increment in Lewis Number.

Fig. 3 illustrates the effect of Biot number on the thermal boundary layer. As expected, the stronger convection results in higher surface temperatures, causing the thermal effect to penetrate deeper into the quiescent fluid. The temperature profiles depicted in Fig. 4 show that as the Prandtl number increases, the thickness of the thermal boundary layer decreases as the curve become increasingly steeper. As a consequence , the reduced Nusselt number, being proportional to the initial slope, increases. This pattern is reminiscent of the convective of the free convective boundary layer flow in a regular fluid[20].Fig 5 shows that the effect of magnetic number on the temperature profiles is noticeable only in region close to the sheet as the curves tend to merge at larger distances from the sheet.

Fig 6,reveals the effect made by the viscous dissipation on temperature profile. These profiles are well behaved and very little change occurs in the shapes of profiles with the varying parameter Eckert number Ec. The different values of Ec contribute little to the thickness of the thermal boundary layer.

Concentration profiles.

The effect of nanoparticle concentration profiles, is shown in Fig. 7. Unlike the temperature profiles, the concentration profiles are only slightly affected by the strength of the Brownian motion and thermophoresis. A comparison of Fig. 3 and Fig. 8 shows that the Lewis number significantly affected the concentration distribution (Fig. 8),but has little influence on the temperature distribution (Fig. 3). For a base fluid of certain kinematic viscosity ν ,a higher Lewis number implies a lower Brownian diffusion coefficient D_B (see Eq.(4.13)) which must result in a shorter penetration depth for the concentration boundary layer. This is exactly what we see in Fig. 8 it was observed in Fig. 4 that as the convective heating of the sheet is enhanced i.e. Bi increases, the thermal penetration depth increases. Because the concentration distribution is driven by the temperature field, one anticipates that a higher Biot number would promote a deeper penetration of the concentration. This anticipation is indeed realized in Fig. 9 which predict higher concentration at higher values of the Biot number. A comparison of Fig6 and fig. 10 shows that the Magnetic number significantly affected the concentration distribution(Fig.10), but has little influence on the temperature distribution(Fig.6).

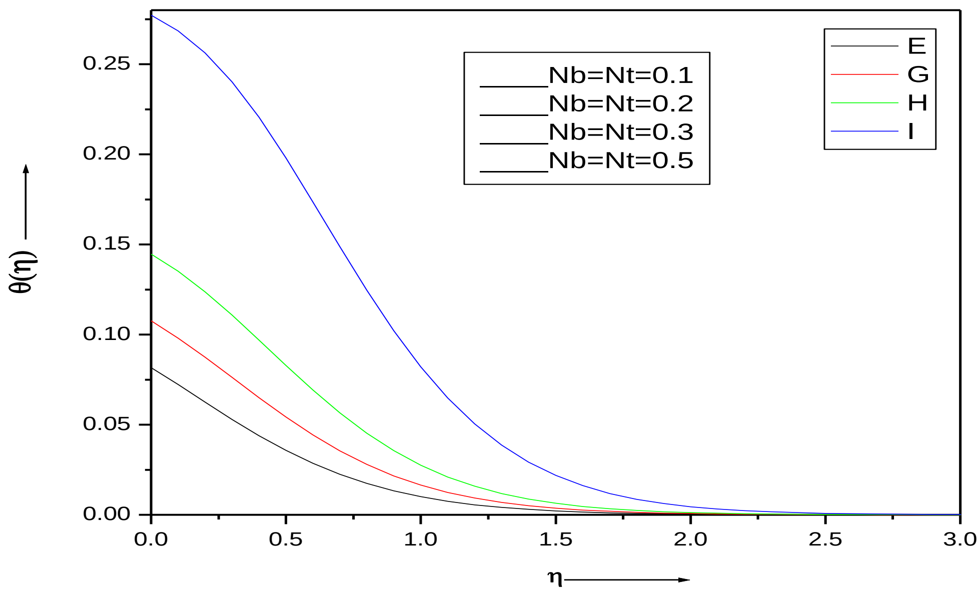

Fig. 1. Effect of Nt **and** Nb **on temperature profiles when**

$Le = 5, Pr = 5, Ec = 5, Bi = 0.1, M = 2.$.

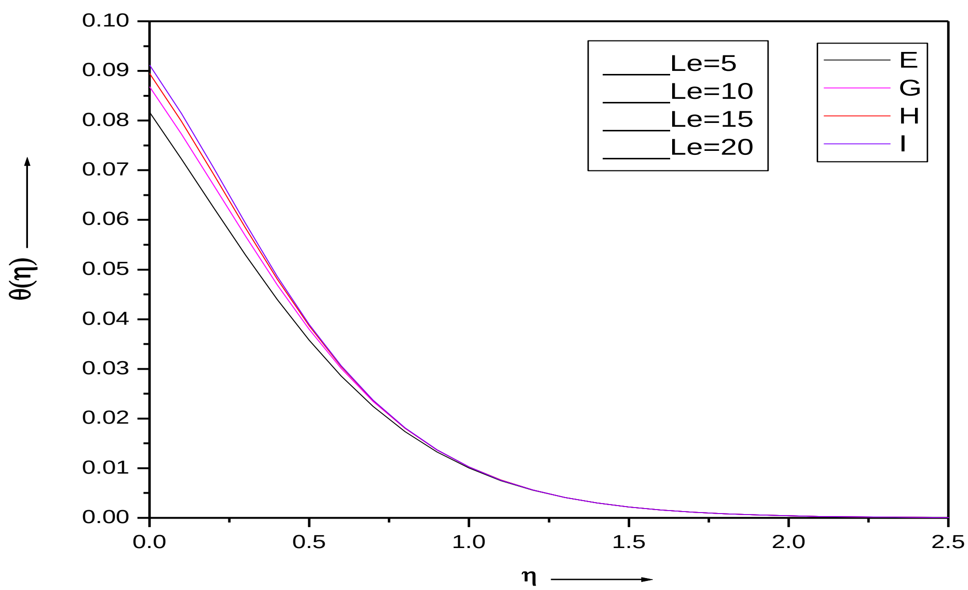

Fig. 2. Effect of Le **on temperature profiles when**

$Nt = Nb = 0.1, Pr = 5, EC = 5, Bi = 0.1, M = 2.$

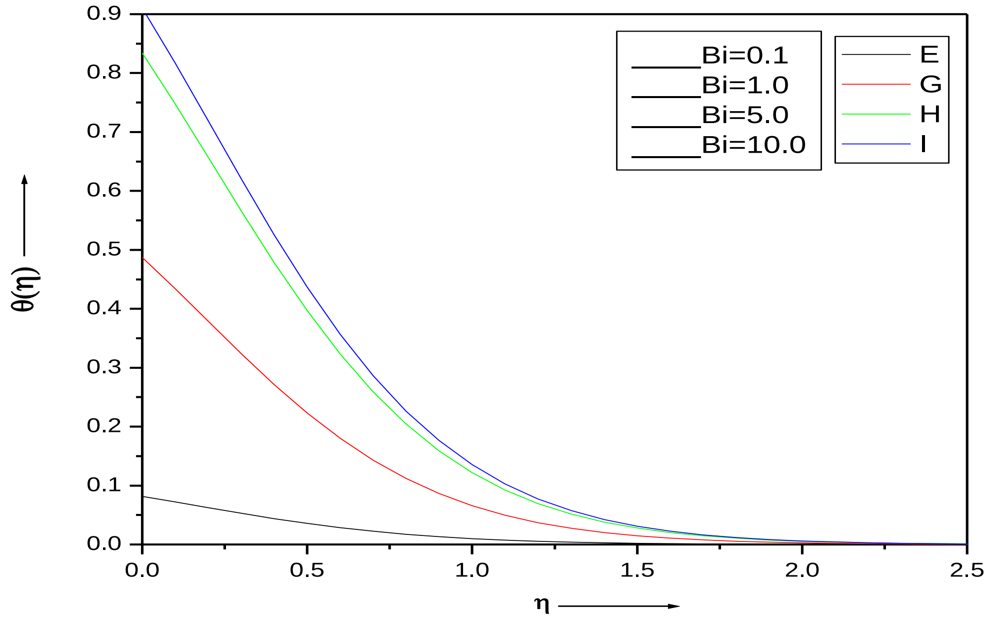

Fig. 3. Effect of Bi **on temperature profiles when**

$Nt = Nb = 0.1, Pr = Le = 5, M = 2, Ec = 5.$

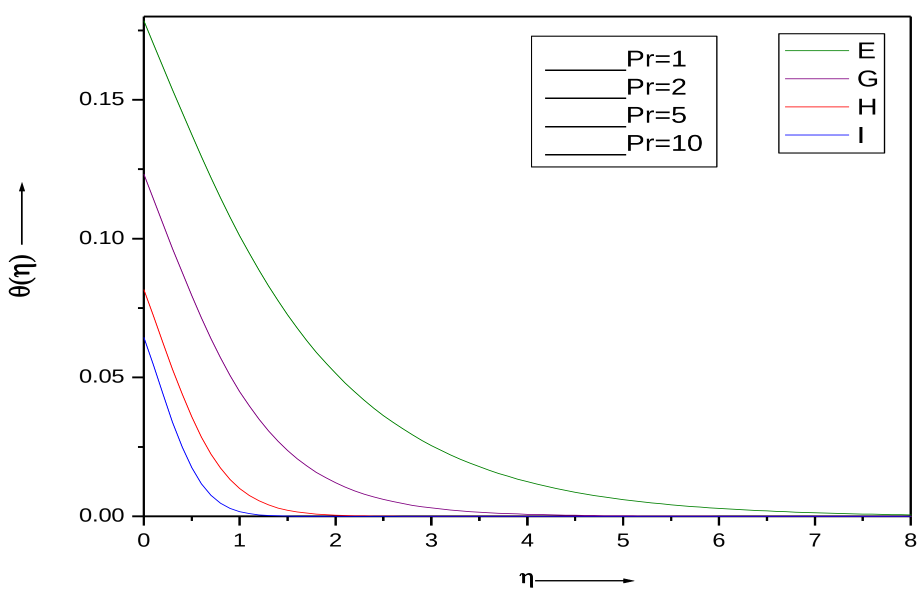

Fig. 4. Effect of Pr **on temperature profiles when**

$Nt = Nb = Bi = 0.1, Le = 5, EC = 5, M = 2.$

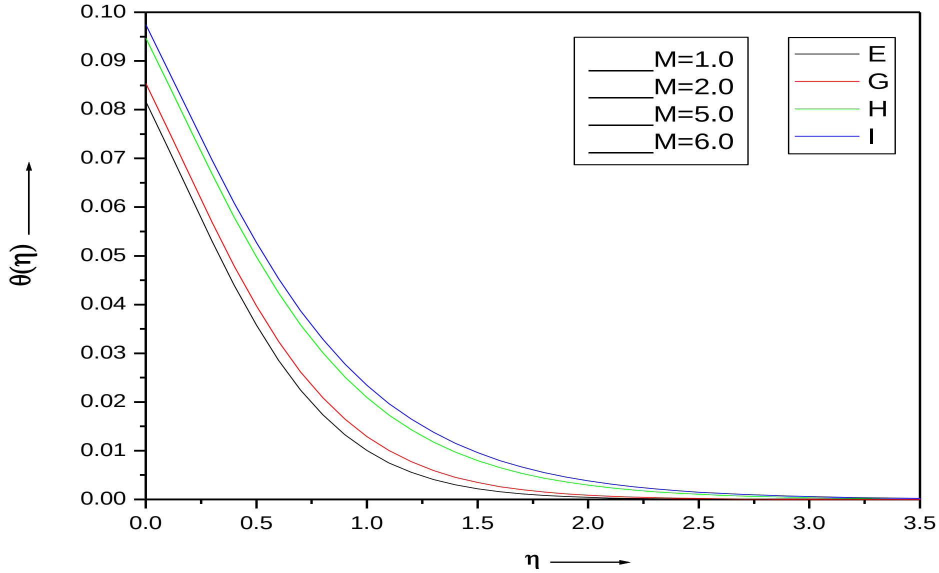

Fig. 5. Effect of M on temperature profiles when

$Nt = Nb = Bi = 0.1, Le = Pr = 5 = EC = 5.$

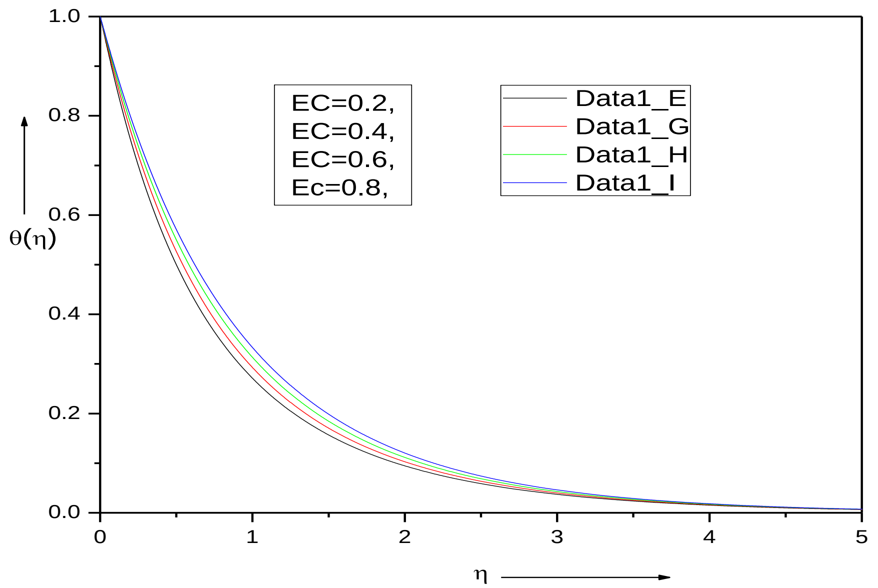

Fig. 6 Effect of Eckert Number on Temperature profile for different values of

$Pr = 1.0, Nt = Nb = 5, Bi = 0.5, M = 2.$

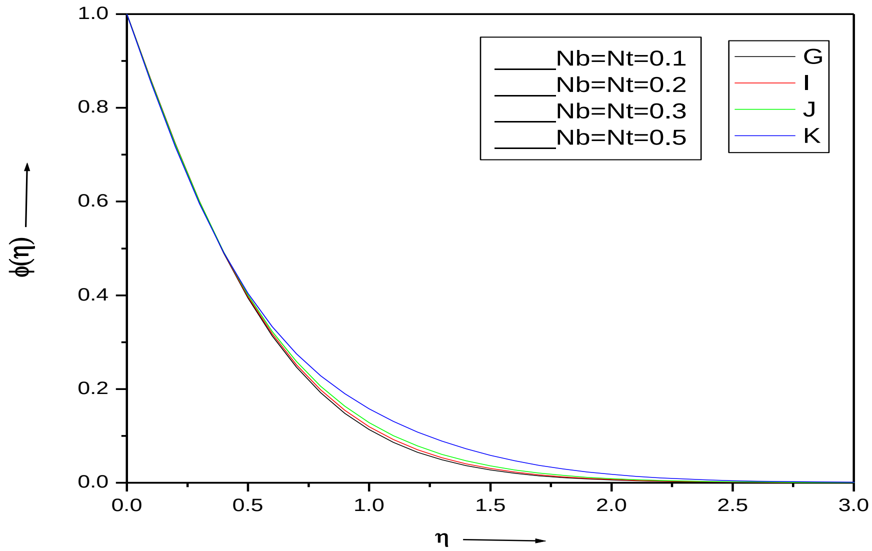

Fig. 7. Effect of Nt **and** Nb **on concentration profiles when**

$Le = 5, Pr = 5, Bi = 0.1, Ec = 5.$

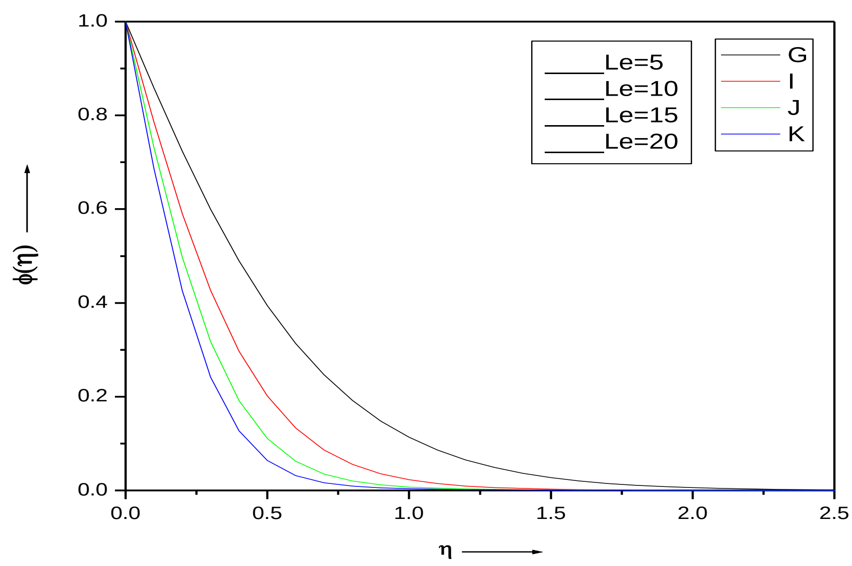

Fig. 8. Effect of Le **on concentration profiles when**

$Nt = Nb = 0.1, Pr = 5, Ec = 5, Bi = 0.1.$

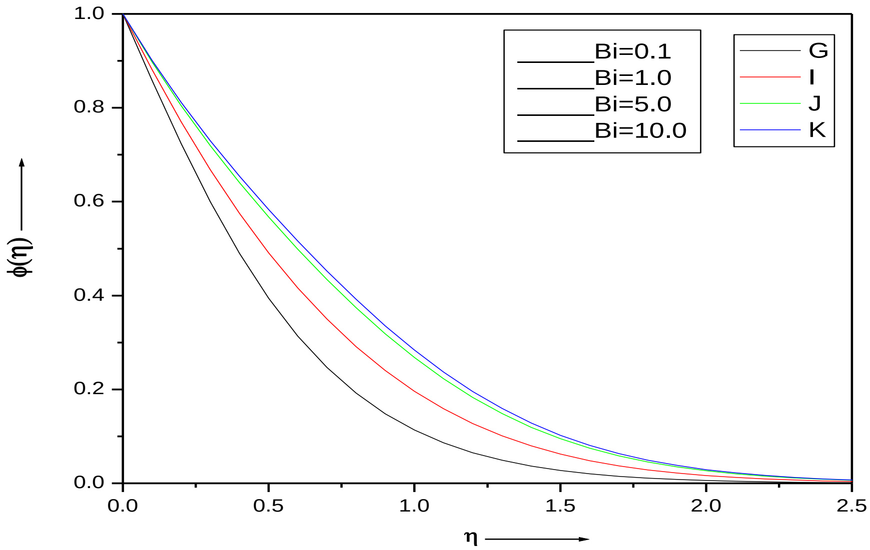

Fig. 9. Effect of Bi **on concentration profiles when** $Nt = Nb = 0.1, Pr = Le = Ec = 5.$

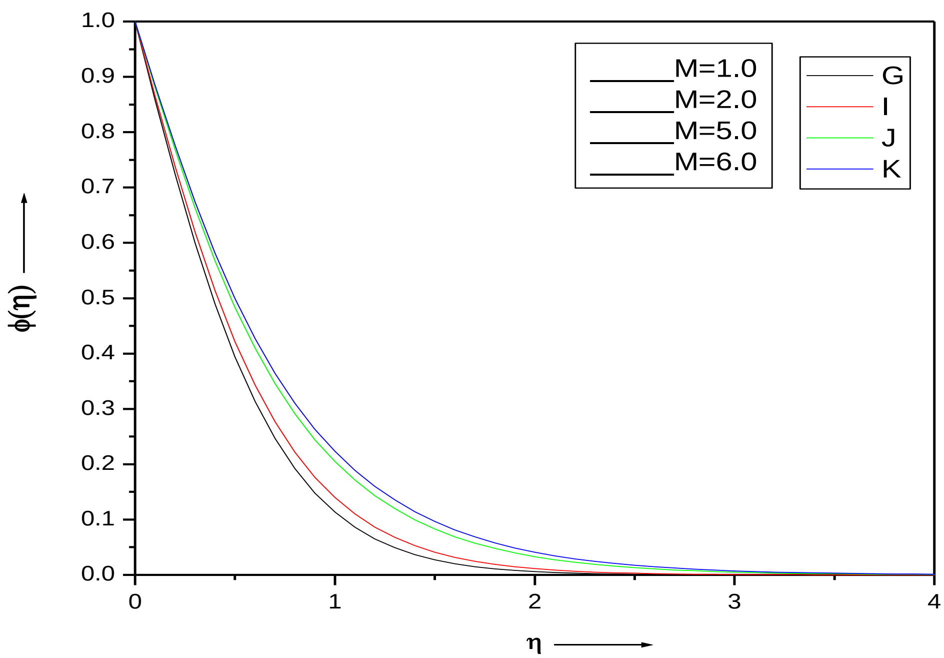

Fig. 10. Effect of M **on concentration profiles when**

$Nt = Nb = Bi = 0.1, Le = Pr = 5 = Ec = 5.$

Table 1

Comparison of results for the reduced Nusselt number $-\theta'(0)$ and the reduced Sherwood number $\varphi'(0)$ with Khan and Pop[15]

Nb	Nt	Nur	Shr	Nur present	Shr present
0.1	0.1	0.9524	2.1294	0.5230	2.0507
0.2	0.1	0.5056	2.3819	0.3561	2.2346
0.3	0.1	0.2522	2.4100	0.2082	2.2797
0.4	0.1	0.1194	2.3997	0.1077	2.2846
0.5	0.1	0.0543	2.3836	0.0513	2.2767
0.1	0.2	0.6932	2.2740	0.4761	1.9851
0.1	0.3	0.5201	2.5286	0.44235	2.0231

4. Conclusion

A numerical study of the boundary layer flow in a nanofluid induced as as a result of motion of a linearly stretching sheet has been performed. The use of a convective hating boundary condition instead of a constant temperature or a constant heat flux makes this study more general novel. The following conclusions are derived

1. The transport of momentum, energy and concentration of nanoparticles in the respective boundary layers depends on seven parameters: Brownian motion parameter Nb, thermophoresis parameter Nt, Prandtl number Pr, Lewis number Le, convection Biot number Bi, Magnetic parameter M and Eckert number Ec.
2. For infinitely large Biot number characterizing the convective heating (which corresponds to the constant temperature boundary condition), the present results and those reported by Khan and Pop [27] match up to four places of decimal.
3. For a fixed Pr, Le, Ec and Bi the thermal boundary thickens and the local temperature rises as the Brownian motion and thermophoresis effects intensify. A

similar effect on the thermal boundary is observed when Nb, Nt, Le and Bi are kept fixed and the Prandtl number Pr is increased or when Pr, Nb, Nt and Le are kept fixed and the Biot number is increased. However, when Pr, Nb, Nt and Bi are kept fixed, and the Lewis number is increased, the temperature distribution is affected only minimally.

4. With the increase in Bi, the concentration layer thickens but the concentration layer becomes thinner as Le increases.
5. For fixed Pr, Le and Bi, the reduced Nusselt number decreases but the reduced Sherwood number increases as Brownian motion and thermophoresis effects intensify.

Nomenclature

B_i Biot number

a a positive constant associated with linear strecthing

D_B Brownian diffusion coefficient

D_T Thermophoretic diffusion coefficient

$f(\eta)$ Dimensionless steam function

g Gravitational acceleration

h Convective heat transfer coefficient

k Thermal conductivity of the nanofluid

Le Lewis number

Nb Brownian motion parameter

Nt Thermophoresis parameter

Nu Nusselt number

Nur	Reduced Nusselt number
Pr	Prandtl number
p	pressure
q''_m	Wall mass flux
q''_w	Wall heat flux
Re_x	Local Reynolds number
Sh	Sherwood number
Shr	Reduced Sherwood number
M	Magnetic number
T	Local fluid Temperature
T_f	Temperature of the hot fluid
T_w	Sheet surface (wall) temperature
T_∞	Ambient temperature
u, v	Velocity components in x and y directions
C	nanoparticle volume fraction
C_w	Nanoparticle volume fraction at the wall
C_∞	Nanoparticle volume fraction at large values of y(ambient)
Ec	Eckert number

Greek symbol

α	Thermal diffusivity of the base fluid

η Similarity variable

θ Dimensionless temperature

φ Dimensionless volume fraction

μ Absolute viscosity of the base fluid

υ Kinematic viscosity of the base fluid

ρ_f Density of the base fluid

ρ_p Nanoparticle mass density

$(\rho c)_f$ Heat capacity of the base fluid

$(\rho c)_p$ Heat capacity of the nanoparticle material

$\tau = (\rho c)_p / (\rho c)_f$

ψ Stream function

References

[1]. T. Altan, S. Oh, H. Gegel, Metal foming fundamentals and Applications. American Society of Metals, Metals Park, OH, 1979.

[2]. E. G. Fisher, Extrusion of plastics, Wiley, New York, 1976.

[3]. Z. Tidmore, I. Klein, Engineering Principles of Plasticating Extrusion, Polymer Science and Engineering Series. Van Norstrand, New York, 1970.

[4]. I. J. Crane. Flow past a stretching plane, J. Appl. Math. Phys. (ZAMP) 21 (1970) 645-647.

[5]. J. L. Grupka, K.M. Bobba, Heat transfer characteristics of a continuous stretching surface with variable temperature, J. Heat transfer 107 (1985) 248-250.

[6]. P. S. Gupta, A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Eng. 55 (1977) 744-746.

[7]. H. I. Andersson, Slip flow past a stretching surface, Acta Mech. 158 (2002) 121-125.

[8]. B. K. Dutta, P. Roy, A. S. Gupta, Temperature field in flow over a stretching sheet with uniform heat flux, Int. Commun. Heat Mass Transfer 12 (1985) 89-94.

[9]. T. Fang, Flow and heat transfer characteristics of boundary layer over a stretching surface with a uniform-shear free stream, Int. J. Heat Mass Transf. 51 (2008) 2199-2213.

[10]. E. Magyari, B. Keller, Exact solutions of boundary layer equations for a stretching wall, Eur. J. Mech. B-Fluids 19 (2000) 109-122.

[11]. F. Labropulu, D. Li, I. Pop, Non orthogonal stagnation point flow towards a stretching surface in a non-Newtonian fluid with heat transfer, Int. J. Therm. Sci. 49 (2010) 1042-1050.

[12]. K. V. Prasad, K. Vajravelu, P.S. Dutta, The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet, Int. J. Therm. Sci. 40 (2010) 603-610.

[13]. B. C. Sakiadas, Boundary layer behavior on continuous solid surfaces: I Boundary layer equations for two dimensional and flow, AIChE J.7(1961) 26-28.

[14]. E. Schmidt, W. Beckmann, Das Temperature-und Geschwindikeitsfeld voneiner warme abgebenden senkrechten platte bei naturlicher konvection, II. Die Versuche und ihre Ergibnisse, Forcsh, Ingenieurwes 1 (1930) 391-406.

[15]. W. A. Khan , I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, Int. J. Heat Mass Transfer . 53 (2010) 2477-2483.

[16]. A. V. Kuznetsov, D. A. Nield, Natural convective boundary-layer flow of a nanofluid past a vertical plate, Int. J. Therm. Sci. 49 (2010) 243-247.

[17]. O.D. Mankinde, A. Aziz, Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition ,Int. J. Therm. Sci. 50(2011) 1326-1332.

[18]. C. Y. wang, Free convection on a vertical stretching surface, J. appl. math. Mech. (ZAMM) 69 (1989) 418-420.

[19]. R. S. R. Gorla, I. Sidawi, free convection on a vertical stretching surface with suction and blowing, Appl. Sci. Res. 52 (1994) 247-257.

[20] K.A.Helmy, Solution of the boundary layer equation for a power law fluid in magneto hydrodynamics,Acta Mech,102(1994)25-37.

[21] T.C.Chiam,hydromagnetic flow over a surface stretching with a power law velocity, Int.J.Engg.Sci,33(1995)429-435.

[22]A.Ishak,r.Nazar,I.Pop,Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet, Heat Mass Transfer 44(2008)921-927.

[23] S.S.Nourazar,M.H.Matin,M.Simiari,The HPM applied to MHD nanofluid flow over a horizontal stretching plate,J.Appl.Math(2011)810-827.

[24] M.A.A. Hamad, Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field, Int. Commun. Heat Mass Transfer 38 (2011) 487–492.

[25] P. Rana, R. Bhargava, O.A. Beg, Finite element simulation of unsteady magnetohydrodynamic transport phenomena on a stretching sheet in a rotating nanofluid, J. Nanoeng. Nanosyst. 227 (2011) 77–99.

[26] Wang XQ, Mujumdar AS. A review on nanofluids – Part I: Theoretical and numerical investigations. Braz J Chem Eng 2008;25:613–30.

[27] Kakaç S, Pramuanjaroenkij A. Review of convective heat transfer enhancement with nanofluids. Int J Heat Mass Trans 2009;52:3187–96.

[28] Chandrasekar M, Suresh S, Senthilkumar T. Mechanisms proposed through experimental investigations on thermophysical properties and forced convective heat transfer characteristics of various nanofluids – a review. Renew Sust Energy Rev 2012;16:3917–38.

[29] Wu JM, Zhao J. A review of nanofluid heat transfer and critical heat flux enhancement-Research gap to engineering application. Prog Nucl Energy 2013;66:13–24.

[30] Olanrewaju AM, Makinde OD. On boundary layer stagnation point flow of a nanofluid over a permeable flat surface with Newtonian heating. Chem Eng Commun 2013;200:836–52.

[31] Ibrahim W, Makinde OD. The effect of double stratification on boundary-layer flow and heat transfer of nanofluid over a vertical plate. Comput Fluids 2013;86:433–41.

[32] Mutuku WN, Makinde OD. Hydromagnetic bioconvection of nanofluid over a permeable vertical plate due to gyrotactic microorganisms. Comput Fluids 2014.

[33] Njane M, Mutuku WN, Makinde OD. Combined effect of Buoyancy force and Navier slip on MHD flow of a nanofluid over a convectively heated vertical porous plate. Sci World J 2013.

[34] Makinde OD. Computational modelling of nanofluids flow over a convectively heated unsteady stretching sheet. Curr Nanosci 2013;9:673–8.

[35] Makinde OD. Effects of viscous dissipation and Newtonian heating on boundary-layer flow of nanofluids over a flat plate. Int J Numer Methods Heat Fluid Flow 2013;23:1291–303.

[36] Makinde OD, Khan WA, Aziz A. On inherent irreversibility in Sakiadis flow of nanofluids. Int J Exergy 2013;13:159–74.

[37] Khan WA, Pop I. Boundary-layer flow of a nanofluid past a stretching sheet. Int J Heat Mass Trans 2010;53:2477–83.

[38] Hassani M, Tabar MM, Nemati H, Domairry G, Noori F. An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. Int J Therm Sci 2011;50:2256–63.

[39] Kahar RA, Kandasamy R, Muhaimin. Scaling group transformation for boundary-layer flow of a nanofluid past a porous vertical stretching surface in the presence of chemical reaction with heat radiation. Comput Fluids 2011;52:15–21.

[40] Makinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Int J Therm Sci 2011;50:1326–32.

[41] Rana P, Bhargava R. Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: a numerical study. Commun Nonlinear Sci Numer Simul 2012;17:212–26.

[42] Noghrehabadi A, Pourrajab R, Ghalambaz M. Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature. Int J Therm Sci 2012;54:253–61..

[43] Noghrehabadadi A, Ghalambaz M, Ghanbarzadeh A. Heat transfer of magnetohydrodynamic viscous nanofluids over an isothermal stretching sheet. J Thermophys Heat Transfer 2012;26:686–9.

[44] Noghrehabadi A, Pourrajab R, Ghalambaz M. Flow and heat transfer of nanofluids over stretching sheet taking into account partial slip and thermal convective boundary conditions. Heat Mass Transfer 2013;49: 1357–66.

[45] Noghrehabadi A, Saffarian M, Pourrajab M, Ghalambaz M. Entropy analysis for nanofluid flow over a stretching sheet in the presence of heat generation/ absorption and partial slip. J Mech Sci Technol 2013;27:927–37.
[46] Ibrahim W, Shankar B. MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. Comput Fluids 2013;75:1–10.
[47] Nadeem S, Mehmood R, Akbar NS. Non-orthogonal stagnation point flow of a nano non-Newtonian fluid towards a stretching surface with heat transfer. Int J Heat Mass Transfer 2013;57:679–89.
[48] Nadeem S, Haq RU, Akbar NS, Lee C, Khan ZH. Numerical study of boundary layer flow and heat transfer of Oldroyd-B nanofluid towards a stretching sheet. PLoS ONE 2013;8.
[49] Chandrasekar M, Suresh S. A review on the mechanisms of heat transport in nanofluids. Heat Transfer Eng 2009;30:1136–50.
[50] Khanafer K, Vafai K. A critical synthesis of thermophysical characteristics of Nanofluids. Int J Heat Mass Transfer 2011;54:4410–28.
[51] Buongiorno J. Convective transport in nanofluids. J Heat Trans-T ASME 2006;128:240–50.