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MHD FLOW OVER A STRETCHING CONVECTIVE SURFACE WITH DUFOUR EFFECT, SLIP AND RADIATIVE HEAT

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ABSTRACT

We have analyzed MHD two dimensional stagnation point flow of nanofluid over a stretching surface. In this problem, the Oberbeck – Boussinesq approximation is taken into account. A mathematical formulation has designed for velocity, momentum, temperature and concentration profiles. The governing partial differential equations are reduced to a system of ordinary differential equations using Shooting technique together with Runge -Kuttaiteration scheme. The numerical results of the flow characteristics are presented graphically.

Keywords: Stagnation point, Nanofluid, Magnetohydrodynamics (MHD), Magnetic Field parameter (M), , Dufour number (Du), Richardson numbers ($Ri_{T,}$ Ri_C), Brownian motion parameter (Nb).

1. Introduction

In fluid dynamics a stagnation point is a point in a flow field where the local velocity of the fluid is zero. Stagnation points exist at the surface of objects in the flow field, where the fluid is brought the rest by the object. MHD stagnation point flow behavior over a stretching convective surface is an important role on several engineering, science and industrial applications. Cooling and metal foundries, nuclear power plants, microelectronics, glass manufacturing and crude oil purifications etc are some applications of these fields. The problem of MHD stagnation point flow of a nanofluid over an exponentially stretching sheet with an effect of chemical reaction, heat source and suction/ injunction investigated by Achi Reddy and Shankar [1].Christian *et al.* [2] have analyzed MHD boundary layer stagnation point flow with radiation and chemical reaction towards a heated shrinking porous surface. Emmanuel and Yakubu [3] have studied MHD thermal stagnation point flow towards a stretching porous surface. Ibrahim *et al.* [4] focused on MHD stagnation point flow of nanofluid past a stretching sheet with convective boundary condition. Imran *et al.* [5] established Radiation effect on MHD stagnation point flow of a nanofluid over an exponentially stretching sheet. Mahantha *et al.* [6] discussed MHD stagnation point flow of a nanofluid with velocity slip, Non-linear radiation and Newtonian heating. Mabood *et al.* [7] explained MHD stagnation point flow heat and mass transfer of nanofluids in porous medium with radiation, viscous dissipation and chemical reaction. Mohammad *et al.* [8] investigated MHD convective stagnation point flow of nanofluid over a shrinking surface with thermal radiation, heat radiation and chemical reaction. Stagnation point flow towards a stretching surface studied by Mahapatra and Gupta [9].Sreenivasulu and Reddy [10] have analyzed the effects of thermal radiation and chemical reaction on MHD stagnation point flow of a nanofluid over a porous stretching sheet embedded on a porous medium with heat absorption/ generation.

2. Flow analysis

Consider the steady, two dimensional, incompressible stagnation point flow of hydromagnetic nanofluidover an exponentially stretching convective surface in the presence of slip and radiative heat at y=0. Let x and y axis are taken along the sheet surface and normal surface respectively. Fig .1 expresses the physical model and the coordinate system of stretching convective surface. Assume that the tangential velocity of the stretching surface is u_w and the free stream velocity U_∞ .

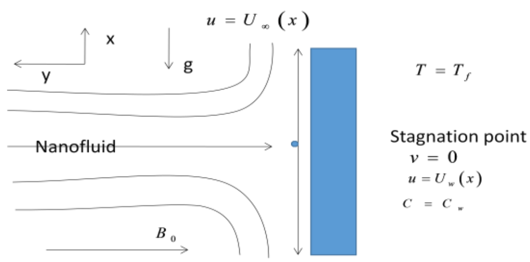

Fig 1. Physical and coordinate system

Let B_0 is uniform magnetic field of strength. The tangential fluid temperature T_∞ and T_f . Then under the Boussinesq approximation the basic equations conservation of mass, momentum, energy and concentration for nanofluids in Cartesian coordinates x and y can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\upsilon_f\frac{\partial^2 u}{\partial y^2}-\left(\frac{\sigma B_0^2}{\rho_f}\right)(u-U_\infty)+g\beta_T(\mathrm{T}-\mathrm{T}_\infty)+\mathrm{g}\,\beta_C(\mathrm{C}-\mathrm{C}_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(1+\frac{16\sigma^* T_\infty^3}{3kk^*}\right)\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\}+\frac{Q}{(\rho c_p)_f}(T-T_\infty)+D_C\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial^2 T}{\partial y^2}\right)\right]-\delta(\mathrm{C}-\mathrm{C}_\infty) \tag{4}$$

With the boundary conditions

$$u = U_w(x) = ax + N\mu \frac{\partial u}{\partial y},\ v = 0\ ,\ -k\frac{\partial T}{\partial y} = h_f\,(T_f - T),\ C = C_w \text{ at } y = 0$$

$$u \to U_\infty(x) = bx,\ v \to 0,\ C \to C_\infty,\ T \to T_\infty \text{ as } y \to \infty \tag{5}$$

Where (u, v) are the nanofluid velocity components in the (x, y) directionsrespectively. C_w, C_∞ are the plate surface concentration and free stream concentration, ρ_f is the fluid density, c_p is specific heat at constant pressure , g is the gravitational acceleration , β_T, β_c are the thermal and solutal expansion coefficients respectively, D_B is the Brownian diffusion coefficient , D_T is the thermo-phoretic diffusion coefficient , D_c is the concentric diffusion coefficient, Q is the heat generation/ absorption coefficient, α is the thermal diffusion coefficient, k is the thermal conductivity coefficient , σ electrical conductivity, σ^* Stefan-Boltzman constant, k^* mean absorption coefficient , δ is the reaction rate, a, b are positive constants with dimensions of inverse time, N is a slip constant, μ is the base fluid dynamic viscosity and υ_f the base fluid kinematic viscosity.

3.Mathematical Formulation

The similarity transformations are

$$\eta = y\sqrt{\frac{a}{\upsilon_f}},\ u = axf'(\eta),\ v = -\sqrt{(a\upsilon_f)}\, f(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \qquad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

Using the above non-dimensional variables (6), equations (2) to (4) along the boundary conditions (5) the nonlinear, non-dimensional coupled ordinary differential equations have been obtained as,

$$f''' + ff'' - f'^2 + M\left(r - f'\right) + r^2 + Ri_T\theta + Ri_C\phi = 0 \tag{7}$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + pr\left(f\theta' + Nb\theta'\phi' + Nt\theta'^2 + S\theta + D_f\phi''\right) = 0 \tag{8}$$

$$\phi'' + Sc\left(f\phi' - \gamma\phi\right) + \frac{Nt}{Nb}\theta'' = 0 \tag{9}$$

The corresponding boundary conditions are

$$f(0) = 0,\ f'(0) = 1 + \beta f''(0),\ \theta'(0) = -Bi\left(1 - \theta(0)\right),\ \phi(0) = 1 \text{ at } \eta \to 0$$

$$f'(\infty) \to r,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty \tag{10}$$

The Non-dimensional parameters are defined as

$$M=\frac{\sigma B_0{}^2}{\rho_f a}\ r=\frac{b}{a}\ Nr=\frac{4\sigma^* T_\infty^3}{k^* k_f}\ pr=\frac{\upsilon_f}{\alpha_f}\ Bi=\frac{h_f}{k}\sqrt{\frac{\upsilon_f}{a}}\qquad Nt=\frac{\tau D_T\left(T_f-T_\infty\right)}{\upsilon_f T_\infty}\ Re_x=\frac{U_\infty x}{\upsilon_f}$$

$$Nb=\frac{\tau D_B\left(C_w-C_\infty\right)}{\upsilon_f}\quad D_f=\frac{D_C\left(C_w-C_\infty\right)}{\upsilon_f\left(T_f-T_\infty\right)}\ Sc=\frac{\upsilon_f}{D_B}\ \ \gamma=\frac{\delta}{a}\ \ S=\frac{Q}{\left(\rho c_p\right)_f a\left(T_f-T_\infty\right)}$$

$$\beta=N\mu\sqrt{\frac{a}{\upsilon_f}}\quad Ri_T=\frac{Gr_T}{\mathrm{Re}_x^2}\ Ri_C=\frac{Gr_C}{\mathrm{Re}_x^2}\ Gr_T=\frac{\beta_T g(\mathrm{T}_f-\mathrm{T}_\infty)\,\mathrm{x}^3}{\upsilon_f^2}\ Gr_C=\frac{\beta_C g(\mathrm{C}_w-C_\infty)\,\mathrm{x}^3}{\upsilon_f^2}\qquad (11)$$

Where M is the Magnetic field parameter, Ri_T, Ri_C are the thermal and mass Richardson numbers, Gr_T, Gr_C are respective Grashof numbers, r is the velocity ratio parameter, Nr radiation parameter, D_f is Dufour number , Pr is Prandtl number, Bi is Convection heat transfer parameter, Nt Brownian motion parameter, Nb is the thermophoresis parameter, Sc is the Schmidt number, γ is the scaled chemical reaction parameter, S is the Dimensionless heat generation or absorption coefficient, and β is the velocity slip parameter.

The local skin-friction coefficient (C_{fx})

$$C_{fx}=\frac{\tau_w}{\rho_f\, u_w^2/2}=\mathrm{Re}_x^{-1/2}\ f''(0)\qquad (12)$$

The local Nusselt number (Nu_x)

$$Nu_x=\frac{q_w x}{k_f(\mathrm{T}_f-\mathrm{T}_\infty)}=-\mathrm{Re}_x^{-1/2}\,\theta'(0)\qquad (13)$$

The Sherwood number (Sh_x)

$$Sh_x=\frac{xq_m}{D_B(\mathrm{C}_w-\mathrm{C}_\infty)}=-\mathrm{Re}_x^{-1/2}\,\phi'(0)\qquad (14)$$

Where the surface shear stress τ_w, the flux q_w and q_m are given as

$$\tau_w=\mu_f\left(\frac{\partial u}{\partial y}\right)_{y=0}\qquad q_w=-\left(\left(k_f+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial T}{\partial y}\right)_{y=0}\quad q_m=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}\qquad (15)$$

Where $\mathrm{Re}_x=\left(ax^2/\upsilon_f\right)$ is the local Reynolds number.

4. Numerical Analysis

In this paper the set of Non-dimensional Non-linear couple boundary layer equations with boundary conditions does not possess a closed form analytical solution. The governing partial differential equations can be converted to closed form equations by using Shooting method then it has been solved numerically by Runge- Kutta integration technique. The entire numerical analysis is done by using Mathematica computer language. From the process of numerical computation the fluid velocity, the temperature, the concentration, the Skin friction coefficient, the Nusselt number and Sherwood number are proportional to $f^{'}(\eta),\theta(\eta),\phi(\eta)$.

5. Results and Discussion

The dimensionless velocity profile for various values of Richardson number (Ri_T) ,Nanofluid parameter (Nt) and Magnetic Field parameter (M) are shown in figure 5.1.From this figure,5.1 (a) illustrate the velocity field increases with increasing values of Richardson number (Ri_T) . Fig.5.1 (b) illustrate the velocity field increases with increasing values of Nanofluid parameter (Nt) and Fig.5.1 (c) illustrate the velocity field decreases while the increase of Magnetic Field parameter (M).

The dimensionless Temperature profile for various values of Nanofluid parameter (Nt), Heat generation or absorption coefficient (S) and Dufour Number (D_f) are shown in figure 5.2.From this figure, 5.2 (a) illustrate the temperature field increases with increasing values of Nanofluid parameter (Nt). Fig.5.2 (b) illustrate the temperature field increases with increasing values of Heat generation or absorption coefficient (S) and Fig.5.2 (c) illustrate the temperature field decreases while the increase of Dufour Number (D_f).

The dimensionless Concentration profile for various values of Nanofluid parameter (Nt), Chemical reaction parameter (γ), and Richardson number (Ri_c) are shown in figure 5.3. From this figure, 5.3 (a) illustrate the concentration field increases with increasing values of Nanofluid parameter (Nt). Fig.5.3 (b) illustrate the concentration field increases with increasing values of Chemical reaction parameter (γ).Fig.5.3 (c) illustrate the concentration field decreases while the increase of Richardson number (Ri_c).

The dimensionless Skin friction coefficient (C_f) for various values of Richardson number (Ri_c), Magnetic Field parameter (M) and Dufour Number (D_f) are shown in figure 5.4. From this figure, 5.4 (a) illustrate the skin friction coefficient (C_f) increases with increasing values of Richardson number (Ri_c). 5.4 (b) illustrate the skin friction coefficient (C_f) increases with increasing values of Magnetic Field parameter (M) and 5.4 (c) illustrate the skin friction coefficient (C_f) increases with increasing values of Dufour Number (D_f).

The dimensionless Nusselt Number (Nu) for various values of Heat generation or absorption coefficient (S), Nanofluid parameter (Nt) and Dufour Number (D_f) are shown in figure 5.5. From this figure, 5.5 (a) illustrate the Nusselt Number (Nu) increases with increasing values of Heat generation or absorption coefficient (S). Fig.5.5 (b) illustrate the Nusselt Number (Nu) increases with increasing values of Nanofluid parameter (Nt). Fig.5.5 (c) illustrate the Nusselt Number (Nu) decreases while the increase of Dufour Number (D_f).

The dimensionless Sherwood Number (Sh) for various values of Dufour Number (D_f), Richardson number (Ri_c) and Nanofluid parameter (Nt) are shown in figure 5.6. From this figure, 5.6 (a) illustrate the Sherwood Number (Sh) increases with increasing values of Dufour Number (D_f). Fig.5.6 (b) illustratethe Sherwood Number (Sh) decreases while the increasing values of Richardson number (Ri_c). Fig.5.6 (c) illustrate the Sherwood Number (Sh) decreases while the increasing values of Nanofluid parameter (Nt).

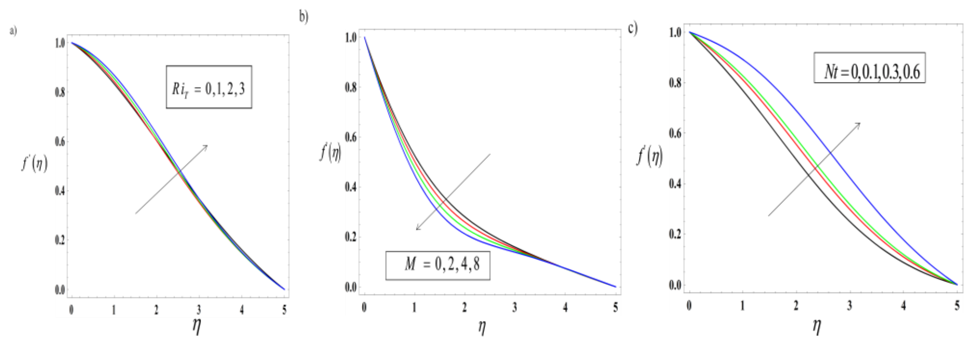

Fig .5.1.Effects of (a) Richardson number (Ri_T), b) Magnetic field parameter (M), c) Nanofluid parameter (Nt) on velocity profiles.

$(\Pr = 6.2, Nr = 0.1, \beta = Nt = Nb = 0.5, Sc = 1, \gamma = 0.1, S = 0.1, r = 1, M = 0.5$

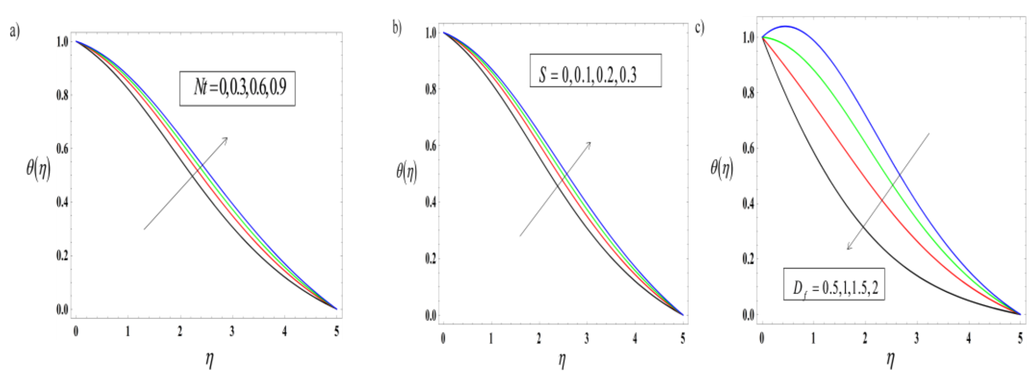

Fig .5.2.Effects of(a) Nanofluid parameter (Nt), (b) Heat generation or absorption parameter (S), c) Dufour Number (D_f) on Temperature profiles.

$(\Pr = 6.2, \beta = 0.5, Sc = 1, \gamma = 0.1, M = 1, Bi = 1)$

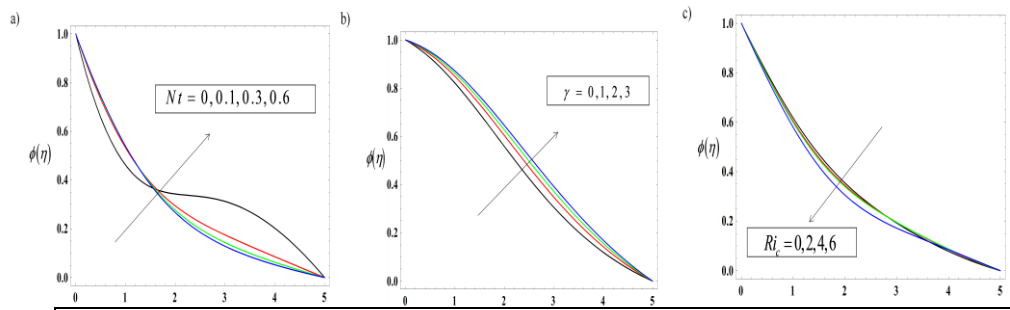

Fig .5.3.Effects of (a) Nanofluid parameter (Nt), (b) Chemical reaction parameter (γ), (c) Richardson number (Ri_c) on Concentration profiles.

$(\Pr = 6.2, \beta = 0.5, Sc = 1, S = 0.1, \mathrm{Bi} = 1)$

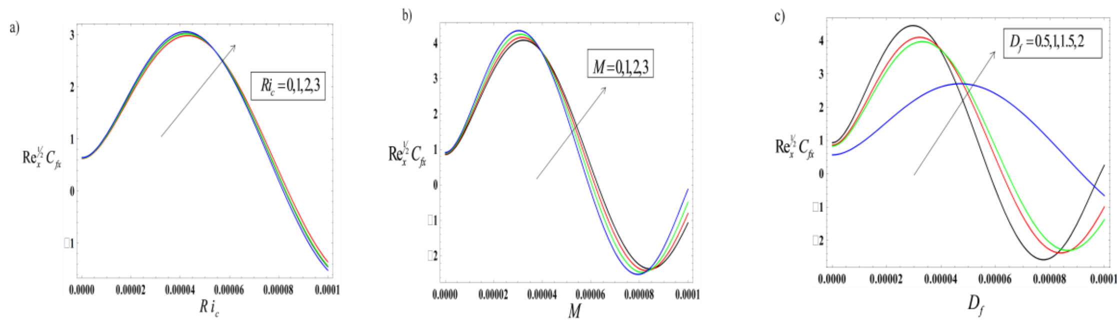

Fig .5.4.Effects of (a)Richardson number (Ri_c), (b) Magnetic field parameter (M), (c) Dufour Number (D_f) on Skin friction coefficient (C_f).

$(\Pr = 6.2, \mathrm{Nr} = 0.1, S = 0.1, \mathrm{Bi} = 1)$

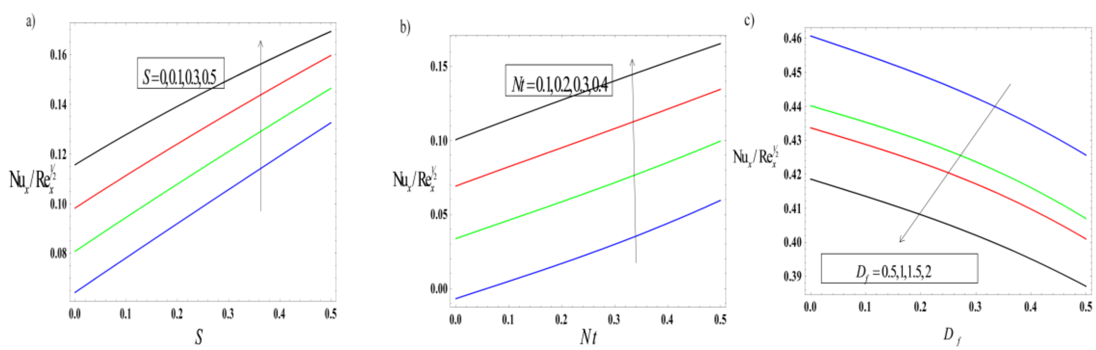

Fig .5.5.Effects of (a) Heat generation parameter (S), (b) Nanofluidparameter (Nt), (c) Dufour Number (D_f) on Nusselt number (Nu).

$(\Pr = 6.2, \beta = 0.5, Sc = 1, \mathrm{Bi} = 1)$

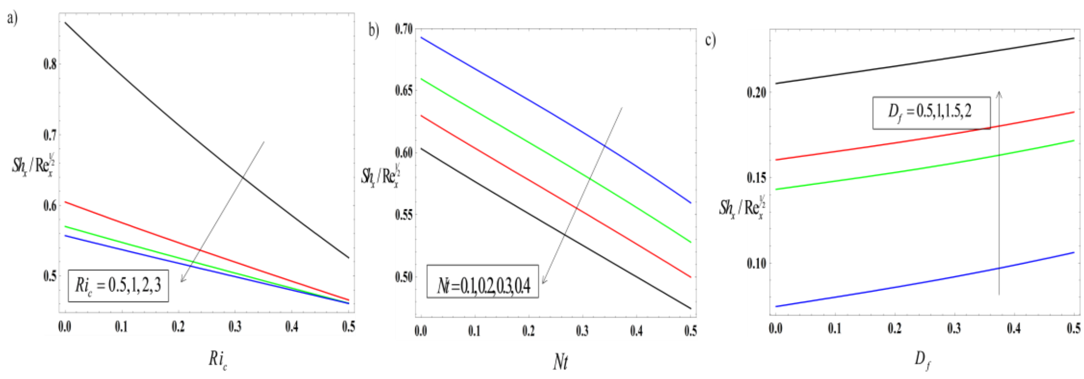

Fig .5.6.Effects of (a) Richardson number (Ri_c), (b) Nanofluidparameter (Nt), (c) Dufour Number (D_f) on Sherwood number (Sh).

$$(\Pr = 6.2, \beta = 0.5, Sc = 1, \mathrm{Bi} = 1)$$

6. Conclusion

In this study, MHD two dimensional Stagnation point flow of nanofluid over a stretching surface are investigated. Specific results have been obtained using Shooting method for different values of pertinent parameters as defined in the transformed governing equations.The following results can be summarized.

- The velocity field increases with increasing values of Richardson number (Ri_T) and Nanofluid parameter (Nt). But decreases while the increase of Magnetic Field parameter (M).
- The temperature field increases with increasing values of Nanofluid parameter (Nt) and Heat generation or absorption coefficient (S). But decreases while the increase of Dufour Number (D_f).
- The concentration field increases with increasing values of Nanofluid parameter (Nt) and Chemical reaction parameter (γ). But decreases while the increase of Richardson number (Ri_c).
- The skin friction coefficient (C_f) increases with increasing values of Richardson number (Ri_c), Magnetic Field parameter (M) and Dufour Number (D_f).
- The Nusselt Number (Nu) increases with increasing values of Heat generation or absorption coefficient (S) and Nanofluid parameter (Nt). But decreases while the increase of Dufour Number (D_f).
- The Sherwood Number (Sh) increases with increasing values of Dufour Number (D_f). But decreases while the increasing values of Richardson number (Ri_c) and Nanofluid parameter (Nt).

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