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Differential Equation

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Definition: - The equation which expresses the relationship between dependent variable, independent variables and the derivatives of dependent variables with respect to independent variable is called differential equation.

Classification of differential equations: -

During study of a differential equation, learn to look at an equation and classify it into a certain group. The reason is that the techniques for solving differential equations common to these various classification groups. Sometimes transform an equation of one type into an equivalent equation of another type so that easier solution techniques are used

1. Ordinary differential equation.
2. Partial differential equation.

1. **Ordinary differential equation**:-A differential equation which expresses relationship b/w one or more dependent variable which are function of a single independent variable and the total derivatives of dependent variable with respect to independent variable i.e., the differential equation involving only total derivatives. That is "An ordinary differential equation contains differentials with respect to only one variable."
2. **Partial Differential Equation**: - A differential equation which contains the derivatives of one or more dependent variables with respect to more than one independent variable i.e. the differential equation involving partial derivatives "Partial differential equation contains differentials with respect to several independent variables".

Classification of ordinary differential equation: -

Ordinary equation can be classified into: -

1. **Simple/Ordinary differential equations**: - An ordinary differential equation which contains one dependent variable and derivatives of dependent variables with respect to independent variable.

   For example: - Ley y=y(x), x is independent variable then $\frac{d^3y}{dx^3} + \sin x \frac{dy}{dx} = \cos y$ is a simple/ordinary differential equation.

2. **System of differential equation**: - A differential equation which contains one independent variable, more than one dependent variable and derivatives of dependent variable with respect to independent variables.

   For example: - Ley y=y(x) and z=z(x) are two dependent variables of a single independent variable x then $\frac{dz}{dx} + \frac{dy}{dx} = \sin x$ and $\frac{dy}{dx} + x\frac{dz}{dx} = \cos x$ together is a system of the differential equation.

Classification of Partial differential equation: -

1. **Simple Partial differential equation**: - A differential with one independent variable which is the function of more than one independent variable.

   For example: - Let z=z(x,t), where x and t are independent variables then

   $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$ is a Partial differential equation.

2. **System of Partial differential equation**: - A differential equation which contain more than one independent variable and their derivatives with respect to independent variables.

   For example: - Let y=y(x,t) & z=z(x,t), where x, t are independent variables then $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0, \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 z}{\partial x^2} = 0$ together is a system of Partial differential equation.

Order of Differential equation: -

The order of the highest ordered derivative occurs in a differential equation is called the order of the differential equation. The order of the differential equation always exists. Forexample:-

1) $Y = \sin \frac{dy}{dx}$ is of order 1
2) $\log\left(1 + \frac{dy}{dx}\right) = c$ is order of 1
3) $e^{\frac{d^3y}{dx^3}}$-$\frac{d^2y}{dx^2}$ + xy =0
4) The order of the equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}$+ $\left(\frac{d^3y}{dx^3}\right)^{3/2}$=0 is of order 3

Degree of Differential equation: -

The degree of the differential equation is the highest exponent of the highest derivatives which occurs in it, after the differential equation has been made free from radicals and fractions as per the derivatives are concerned. The degree of differential equations need not exist. For examples:

1) The degree of $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right) + y = 3e^x$is 3
2) The degree of $\left(\frac{d^3y}{dx^3}\right)^2 - 7\left(\frac{dy}{dx}\right)^2 = 3\sin x$is 2
3) The degree of $\left(\frac{d^3y}{dx^3}\right)^{2/3} + \left(\frac{d^3y}{dx^3}\right)^{3/2}$=0 is 9
4) $y = \sin\left(\frac{dy}{dx}\right)$ degree of the equation does not exist.

The degree of $\log\left(1 + \frac{dy}{dx}\right) = c$ does not exist.

Family of curves: -

A family of curves is a set of curves, each of which is given by a function of parameterization in which one or more of the parameters are variables. That is the n-parameters family of curves is a set of relation of the form $\{(x, y)|g(x, y, c_1, c_2, c_3, \dots\dots.., c_n)\}$, where g is real value function of x,y,$c_1, c_2, \dots\dots\dots, c_n$& each $c_i$is parameter. Families of curves appear frequently in solutions of thedifferential equation. For example:-

Family of the circle whose center is $(c_1, c_2\ )$ and radius is 3 i.e.$\{(x, y): (x - c_1)^2 +$ $x{-}c22{=}9$ is two parameter family of curves.

Formation of Differential equation:-

Let $F(x, y, c_1, c_2, \dots\dots., c_n) = 0$ be n-parameter family of curves, where x is independent variables, y dependent variable. Then the differential equation whose solution is $F(x, y, c_1, c_2, \dots\dots., c_n) = 0$ is obtained by eliminating n arbitrary constants$c_1, c_2, \dots\dots., c_n$.

Method to eliminate arbitrary constants:-

Let $F(x, y, c_1, c_2, \dots\dots., c_n) = 0$ ………… (1) be n-parameter family of curves.

Differentiating (1)w.r.t. x (n- times), we get

$\emptyset_1(x, y, y', c_1, c_2, \dots\dots., c_n) = 0$ ………… (1)

$\emptyset_2(x, y, y', c_1, c_2, \dots\dots., c_n) = 0$ ………… (2)

$\emptyset_3(x, y, y', y'', c_1, c_2, \dots\dots., c_n) = 0$ ………… (3)

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$\emptyset_n(x, y, y', y'', \ldots \ldots y^{(n)}, c_1, c_2, \ldots \ldots., c_n) = 0$ ………… (n)

Where $\emptyset_i$ denotes the i^{th} derivative of $F(x, y, c_1, c_2, \ldots \ldots., c_n) = 0$ Eliminating n-unknowns using (n+1) equations, we get required differential equation.

Linear differential equation:-

A linear differential equation is any differential equation that can be written in the following form

$a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots\ldots\ldots\ldots. + a_{n-1}(x)y' + a_n(x)y = \emptyset(x)$, where $a_i\ \forall\ i$ are functions of x.

The important thing to note about linear differential equation is that there are no products of the dependent variable and its derivatives or no products of the derivatives and neither the dependent variable not its derivatives occur to any power other than the first power.

For example:-$x^2y'' + xy' + y = e^x$ ios linear differential equation of second order.

Solution of differential equation: -

Let $F(x, y, y^1, \ldots, y^{(n)})=0$ be the n^{th} order differential equation then a real or complex valued function Q(x) of real variable x is a solution of$F(x, y, y^1, \ldots, y^{(n)})$ if

I. $Q(x), Q^1(x), \ldots Q^{(n)}(x)$ exist , where $Q^{(k)}(x)$ denotes the k^{th} derivative of Q(x) with respect to x.
II. $Q^{(n)}(x)$ satisfies the differential equation i.e.$F(x,Q(x), Q^1(x), \ldots Q^{(n)}(x))=0$

Classification of solution of the differential equations:-

Let $F(x, y, y^1, \ldots, y^{(n)})=0$ be n^{th} order differential equation.

1. **General Solution**: - Any solution of the differential equation which contains n "independence of arbitrary constants" is called general solution.

   Independence of arbitrary constants: - The arbitrary constants appearing in the general solution must be independent and should not be reducible to a fewer number of equivalent constants.
2. **Particular solution/ integral curve**:- Any solution which can be obtained from general solution by taking some particular value of arbitrary constants.
3. **Singular solution**:- A singular solution in this stronger sense is often given as tangent to every solution from family of solutions. A singular solution is the envelope of the family of solutions cannot be obtained from general solution by giving particular values of arbitrary constants.

The singular solution is related to the general solution by its being what is called the envelope of that family of curves representing the general solution. An envelope is defined as the curve that is tangent to a given family of curves.

Envelope:- An envelope of a family of curves in the plane is a curve that is tangent to each member of the family at some point.

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