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MHD BOUNDARY LAYER VISCOUS INCOMPRESSIBLE FLUID FLOW AND HEAT TRANSFER OVER A NONLINEAR STRETCHING SURFACE WITH VARIABLE SURFACE TEMPERATURE AND PARTIAL SLIP

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ABSTRACT

The present study concerns to boundary layer viscous fluid flow and heat transfer over a nonlinear stretching surface with the effect of variable surface temperature, partial slip, and magnetic field.The governing boundary value problem,consisting of a set of nonlinear partial differential equations are transformed into a set of nonlinear ordinary differential equations and are solved using Runge-Kutta fourth order method.The Effects of various flow and heat transfer charecteristics are analysed and the results are suitably interpreted graphically.

KEYWORDS: Viscous incompressible fluid;Nonlinear stretching surface;MHD;boundary layer;Variable surface temperature;Partial slip.

I INTTRODUCTION

Sakiadis [1], was the first to investigate the boundary layer flow past a moving solid surface of a viscous fluid with a constant velocity. Later, the numerical results of Sakiadis [1] were confirmed by Tsou *et al*. [2] analytically and experimentally.In recent years,

the flow and heat transfer over a stretching sheet immersed in a Newtonian fluid in the presence of magnetic field has received great attention because of its important applications in metallurgical industry which involves the cooling of continuous strips and filaments drawn through a quiescent fluid. The problem of flow and heat transfer over stretching surface find applications to polymer technology, where one deals with stretching of plastic sheets. In particular , many metallurgical processes invove cooling of continuos strips or filaments by drawing them through a quiscent fluid and that in process of drawing, these strips are sometimes stretched.In the case of annealing and thinning of copper wires, the properties of the final product depend to a great extent on the rate of cooling. In view of such applications a problem of this kind is considered in this present work. The process of suction and injection has also its importance in many engineering applications such as the design of thrust bearing, radial diffusers and thermal oil recovery. Blowing is used to add reactants, cool the surfaces, prevent corrosion or scaling and reduce the drag, Labropulu etal[3]

The problem of non-linear stretching sheet for different cases of fluid flow has also been analyzed by different researchers. Vajravelu [4]examined fluid flow over a nonlinearly stretching sheet. Cortell [5] has worked on viscous flow and heat transfer over a non-linearly stretching sheet. Cortell [6] further investigated on the effects of viscous dissipation and radiation on the thermal boundary layer, over a non-linearly stretching sheet. Raptis et al [7] studied viscous flow over a non-linear stretching sheet in the presence of a chemical reaction and magnetic field. Abbas and Hayat [8] addressed the radiation effects on MHD flow due to a stretching sheet in porous space. Cortell [9] investigated the influence of similarity solution for flow and heat transfer of a quiescent fluid over a non-linear stretching surface. Awang and Kechil [10] obtained the series solution for flow over nonlinearly stretching sheet with chemical reaction and magnetic field.

The no slip boundary condition(the assumption that a liquid adheres to a solid boundary) is one of the central tenets of the Navier Stokes theory. However, there are situations wherein this condition does not hold.The inadequacy of the no slip condition is evident for most Newtonian as well as non-Newtonian fluids.For,example, polymer melts often exhibit macroscopic wall slip and that in general is governed by a nonlinear and monotone relation between the slip velocity and the traction.This may be important in shear skin, spurt and hysterisis effects, and also the fluids which exhibit slip boundary condition have important technological applications such as in the polishing of artificial heart valves and internal cavities. Navier[11] proposed a slip boundary condition wherein the slip depends linearly on shear stress.

In the recent years,micro-scale fluid dynamics in the Micro-Electro-Mechanical Systems (MEMS)received much attention in research.Because of the micro-scale dimensions,the fluid flow behavior belongs to the slip fow regime and greatly differs from the traditional flow .For the fow in the slip regime,the fluid motion still obeys the Navier Stokes equations,but with slip velocity or temperature boundary conditions.

In addition, Partial velocity slip may occur on the stretching boundary when the field is particulate such as emulsions, suspensions, foams and polymer solutions.

In certain cases, partial slip between the fluid and the moving surface may occur. Examples include situations when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. Also the extrudate may be rough or porous where the no slip condition is considered. In these cases the proper boundary condition is well described by Navier's condition.

All the investigators restricted their analysis to viscous/visco-elastic flow over a non-linear stretching sheet. However, the intricate problem of flow and heat transfer over non-linear stretching sheet with the effects of buoyancy and partial slip is yet to be studied. This has several industrial applications. Hence the present paper aims in the investigation of the same. The combined effect of all the above-mentioned parameters has not been reported so far, in the literature, which makes the present problem unique

II Problem formulation

Consider a steady, two-dimensional free convection flow adjacent to a nonlinear stretching vertical sheet immersed in an incompressible electrically conducting viscous fluid of temperature T_∞. The stretching velocity $U_w(x)$ and the surface temperature $T_w(x)$ are assumed to vary linearly with the distance x from the leading edge, i.e. $U_w(x) = ax^m$ and $T_w(x) = T_\infty + bx^s$, where a and b are constants with $a > 0$, $b \geq 0$, and $B(x) = Bx^{\frac{m-1}{2}}$.

The boundary layer equations of motion and heat transfer are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}B(x)u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

and are subjected to the following boundary conditions

$$u(x, y) = L\frac{\partial u}{\partial y} + ax^m \ , v = v_w(x), , T = T_w + bx^s \quad \text{at} \quad y = 0,$$

Where $v_w(x) = -f_w\sqrt{\frac{\nu a(m+1)}{2}}x^{\frac{m-1}{2}}$

$$u \to 0, , T \to T_\infty \quad \text{as} \quad y \to \infty, \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively. Further, μ, ρ, α, and T, are the dynamic viscosity, fluid density, thermal diffusivity, and fluid temperature in the boundary layer, respectively.

A common feature of all these analyses is the assumption that the flow field obeys the conventional no-slip condition at the sheet that is the velocity component $u(x,y)$ parallel with the sheet becomes equal to the sheet velocity ax^m at the sheet. In certain situations, however, the assumption of no-slip does no longer apply and should be replaced by a partial slip boundary condition which relates the fluid velocity u to the shear rate $\frac{\partial u}{\partial y}$ at the boundary. Here L is the slip length, and y denotes the coordinate perpendicular to the surface. This slip-flow condition was first introduced by C-L.M.H Navier more then a century ago and has more recently been used in studies of fluid flow past permeable walls, slotted plates, rough and coated surfaces, and gas and liquid flow in micro devices. The no-slip boundary condition is known as the central tenets of the Navier-Stokes theory. But there are situations wherein such condition is not appropriate .Especially; no slip condition is inadequate for most non-Newtonian fluids. For example polymer melts often exhibit macroscopic wall slip and that in general is governed by a non-linear and monotone relation between the slip velocity and traction. The fluids exhibiting boundary slip find applications in technology such as in the polishing of artificial heart valves and internal cavities. Navier suggested a slip boundary condition in terms of linear shear stress.

The momentum, and energy equations, (2),(3), and (4) can be transformed into the corresponding nonlinear ordinary differential equations by the following similarity transformation:

$$\eta = \left(\frac{(m-1)a}{2\nu}\right)^{1/2} x^{\frac{m-1}{2}} y, \qquad u(x,y) = ax^m f'(\eta),$$

$$v(x,y) = \left(\frac{(m+1)\nu a}{-2}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left(\frac{m-1}{m+1}\eta f'(\eta) + f(\eta)\right) \tag{5}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{6}$$

Where $T_w(x) = T_\infty + bx^s$, b is dimensional constant and n is the index

Of power law variation of temperature.

The transformed nonlinear ordinary differential equations are

$$f''' = ff'' - \left(\frac{2m}{m+1}\right) f'^2 - Mf', \tag{7}$$

$$where\ M = \frac{2\sigma B^2}{\rho a(1+m)}$$

$$\theta'' = -P_r[\, f\theta' - (\frac{2s}{m+1}) f'\theta], \tag{8}$$

Boundary conditions (6.2.4) becomes

$$f(0) = f_w \quad f'(0) = 1 + \gamma f''(0), \qquad \theta(0) = 1, f'(\infty) = 0, \theta(\infty) = 0. \tag{9}$$

Where $\gamma = L\sqrt{\mathrm{Re}_x(1+m)},\ \mathrm{Re}_x = \frac{ax^{m-1}}{\nu}$,

III Numerical Solution

The nonlinear boundary value problem represented byEqs.(7) to(9) is solved numerically using Fourth-orderRunge Kutta shooting technique.Making an initial guess for the values of $f''(0) and \theta'(0)$ to initiate the shooting process is very crucial in this process.The success of the procedure depends very much on how good this guess is. Numerical solutions are obtained for several values of the physical parameters i.e. magnetic parameter M, suction/injection parameter f_w , stretching parameter m, Prandtl number Pr, and slip parameter γ .

V**Results and discussion**

The nonlinear ordinary differential equations (7) - (8) subject to the boundary conditions (9) have been solved numerically using fourth order Runge-kutta shooting Technique.In order to have a physical point of view of the problem, numerical calculations were carried out for different values of magnetic parameter M, suction/injection parameter f_w , stretching parameter m ,Prandtl number Pr, and slip parameter γ .

The influences of the magnetic parameter M on the velocity and temperature profiles are depicted in fig1. It can be seen that increasing M is to reduce the velocity distribution in the boundary layer which results in thinning of the boundary layer thickness, and hence induces an increase in the absolute value of the velocity gradient at the surface

The influence of suction/ injection parameter f_w, over the dimensionless longitudinal velocity profile is shown in fig 2 and it is noticed that longitudinal velocity decreases with suction and increases with injection. It should be noted that in Figure 2,the boundary-layer assumptions do not permit a solution of the boundary-layer equation for large f_w, because θ will approach a constant value of 1, and the boundary layer is almost literally blown off the surface, similar to that of stationary plate with injection (Burmeister [1983]; Kays andCrawford [1987]).

Fig 3, shows the effect of suction/injection on dimensionless temperature profile and it is observed that there is decrease in temperature in the thermal boundary layer resulting in thinning of thermal boundary layer thickness in the case of suction and the reverse trend is observed for injection. Further it is clear that suction ($f_w < 0$) enhances the heat transfer coefficient much better than injection ($f_w > 0$), and the thickness of the thermal boundary layer is reduced. Thus, suction can be used as a means for cooling the surface much faster than injection.

Figs.4 and 5 describe respectively the behaviors of the longitudinal velocity profile and temperature profile for different values of power law stretching parameter m and it is noticed that increase in m results in decrease of longitudinal velocity profile which is more pronounced for small values of m, where as temperature profile increases with the increase of stretching parameter m. It is observed that the variation of the sheet temperature has a substantial effect on the thermal boundary layer. This effect is more pronounced when sheet temperature varies in the direction of highest stretching rate.

An increase in Prandtl number Pr is associated with a decrease in the temperature distribution which is displayed in Fig. 6, which is consistent with the fact that thermal boundary layer thickness decreases with increase in the values of prandtl number. The rate of heat transfer increases with the increasing values of Prandtl number.The boundary layer edge is reached faster as Pr increases.

Dimensionless velocity profile $f'(\eta)$ is presented in fig.7 for some different values of the slip parameter γ. It is readily seen that γ has a substantial effect on the solutions. In fact, the amount of slip $1+\gamma f''(0)$ increases monotonically with γ from the no-slip solution for $\gamma = 0$ and towards full slip as γ tends to infinity. The latter limiting case implies that the frictional resistance between the viscous fluid and the surface is eliminated, and the stretching of the sheet does no longer impose any motion of the fluid The velocity and temperature profiles presented in figs. 1-7, show that the far field boundary conditions are satisfied asymptotically, which support the validity of the numerical results presented.

V Conclusions

We have theoretically studied the problem of steady two-dimensional free convection flow adjacent to a nonlinearly stretching vertical sheet immersed in an viscous incompressible fluid. The governing partial differential equations are transformed, using similarity transformation, to a system of nonlinear ordinary differential equations, before being solved numerically by the fourth order Runge –Kutta shooting method. The effects of the governing parameters like magnetic parameter M, suction/injection parameter f_w, stretching parameter m, Prandtl number Pr, and slip parameter γ on flow and heat transfer are thoroughly discussed with the aid of graphs. The effects of Magnetic parameterM,suction parameter f_w,nonlinear stretching parameter is to deccelerate dimensionless longitudinal velocity in the boundary layer and an opposite trend is noticed for slip parameter γ. Further it is noticed that temperature profile increases with increase of stretching parameter m and suction parameter $-f_w$, and decreases with increase of Prandtl number pr.

REFERENCES

[1] **Sakiadis, B.C**, "Boundary-layer Behavior on Continuous Solid Surfaces: I BoundaryLayer Equations for Two Dimensional and Axisymmetric Flow",AIChE J ., (7**)(** 1961) pp. 26-28.

[2] **F. K. Tsou, E. M. Sparrow, R. J. Goldstein**, "Flow and heat transfer in the boundary layer on a continuous moving surface", *Int. J. Heat Mass Transfer.,* vol. 10, (2) (1967)219-235,

[3] **Labropulu F, Li D, and Pop I,** Non-orthogonal stagnation-point flow towards a stretching surface in a non-Newtonian fluid with heat transfer. *Int. J. Therm. Sci.*49, **(2010** 1042-1050.

[4]K. Vajravelu, Fluid flow over a nonlinearly stretching sheet, Appl.Math.Comput, 181(2006) 609 – 618.

[5]Cortell, R, Effects of viscous dissipation and work done by deformation on the MHDflow And heat transfer of a viscoelastic fluid oever a stretching sheet. Physics Letters A, 357(2006)298-305.

[6]**R. Cortell**, MHD flow and heat transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet subject with chemically reactive species, Chem. Engng. Processing, 46(8) (2007) 721-728.

[7]A. Raptis and C. Perdikis, Viscous flow over a non-linear stretching sheet in the presence of a chemical reaction and magnetic field. Int. J. Non-Linear Mech, 41 (2006)527– 529.

[8]Zaheer Abbas and Tasawar Hayat, Radiation effects on MHD flow in porous space, Int. J. Heat Mass Transfer, 51(2008) 1024 – 1033.

[9] R. Cortell, Similarity solution for flow and heat transfer of a quiescent fluid over a non-linearly stretching surface, Journal of materials processing technology, 203(2008) 176 –183.

[10] Awang Kechil, S. and I. Hashim, 2008. SeriesSolution of Flow over Nonlinearly Stretching Sheet with Chemical Reaction and Magnetic Field,Physics Letter A, (372)2258-2263.

[11]Navier , C. L. M. H. (1827). Sur les lois du mouvement des fluides , *Mem. Acad. R. Sci. Inst. Fr.* , 6 , 389 – 440 .

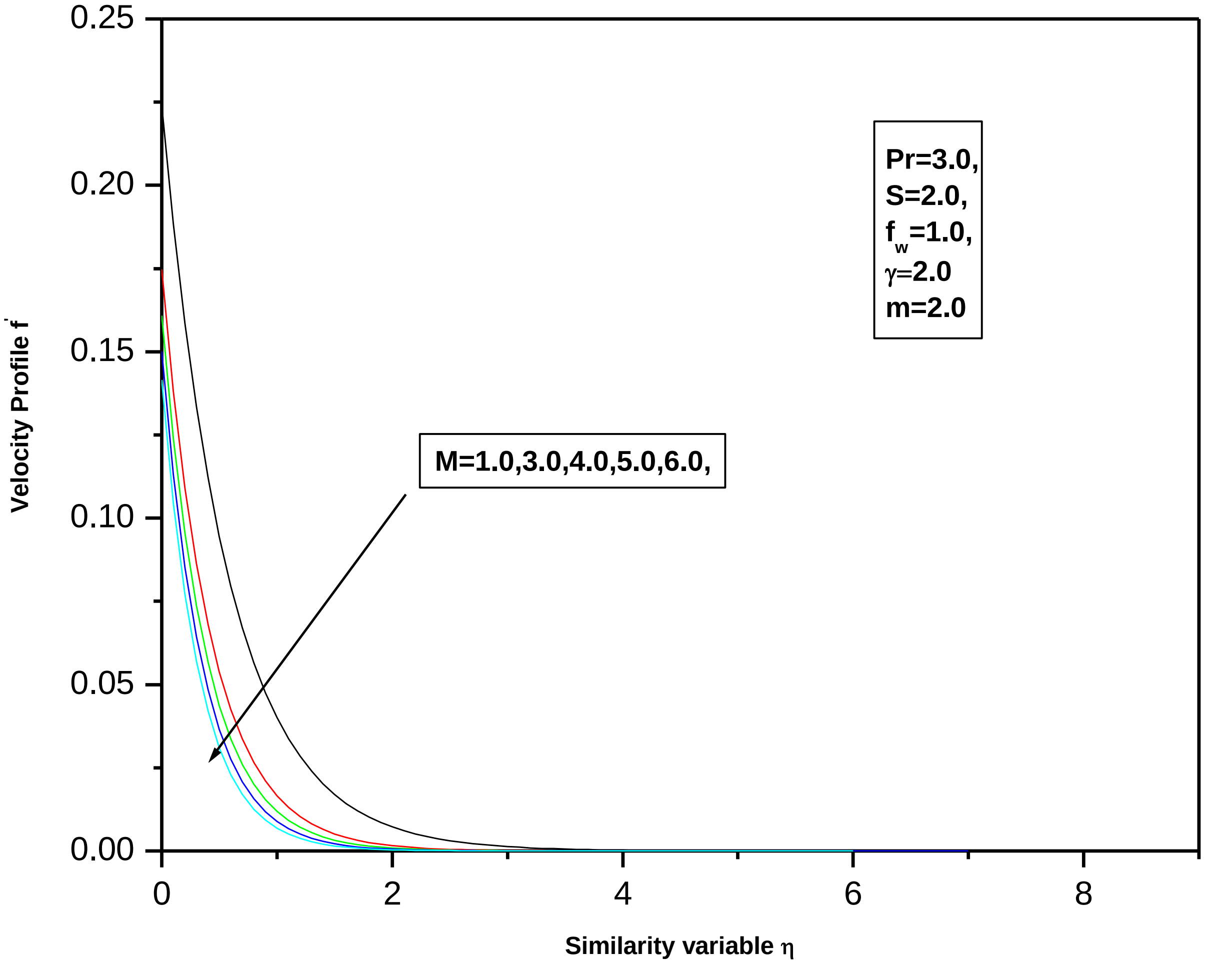

Fig 1, Velocity profile f^{I} versus Similarity variable η. for different values of magnetic parameter

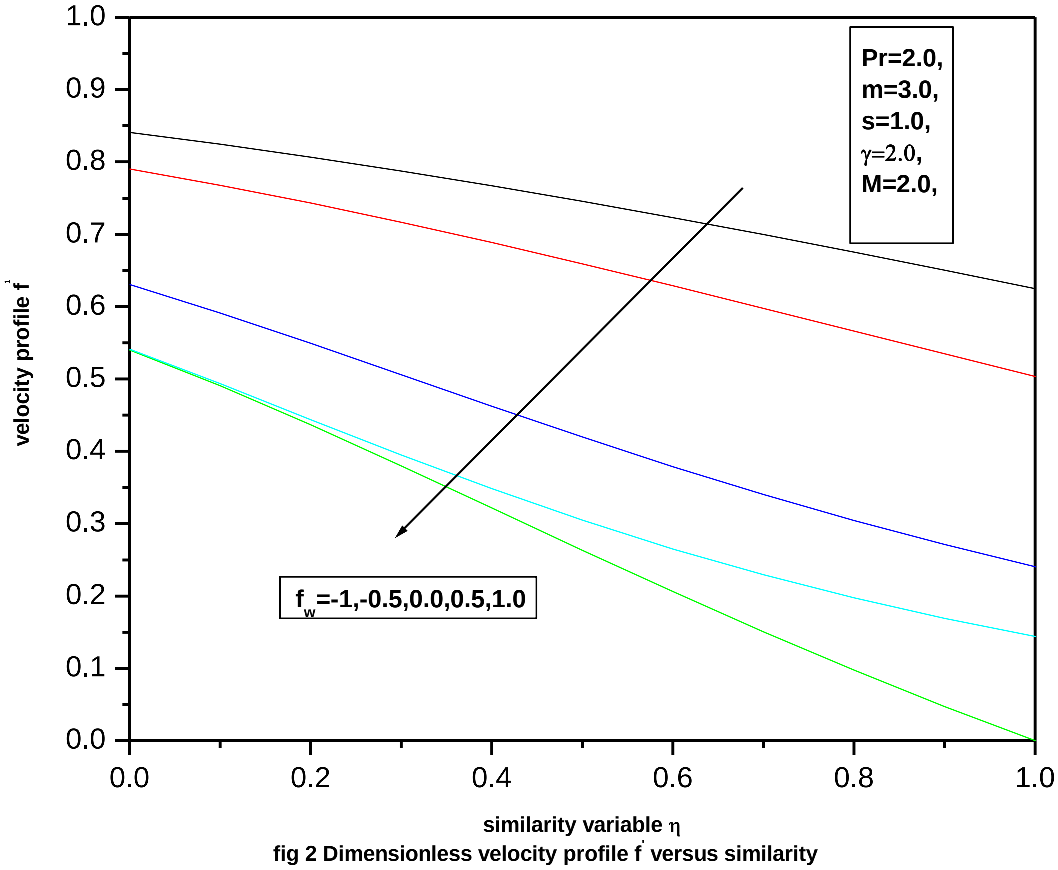

fig 2 Dimensionless velocity profile f' versus similarity variable η for different values of suction/injection parameter

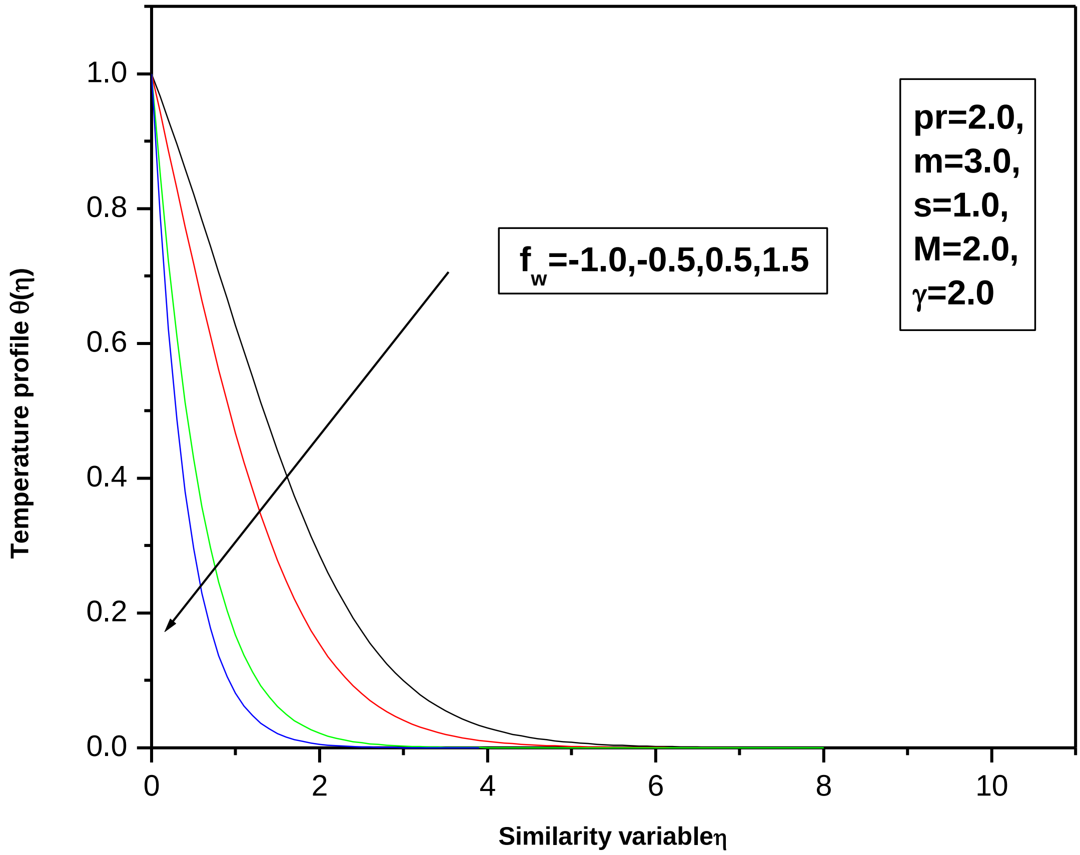

fig 3,Temperature profile θ(η) vs similarity variableη for different values of suction/injection

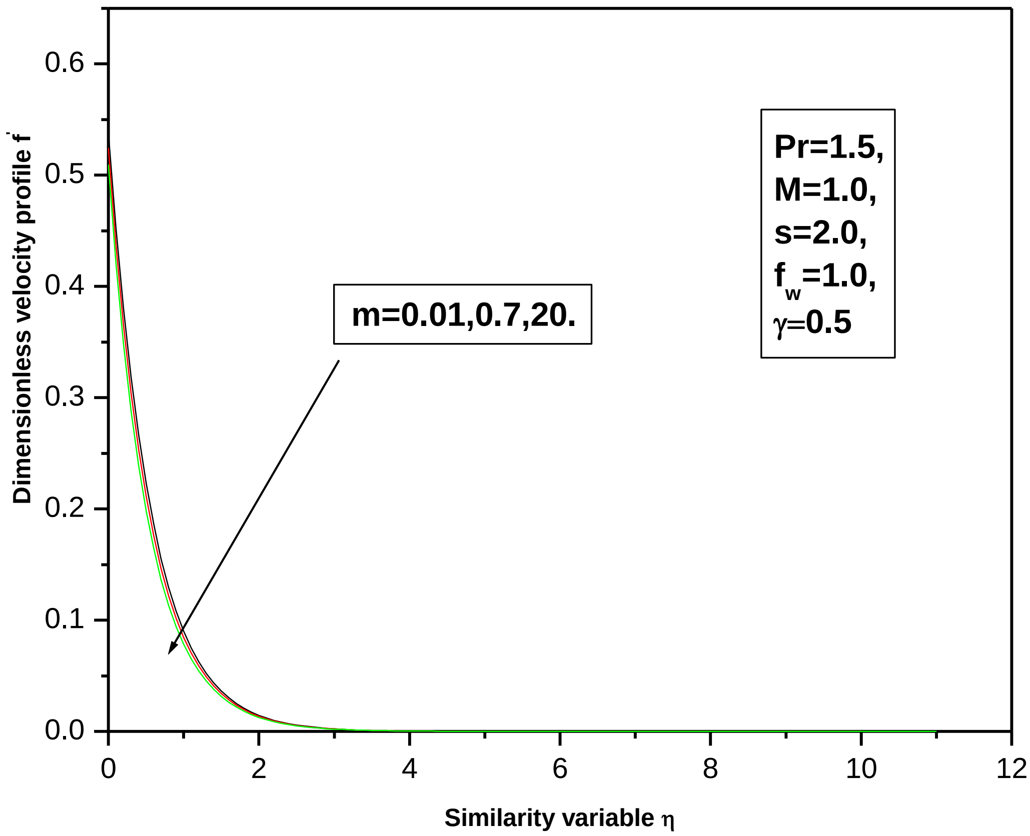

Fig4,Dimensionless velocity profile f' vs similarity variable η
for different values of m

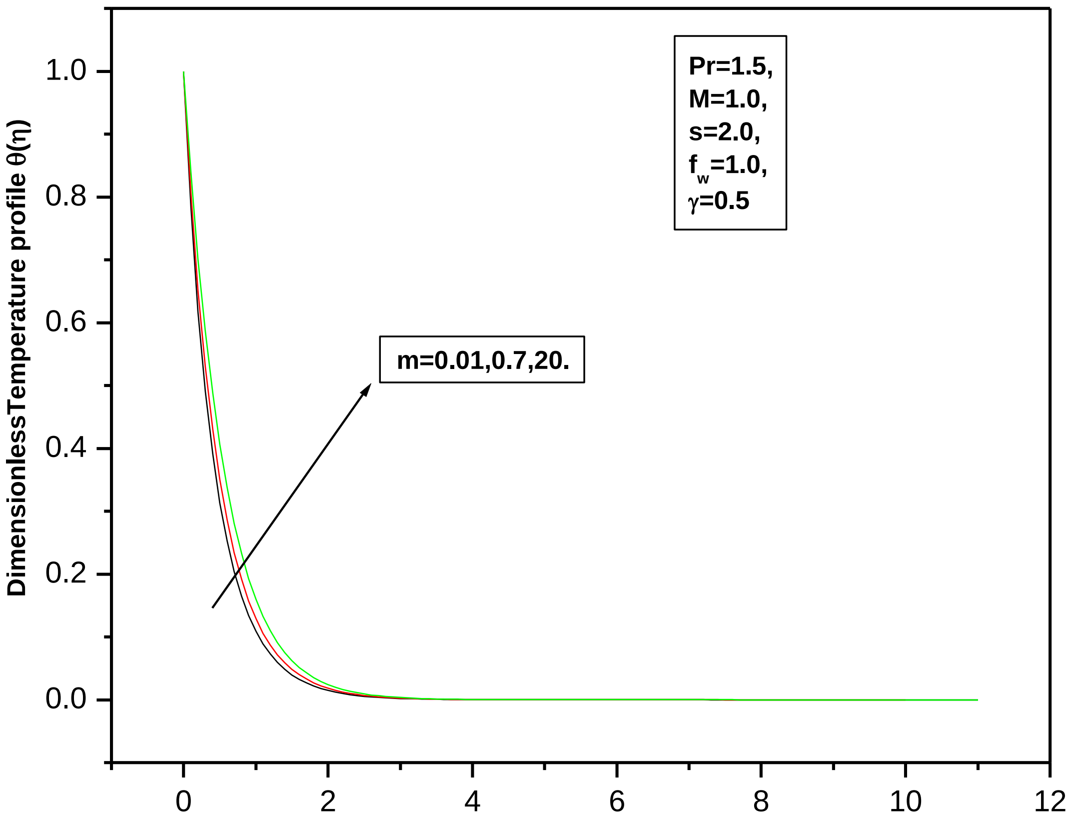

Fig5, Dimensionless temperature profile θ(η) vs similarity variable η for different values of m

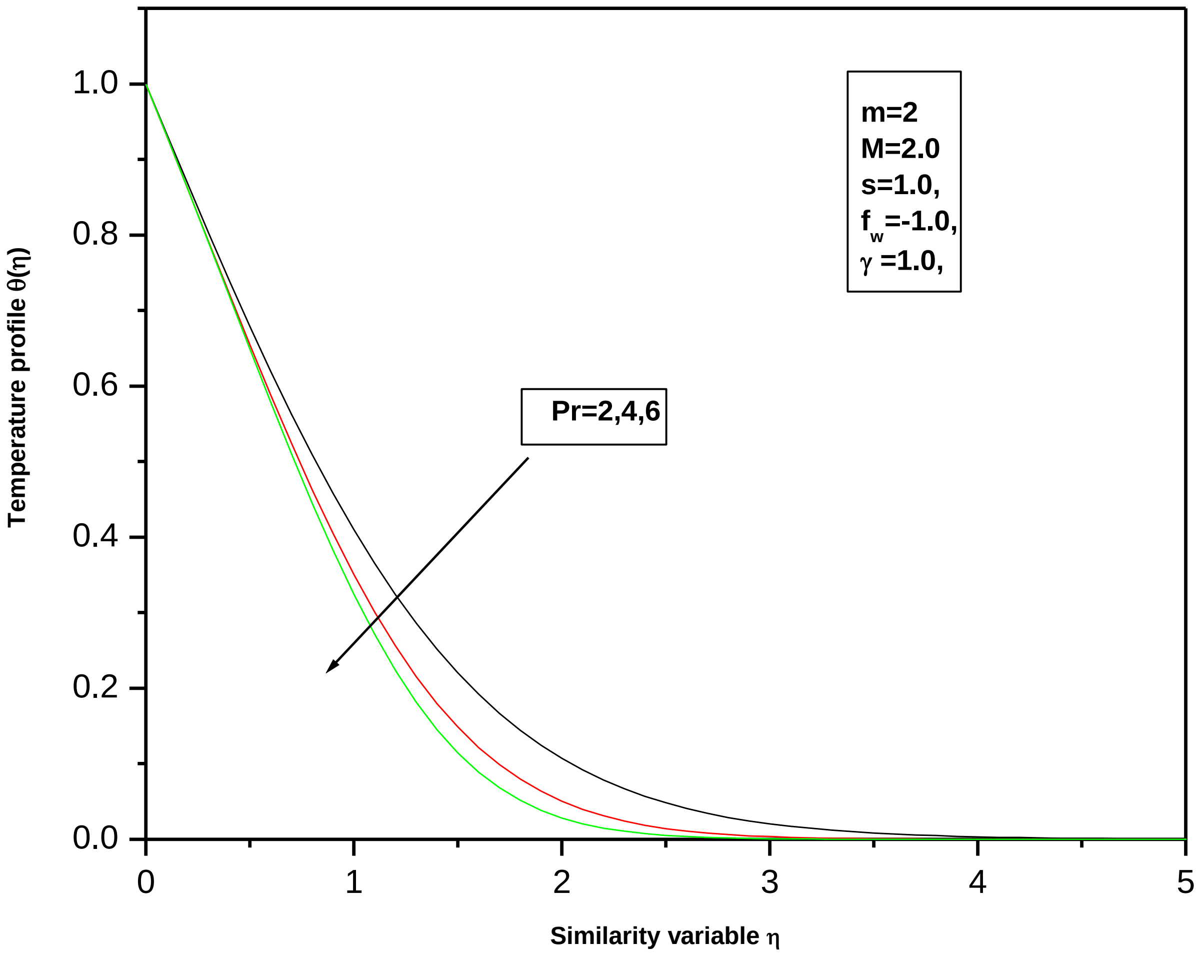

Fig 6, Temperature profile θ(η) versus similarity variable η for different values of Pr

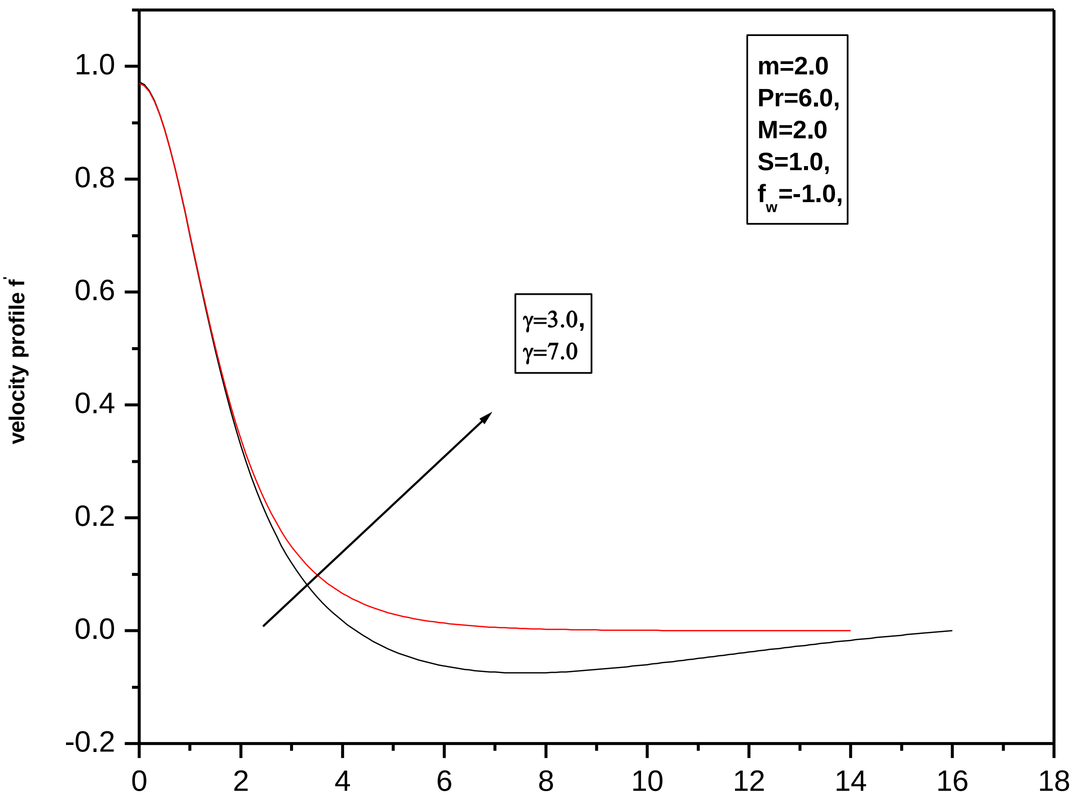

Fig7 Dimensionless velocity profile f'
versus similarity variable η for different values ofslip parameter γ