



APPLICATION OF BAYESIAN ESTIMATION IN ANALYZING CONFIRMATORY FACTOR ANALYSIS MODEL OF PHYSICS CLASSROOM PSYCHO-SOCIAL COVARIATES

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ABSTRACT

The problem of poor achievement of students in Physics warranted the study. Researchers have linked poor Physics achievement to poor quality of instrument and Physics classroom psycho-social covariates (PCSC). The gap which this study filled was to use Bayesian estimation to reaffirm the result of exploratory factor analysis of PCSC and achievement through confirmatory factor analysis. The study adopted an instrumentation design.. The sample for the study comprised 420 SS3 Physics students sampled using random sampling from ten schools in Agbani Education zone. The instrument used to collect data included Physics classroom psycho-social covariates (PCSC) and students' results. The alpha values of PCSC ranged between .67 to .78 for its clusters and .74 for the overall. The data collected were analyzed using regression weights, correlation and deviance information criterion values. The results indicated that the indicators of each latent construct positively loaded on them. There were moderately positive correlation between the underlying constructs. The sub-models were invariant.

KEY WORDS: Confirmatory Factor Analysis, Classroom Psycho-social covariates and Physics achievement.

Introduction

Physics is one of the core science subjects studied in secondary schools. The reason for inclusion of Physics as a school subject is to be able to make the students adapt to an ever-changing science and technological age. Living a life in an ever dynamic and complex age requires the possession and use of specialized skills and tools to be able to paddle through the turbulent waters orchestrated by the fast changing age. For instance, the knowledge gained from Atmospheric Physics is used to debunk students' alternative idiosyncrasies concerning eclipses and image formation. Such questions on stars including: "Why wouldn't the sun, being a burning object decrease in size? and "Why doesn't the sun fall down to the Earth?" are provided using Physics concepts and principles. This kind of knowledge which Physics provides helps the human inhabitants of the Earth to stop living in fear concerning the presence of heavenly bodies which hang in space. When spotted from the surface of the vegetative-ground below, the bodies appear as if they are falling down with a very slow gravitational motion. Thus, their appearances provide a source of worry to the uninformed space onlooker, in terms of feeling unsafe in the planet. Also, with the advent of spaceship which after being launched carries satellites into the weightless orbit of the Earth. What hitherto was considered impossible: cordless communication became a reality. Consequently, people are now able to talk or share information over long distances via phones and the internet respectively. So, the possession of information and communication technology (ICT) skills is a determinant to living a fulfilled life in a modern age. The ICT objects technically referred to as Information Technology (IT) which consists of both the hardware and software all depend on the principles of Physics in their making and working. Electromagnetic data stored in tapes, flashes or compact-discs run are used to influence the way a computer works. In addition, transistors, diodes, capacitors, resistors, transformers, logic gates to mention a few, whose workings obey the principles of Physics are used to make IT devices.

In fact, the principles of Physics run across the length and breadth of everyday real world applications. The knowledge of Mathematical Physics, as a field of study enables an

experimenter to determine physical constants including mass, length of an object, the electromotive force of a cell, focal length of a lens/mirror and the resistivity of constantan wire without directly weighing or measuring them. The technique of indirect measurement of physical quantities by manipulating their associated ancillary parameters to attain a balance is alternatively investigated through linear equations, specifically the slope-intercept form to mention a few benefits of Physics. This technique helps to ensure precision during the manufacture of IT equipment.

In recognition of the importance of Physics to make individuals adapt to the fast changing age, the Federal Government of Nigeria, through the Federal Ministry of Education, FME (2011) has taken the bull by the horn in making sure that Physics is included in the senior secondary school curriculum. The senior secondary school acts as a gateway between Basic Science learnt in Junior secondary school and the Physics studied in tertiary institutions of learning. However, despite the importance of Physics for the well being of individuals and the society, the Nigerian secondary school Physics students have consistently achieved poorly in Physics external examinations. The National Examinations Council, NECO (2014) and West African Examinations Council, WAEC (2016) Chief Examiners' reports in both examination bodies attest to this claim in Physics. Physics Education researchers in a bid to solve the problem of poor achievement in Physics examination, indicated that part of the problem of poor achievement was attributed to the poor quality of the Physics classroom environment (Abuh, 2014) and poor instrumentation (Nworgu, 2014).

The psycho-social climate within the Physics classroom relate to those psychological and socially-related variables within the Physics classroom. Herbert (2004) reported that psycho-social classroom climate influenced learning outcomes. The psycho-social variables of Physics classroom environment including involvement, student-student interaction, teacher-student interaction, satisfaction, task orientation, order and organization, teacher control and innovation meaningfully influenced achievement in Physics (Abuh, 2017). Moreover, the extent of involvement of both the students and their Physics teachers in the classroom activities may go a long way in determining students' success in Physics examinations. The same is true for the extent of students' interactions both within themselves as well as between students and teachers alike. The satisfaction which both the students and teachers derive at the end of the Physics classroom instruction may also count as a variable

influencing Physics achievement. Other variables within the family of psycho-social variables included the orientations which the students or teachers have had prior to entering the classroom in performing certain Physics tasks in the classroom. This is perceived as the world view of the students and teachers. The social order in the Physics classroom, and the way the seats are arranged all contribute to Physics achievement. Teachers' control refers to the ability of the Physics teacher to deploy instructional strategies like use of examples stimulus variation, planned repetition, reinforcement and questioning to make the classroom lively. Innovations mean the Physics teachers' ability to introduce new ideas in the classroom.

The use of instrument with no or defective item banks to collect data is a common practice in the school system (Nworgu & Nnadi, 2014). This kind of instrument exerts negative influence on the students' achievement scores. The use of instruments of poor psychometric quality does not bring out the truly fair ability of the test takers because the scores contain higher measurement errors including instrument errors. The few researchers who understood the need for instrument quality and the rigorous nature of obtaining robust instrument mainly focused on determining the psychometric quality of cognitive tests prior to usage (). In addition, the researchers including () utilized the item response theory (IRT) to determine the item banks of Physics objective tests. The IRT is a modern psychometric theory which measures the item characteristics including item difficulty, discrimination, distracter and carelessness indices as well as person or group characteristics (with respect to the items) like differential item functioning (DIF) and differential distracter functioning (DDF) with lower measurement error relative to the classical test theory. Under the IRT framework, both the item and person parameters are measured without one influencing the other.

There appears to be dearth of studies on measuring the item banks of a four-point Physics environment scale using the IRT approach in addition to determining how the items of the scale have clustered as a confirmatory test to exploratory factor analysis using Bayesian estimation. Bayesian estimation adopts Gibb's sampler, which uses Markov Chain Monte Carlo (MCMC) algorithm, specifically Hamiltonian. From the researchers' experience, Hamiltonian was used because of its robustness in analyzing complex models. It is on this premise that the researchers deemed it fit to develop and test-run a confirmatory

factor analysis (CFA) model of psycho-social variables nested within the Physics classroom and Physics achievement using Bayesian estimation. Put in question form, what is the parsimonious CFA model of Physics classroom psycho-social covariates and achievement?

Purpose of the study

The purpose of the study was to determine the: (i) weights of the indicators of each sub-construct in the model (ii) correlation coefficient among the clusters of Physics classroom environment scale and Physics achievement and (iii) invariance parameter of the overall model.

Research Questions

Two research questions guided the study. They included (i) What are the weights of the indicators of each sub-construct in the model? (ii). What are the correlation coefficients among the clusters of Physics classroom environment scale and Physics achievement in the model?

Hypothesis

The study was also guided by one null hypothesis tested at 95% confidence interval:

H_{01} : There is no significant difference between the constrained and unconstrained models' deviance information criterion values based on students' gender.

Research Method

The design of the study was instrumentation. The population for the study comprised two thousand and twenty two (1223 female and 799 male) senior secondary three Physics students in Agbani Education zone of Enugu state (Planning, Research & Statistics Department, Post Primary Schools Management Board, Enugu, 2015). Simple random sampling, specifically balloting with replacement was used to sample 10 public schools in the education zone from where students' sample of four hundred and twenty (420: 189 male & 231 female) was sampled. SS3 Physics students were used because their external examinations in Physics were set by statutory examination bodies in Nigeria. The examination bodies provide better examination questions relative to classroom teachers. The instrument used to collect data included Physics classroom psycho-social covariates (PCSC)

and students results. PCSC was a 4-point rating scale. It consisted of 9 sub-scales and 36 manifest variables. The original instrument, classroom environment scale (CES) was developed by Rudolf Moos at Stanford university (Trickett & Moos, 2003). It consisted of 9 sub-scales with 41 manifest variables. Due to differential location and culture, 5 manifest variables were dropped on the basis of having poor psychometric qualities during exploratory factor analysis. The sub-scales included: innovation, student-student interaction, teacher-student interaction, satisfaction and task orientation. Others included competition, order and organization, teachers' control and innovation. The internal consistency of the original questionnaire (PCSC) ranged between .46 to .72 whereas the Chronbach's alpha reliability for the whole questionnaire items was .60. The alpha values of PCSC ranged between .67 to .78, while the overall alpha was .74 after the trial test. Also, the confirmatory factor analysis result of the exploratory factor analysis, obtained using maximum likelihood estimation indicated that the correlation between the sub-scales of PCSC ranged between .119 to .693. The values indicated that the sub-scales of PCSC positively correlated, indicating that they measured same underlying construct (Physics classroom environment). The Physics students' 2015/2016 senior secondary school results (WAEC and NECO) formed the achievement part of the study. Both WAEC and NECO students' grades which were released as inverse-stenine grades: A₁, B₂, B₃, C₄, C₅, C₆, D₇, E₈ and F₉ were changed to the following points: 9, 8, 7, 6, 5, 4, 3, 2 and 1 point respectively. To ensure that the dataset had equal scale of measurement, it was normalized using R 3.5.1 software with the aid of its scale functionality. Normalization of the dataset set the mean of the variables to zero and their standard deviation values were used for comparison.

The normalized dataset were exposed to exploratory factor analysis (EFA). The principal component method was adopted in extracting the factors, in addition to the use of rotated factor solution as the display format to ensure sufficient correlation of the factors. Factors were extracted based on ten fixed latent factors at 50 iterations. The nine latent variables originally present in PCSC were extracted. However, only 36 manifest variables loaded distinctly on one latent variable at a time. So, the 5 manifest variables in PCSC whose coefficients were either below the set limit of .35 or that loaded on more than one latent variable at a time were deleted and as such were not included in the main analysis.

Model specification, evaluation, identification and modification. Model specification adopted involved the use of symbols, arrows and curves to represent the manifest, latent and error terms; directional effects and correlation/covariance matrix of the exogenous variables. The measurement model was specified. Both the means and the variances of each construct in the model were respectively constrained to zero and one respectively. The mean of each latent construct was constrained to zero as a necessary pre-condition for Bayesian analysis of the model's structures. Also, the variance of each latent construct was constrained to one for three reasons: (i) so that each weight of the indicator variables could be estimated. (ii) To avoid outrageous weights, with coefficients above 1.00. (iii) To avoid very low correlation coefficients occasioned by outrageous weights. Model constraints including configural invariance (the same factor loading in each sub-group model was equated to 1), metric invariance (configural invariance + setting all the other factor loadings in each subgroup to be equal) and structural invariance (metric invariance + setting all the covariance curves to be equal across the subgroups) testing were done to determine the equivalence of urban and rural sub-group models in producing a unified model. The data were loaded and the models' parameters were initially estimated using maximum likelihood (ML) to help diagnose estimation problems. The model did not run, despite that it was identified. An error message (waiting to accept a transition before beginning burn-in) cropped up. Arbuckle (2013) noted that such an error message usually cropped up when the maximum likelihood estimate was inadmissible as a result of having a check mark placed on admissibility test on the prior tab of the Bayesian structural equation modeling (SEM) options window. The problem was overcome by *unchecking* the admissibility test button, so that the computer programme could accept the first MCMC candidate. Then the model ran. So all the coefficients of the directional effects were reasonable (had linear coefficients ranging from -1 through 0 to 1). See appendixpage..... for the full posterior estimates. The final estimation of the measurement model was done using Bayesian MCMC algorithm, specifically Hamiltonian. Hamiltonian algorithm was chosen because it saved computational time relative to *random-walk* algorithm. During the MCMC sampling process of both the posterior parameter values and the MCMC sample-size, the prior distribution of each parameter was set to normal probability distribution. The reason was to reduce the model parameters' computation complexities as the data collected were tested to be multivariate normal. The default value of

the convergence criteria (1.002) was maintained. However, the sampling process was pulsed when the parameter values appeared to be stable.

Results

The results are presented according to the formulated research questions and hypotheses that guided the study.

Research question 1 (RQ1) sought information on the weights of the indicators of each sub-construct in the model. The data presented in Table 1 were used to answer RQ1.

Table1: Regression weights with standard error (SE) of the indicators of each construct in the model

REGRESSION WEIGHTS	MEAN	SE	REGRESSION WEIGHTS	MEAN	SE
INVOLV1<--INV	0.25	0.00	TASKOR4<--TAS	0.21	0.00
INVOLV2<--INV	0.37	0.00	COMPET1<--COM	0.39	0.00
INVOLV4<--INV	0.32	0.00	COMPET2<--COM	0.27	0.00
SSINTER1<--SSI	0.16	0.00	COMPET3<--COM	0.26	0.00
SSINTER2<--SSI	0.28	0.00	COMPET4<--COM	0.17	0.00
SSINTER3<--SSI	0.43	0.00	COMPET5<--COM	0.26	0.00
SSINTER5<--SSI	0.17	0.00	ODORGA2<--ODO	0.15	0.00
SSINTER6<--SSI	0.21	0.00	ODORGA3<--ODO	0.23	0.00
TSINTER1<--TSI	0.35	0.00	ODORGA4<--ODO	0.20	0.00
TSINTER3<--TSI	0.14	0.00	ODORGA5<--ODO	0.32	0.00
TSINTER4<--TSI	0.15	0.00	TCONTRL1<--TCO	0.18	0.00
SATISFA1<--SAT	0.29	0.00	TCONTRL2<--TCO	0.23	0.00
SATISFA2<--SAT	0.08	0.00	TCONTRL4<--TCO	0.15	0.00
SATISFA3<--SAT	0.30	0.00	INNOVAT1<--INN	0.28	0.00
SATISFA4<--SAT	0.27	0.00	INNOVAT3<--INN	0.40	0.00
TASKOR1<--TAS	0.23	0.00	INNOVAT4<--INN	0.22	0.00
TASKOR2<--TAS	0.14	0.00	WAEC<--PHYACHV	0.24	0.00
TASKOR3<--TAS	0.19	0.00	NECO<--PHYACHV	0.13	0.00

From Table 1, Involvement (INVOLV) 1, 2 and 4 loaded on the construct involvement (INV) with the following weights: .25, .37 and .32 respectively. Student-student interaction (SSINTER) 1, 2, 3, 5 and 6 loaded on student-student interaction (SSI) with the following weights: .16, .28, .43, .17 and .21 respectively. Teacher-student interaction (TSINTER) 1, 3 and 4 loaded on the construct: Teacher-student interaction (TSI) respectively. The indicators

of satisfaction (SAT) including SATISFA1, SATISFA2, SATISFA3 and SATISFA4 loaded on it with the following weights: .29, .08, .30 and .27 respectively. The indicators of task orientation (TAS) including TASKOR1, TASKOR2, TASKOR3 and TSAKOR4 had .23, .14, .19 and .21 as their weights on the construct respectively. The indicators of competition (COM) including COMPET1, COMPET2, COMPET3, COMPET4 and COMPET5 had their weights as .39, .27, .26, .17 and .26 respectively. Other sub-constructs in the confirmatory factor analysis model including order and organization (ODO) had its indicator weights including ODORGA2, ODORGA3, ODORGA4, ODORGA5, loaded as .15, .23, .20 and .32 respectively. While the manifest variables due to the sub-construct- teachers' control including TCONTRL1, TCONTRL2 and TCONTRL4 had their weights as .18, .23 and .15 respectively. The indicators of innovations (INN) including INNOVAT1, INNOVAT3 and INNOVAT4 loaded onto the underlying construct with the following weights: .28, .40 and .22 respectively. However, WAEC and NECO loaded on Physics achievement (PHYACHV) with the following weights: .22 and .24 respectively.

Research question 2 (RQ2) sought information on the correlation coefficients among the clusters of Physics classroom environment scale and Physics achievement in the model. The trace plots in figure 2 were used to answer RQ2.

Table 2: Correlation coefficients of the sub-constructs of Physics classroom environment.

	INN	TCO	ODO	COM	TAS	SAT	TSI	SSI	INV	PHYACHV
INN	1.000									
TCO	.590	1.000								
ODO	.550	.400	1.000							
COM	.430	.590	.510	1.000						
TAS	.520	.420	.390	.180	1.000					
SAT	.650	.410	.440	.540	.550	1.000				
TSI	.550	.470	.130	.460	.290	.420	1.000			
SSI	.710	.630	.060	.150	.330	.380	.380	1.000		
INV	.600	.720	.340	.570	.440	.510	.640	.680	1.000	
PHYACHV	.100	.320	.450	.440	.260	.340	.590	.610	.400	1.000

From Table 2, the bi-variate correlation between the sub-constructs of Physics classroom environment and Physics achievement in the model are reported as follows. Innovations (INN) had a coefficient of .59 with teacher's control (TCO), .55 with order and organization (ODO), .43 with communications (COM). Task orientation (TAS) had .52, satisfaction (SAT) had .65, teacher-student interaction (TSI) had .55, student-student interaction (SSI) had .71, involvement (INV) had .60 while Physics achievement (PHYACHV) had a correlation coefficient of .10 with INN. TCO had a coefficient of .4, .59, .42, .41, .47, .63, .72 and .32 with ODO, COM, TAS, SAT, TSI, SSI, INV and PHYACHV respectively. The correlation between ODO and COM, SAT, TSI, SSI, INV and PHYACHV included .51, .39, .44, .13, .06, .34 and .45 respectively. COM correlated with TAS, SAT, TSI, SSI, INV and PHYACHV with coefficients of .18, .54, .46, .15, .57 and .44 respectively. TAS had correlation coefficients of .55, .29, .33, .44 and .26 with SAT, TSI, SSI, INV and PHYACHV respectively. Also, SAT had coefficients of .42, .38, .51 and .34 with TSI, SSI, INV and PHYACHV respectively. TSI correlated with SSI, INV and PHYACHV and had .38, .64 and .59 as indices of correlation respectively. SSI had correlation of .68 and .61 with INV and PHYACHV respectively. In addition, the coefficient of correlation between INV and PHYACHV was .40.

Hypothesis 1 (H₀₁) sought to determine if any significant difference existed between the constrained and unconstrained models' deviance information criterion values based on students' gender. The data presented in Table 3 were used to test the hypothesis.

Table 3: Deviance information criterion (DIC) values of the constrained and unconstrained urban and rural sub-models tested at 95% confidence interval.

Location	Constrained Model's DIC	Unconstrained Model's DIC	Differences		Interpretation
			(a-d)	(c-b)	
Urban	264.54 (a)	101.01(b)		86.32	Not significantly different
Rural	187.33 (c)	178.8 (d)	85.74		

From Table 3, the sub-model for urban Physics students had a constrained model's DIC value of 264.54 while the rural sub-model had a DIC value of 187.33. For the unconstrained models, the urban sub-model had a DIC value of 101.01 whereas the rural sub-model had a DIC value of 178.80. The difference between constrained and unconstrained urban and rural models is 85.74, while the difference between the constrained and unconstrained rural and urban sub-models is 68.32. This implies that the urban and rural sub-models are equivalent.

Discussion of Findings

The results in Table 1 provided answer to the research question 1. Out of the nine sub-scales of Physics classroom environment questionnaire, the highest positive direct effect on the criterion variable was the path from student-student interaction. The next sub-scales in decreasing magnitude of meaningful paths included order and organization, teacher-student interaction and competitions. However, teacher satisfaction and task orientation had weak and positive direct effects on the criterion variable. Other variables including teachers' control, involvement and innovations had negative direct effects on the criterion variable. Teachers' control had the highest negative direct effect on Physics achievement. Involvement followed and innovation was the least. The weighted path coefficient from involvement and teachers' control to Physics achievement varied slightly by -.01 each. All the other weighted paths did not vary from the estimated path coefficients. The implication for the result is that MCMC based approach to parameter estimation provides reliable estimates due to the fact that it records very low standard error of estimates. From the Table 1, it becomes apparent that the level of students satisfaction and task orientation in the population is low and needs improvement. Other clusters of Physics classroom environment scale that needs serious intervention included the ones with negative direct effects in the model. They are paths from involvement, teachers' control and innovation. However the statuesque should be maintained for student-student interaction, teacher-student interaction, communication and order and organization within the Physics classroom.

From Table 2, the difference between the constrained male and unconstrained female models was positive. Also, the difference between the constrained female and unconstrained male models was positive. This shows that the sub-group models were equivalent. This result is in line to the recommendation made by Zang, Hamagami, Wang,

Nesselroade, and Grimm (2007). The authors noted that sub-group models' equivalence was achieved when the DIC of the constrained model was higher than the DIC value of the unconstrained model.

Recommendation

Based on the findings of this study the following recommendation is made. Researchers in Physics Education should avail themselves of the opportunity to use Physics classroom environment rating scale to collect data, since it has been adjudged psychometrically fit to use.

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Appendix A: Full Posterior Summaries of Estimates.

	Mean	S.E.	S.D.	C.S.	Median	95% Lower bound	95% Upper bound	Skewn ess	Kurtosis	Min	Max	Name
Regression weights												
INVOLV1<--INV	0.25	0.00	0.02	1.01	0.25	0.21	0.29	-0.03	-0.12	0.19	0.32	
INVOLV2<--INV	0.37	0.00	0.02	1.01	0.37	0.33	0.42	0.05	-0.01	0.29	0.46	
INVOLV4<--INV	0.32	0.00	0.02	1.01	0.32	0.28	0.35	0.02	-0.30	0.26	0.37	
SSINTER1<--SSI	0.16	0.00	0.01	1.01	0.16	0.14	0.19	0.12	-0.01	0.12	0.21	
SSINTER2<--SSI	0.28	0.00	0.02	1.01	0.28	0.25	0.32	0.41	0.15	0.23	0.35	
SSINTER3<--SSI	0.43	0.00	0.04	1.01	0.43	0.36	0.52	0.17	-0.11	0.31	0.56	
SSINTER5<--SSI	0.17	0.00	0.01	1.01	0.17	0.14	0.19	0.15	-0.21	0.13	0.21	
SSINTER6<--SSI	0.21	0.00	0.02	1.01	0.21	0.18	0.24	0.02	-0.16	0.16	0.27	
TSINTER1<--TSI	0.35	0.00	0.04	1.00	0.35	0.28	0.43	0.11	0.10	0.21	0.49	
TSINTER3<--TSI	0.14	0.00	0.01	1.01	0.14	0.11	0.17	0.16	0.05	0.10	0.19	
TSINTER4<--TSI	0.15	0.00	0.02	1.01	0.15	0.13	0.18	0.10	-0.36	0.11	0.20	
SATISFA1<--SAT	0.29	0.00	0.04	1.00	0.29	0.22	0.36	0.15	0.57	0.17	0.45	
SATISFA2<--SAT	0.08	0.00	0.01	1.01	0.08	0.06	0.11	-0.01	0.16	0.04	0.12	
SATISFA3<--SAT	0.30	0.00	0.03	1.00	0.30	0.24	0.36	0.11	-0.18	0.21	0.39	
SATISFA4<--SAT	0.27	0.00	0.03	1.00	0.27	0.20	0.33	-0.07	0.07	0.15	0.37	
TASKOR1<--TAS	0.23	0.00	0.02	1.00	0.23	0.18	0.27	-0.03	-0.07	0.16	0.29	
TASKOR2<--TAS	0.14	0.00	0.01	1.00	0.14	0.12	0.17	0.21	0.11	0.10	0.20	
TASKOR3<--TAS	0.19	0.00	0.02	1.01	0.19	0.17	0.22	0.00	0.29	0.13	0.24	
TASKOR4<--TAS	0.21	0.00	0.02	1.01	0.21	0.18	0.25	0.01	0.32	0.15	0.27	
COMPET1<--COM	0.39	0.00	0.03	1.00	0.39	0.33	0.44	0.12	-0.20	0.30	0.48	
COMPET2<--COM	0.27	0.00	0.02	1.01	0.27	0.23	0.31	0.20	0.32	0.19	0.35	
COMPET3<--COM	0.26	0.00	0.02	1.01	0.25	0.23	0.30	0.44	-0.04	0.21	0.32	
COMPET4<--COM	0.17	0.00	0.02	1.01	0.17	0.14	0.21	0.16	0.72	0.11	0.24	
COMPET5<--COM	0.26	0.00	0.02	1.00	0.26	0.22	0.31	0.11	-0.02	0.18	0.35	
ODORGA2<--ODO	0.15	0.00	0.02	1.00	0.15	0.11	0.18	-0.13	-0.02	0.09	0.21	
ODORGA3<--ODO	0.23	0.00	0.02	1.01	0.23	0.20	0.26	-0.21	-0.05	0.17	0.28	

ODORGA4<--ODO	0.20	0.00	0.02	1.01	0.20	0.17	0.24	0.09	-0.43	0.15	0.26
ODORGA5<--ODO	0.32	0.00	0.02	1.01	0.32	0.29	0.37	0.28	-0.11	0.27	0.39
TCONTRL1<--TCO	0.18	0.00	0.02	1.00	0.18	0.13	0.23	0.14	-0.10	0.11	0.27
TCONTRL2<--TCO	0.23	0.00	0.02	1.01	0.23	0.19	0.27	0.28	0.07	0.17	0.32
TCONTRL4<--TCO	0.15	0.00	0.02	1.01	0.15	0.12	0.19	0.10	-0.05	0.10	0.23
INNOVAT1<--INN	0.28	0.00	0.02	1.01	0.28	0.24	0.33	0.18	-0.33	0.22	0.35
INNOVAT3<--INN	0.40	0.00	0.02	1.01	0.40	0.35	0.45	0.04	-0.23	0.32	0.48
INNOVAT4<--INN	0.22	0.00	0.02	1.01	0.22	0.18	0.25	0.09	-0.35	0.16	0.28
WAEC<-- PHYACHV	0.24	0.00	0.02	1.01	0.24	0.20	0.29	0.77	1.02	0.18	0.33
NECO<--PHYACHV	0.13	0.00	0.01	1.01	0.13	0.11	0.15	0.81	1.47	0.10	0.17

Intercepts

INVOLV1	2.48	0.00	0.02	1.01	2.48	2.44	2.52	-0.12	0.01	2.41	2.55
INVOLV2	1.54	0.00	0.03	1.00	1.55	1.48	1.60	-0.15	-0.06	1.45	1.65
INVOLV4	1.56	0.00	0.02	1.01	1.56	1.51	1.60	-0.16	-0.05	1.48	1.64
SSINTER1	1.54	0.00	0.02	1.00	1.54	1.50	1.57	-0.04	-0.05	1.47	1.59
SSINTER2	1.59	0.00	0.03	1.00	1.59	1.54	1.64	0.16	-0.10	1.51	1.67
SSINTER3	1.60	0.00	0.05	1.01	1.60	1.51	1.69	-0.01	-0.13	1.44	1.75
SSINTER5	1.52	0.00	0.02	1.01	1.52	1.49	1.55	-0.10	0.20	1.46	1.58
SSINTER6	2.61	0.00	0.02	1.01	2.61	2.56	2.65	-0.08	0.18	2.53	2.68
TSINTER1	2.41	0.00	0.04	1.00	2.41	2.33	2.50	0.02	0.13	2.25	2.56
TSINTER3	2.51	0.00	0.02	1.00	2.51	2.48	2.55	0.22	0.04	2.45	2.57
TSINTER4	2.42	0.00	0.02	1.01	2.42	2.39	2.45	-0.02	-0.37	2.37	2.47
SATISFA1	2.67	0.00	0.04	1.01	2.67	2.60	2.74	0.07	-0.21	2.56	2.78
SATISFA2	2.50	0.00	0.01	1.00	2.50	2.48	2.52	-0.06	0.02	2.46	2.54
SATISFA3	2.64	0.00	0.03	1.01	2.64	2.58	2.70	0.05	-0.16	2.53	2.73
SATISFA4	2.51	0.00	0.03	1.00	2.51	2.44	2.57	-0.15	-0.11	2.41	2.65
TASKOR1	2.52	0.00	0.02	1.00	2.52	2.47	2.57	0.15	-0.32	2.45	2.60
TASKOR2	2.43	0.00	0.02	1.00	2.43	2.40	2.46	0.00	0.46	2.36	2.48
TASKOR3	2.48	0.00	0.02	1.01	2.48	2.45	2.51	-0.08	0.66	2.42	2.54
TASKOR4	2.63	0.00	0.02	1.01	2.63	2.59	2.67	-0.08	-0.42	2.57	2.69
COMPET1	2.61	0.01	0.04	1.01	2.61	2.54	2.69	0.07	-0.03	2.48	2.74
COMPET2	2.48	0.00	0.02	1.01	2.48	2.43	2.52	0.06	0.31	2.39	2.57
COMPET3	2.43	0.00	0.02	1.01	2.43	2.39	2.47	-0.16	-0.07	2.34	2.49
COMPET4	2.49	0.00	0.02	1.01	2.49	2.46	2.52	0.00	-0.36	2.43	2.54
COMPET5	2.67	0.00	0.03	1.00	2.67	2.62	2.72	-0.05	0.05	2.59	2.76
ODORGA2	2.62	0.00	0.02	1.01	2.62	2.58	2.66	-0.05	-0.25	2.55	2.68

ODORGA3	2.63	0.00	0.02	1.00	2.63	2.59	2.66	-0.02	-0.50	2.57	2.68
ODORGA4	2.41	0.00	0.02	1.01	2.41	2.37	2.44	0.00	-0.32	2.35	2.47
ODORGA5	2.41	0.00	0.03	1.01	2.41	2.36	2.45	-0.05	-0.41	2.32	2.48
TCONTRL1	2.65	0.00	0.03	1.00	2.65	2.60	2.70	0.13	-0.12	2.58	2.76
TCONTRL2	2.61	0.00	0.02	1.01	2.61	2.57	2.66	0.18	-0.36	2.53	2.69
TCONTRL4	2.47	0.00	0.02	1.00	2.47	2.43	2.51	-0.03	-0.01	2.40	2.54
INNOVAT1	2.48	0.00	0.03	1.00	2.48	2.42	2.52	-0.11	-0.28	2.39	2.56
INNOVAT3	2.51	0.00	0.03	1.00	2.51	2.44	2.57	-0.11	0.01	2.40	2.63
INNOVAT4	2.48	0.00	0.02	1.00	2.48	2.43	2.52	-0.28	-0.07	2.41	2.54
WAEC	2.47	0.00	0.02	1.01	2.48	2.43	2.52	-0.18	-0.20	2.39	2.54
NECO	2.30	0.00	0.01	1.01	2.30	2.28	2.32	-0.09	-0.49	2.27	2.34

Covariances

INV<->SSI	0.68	0.01	0.05	1.01	0.68	0.58	0.77	-0.18	-0.02	0.50	0.81
INV<->TSI	0.64	0.01	0.06	1.01	0.64	0.50	0.75	-0.32	0.10	0.41	0.84
INV<->SAT	0.51	0.01	0.07	1.00	0.51	0.37	0.64	-0.08	-0.22	0.26	0.70
INV<->TAS	0.43	0.01	0.06	1.00	0.43	0.29	0.54	-0.28	-0.12	0.20	0.61
INV<->COM	0.57	0.01	0.05	1.01	0.57	0.47	0.66	-0.13	-0.32	0.42	0.73
INV<->ODO	0.34	0.01	0.07	1.01	0.34	0.21	0.48	0.04	-0.25	0.14	0.56
INV<->TCO	0.72	0.00	0.05	1.01	0.72	0.62	0.81	0.05	-0.26	0.57	0.87
INV<->INN	0.60	0.01	0.05	1.01	0.60	0.48	0.70	-0.12	-0.05	0.41	0.79
SSI<->TSI	0.38	0.01	0.08	1.01	0.38	0.21	0.53	-0.32	0.04	0.04	0.61
SSI<->SAT	0.38	0.01	0.08	1.00	0.39	0.22	0.53	-0.26	0.17	0.11	0.64
SSI<->TAS	0.33	0.01	0.06	1.00	0.33	0.20	0.46	-0.12	-0.13	0.12	0.53
SSI<->COM	0.15	0.01	0.08	1.01	0.15	0.00	0.31	0.00	0.25	-0.13	0.40
SSI<->ODO	0.06	0.01	0.07	1.01	0.06	-0.08	0.21	0.03	-0.15	-0.19	0.31
SSI<->TCO	0.63	0.01	0.06	1.01	0.63	0.50	0.75	-0.06	-0.18	0.43	0.82
SSI<->INN	0.71	0.01	0.05	1.01	0.71	0.62	0.79	-0.18	-0.13	0.55	0.85
TSI<->SAT	0.42	0.01	0.09	1.01	0.42	0.25	0.60	0.00	0.10	0.11	0.69
TSI<->TAS	0.29	0.01	0.08	1.00	0.29	0.12	0.45	-0.12	-0.03	0.03	0.53
TSI<->COM	0.46	0.01	0.07	1.00	0.46	0.32	0.59	-0.08	0.23	0.19	0.67
TSI<->ODO	0.13	0.01	0.09	1.01	0.13	-0.06	0.31	0.01	0.09	-0.21	0.44
TSI<->TCO	0.47	0.01	0.09	1.01	0.48	0.28	0.64	-0.20	-0.18	0.20	0.75
TSI<->INN	0.55	0.01	0.08	1.01	0.55	0.35	0.68	-0.64	0.65	0.24	0.76
SAT<->TAS	0.55	0.01	0.07	1.00	0.55	0.42	0.69	-0.04	-0.04	0.29	0.78
SAT<->COM	0.54	0.01	0.07	1.01	0.54	0.38	0.67	-0.37	0.31	0.27	0.76
SAT<->ODO	0.44	0.01	0.08	1.01	0.44	0.27	0.58	-0.16	-0.42	0.15	0.65
SAT<->TCO	0.41	0.01	0.10	1.01	0.41	0.19	0.59	-0.19	-0.25	0.06	0.70

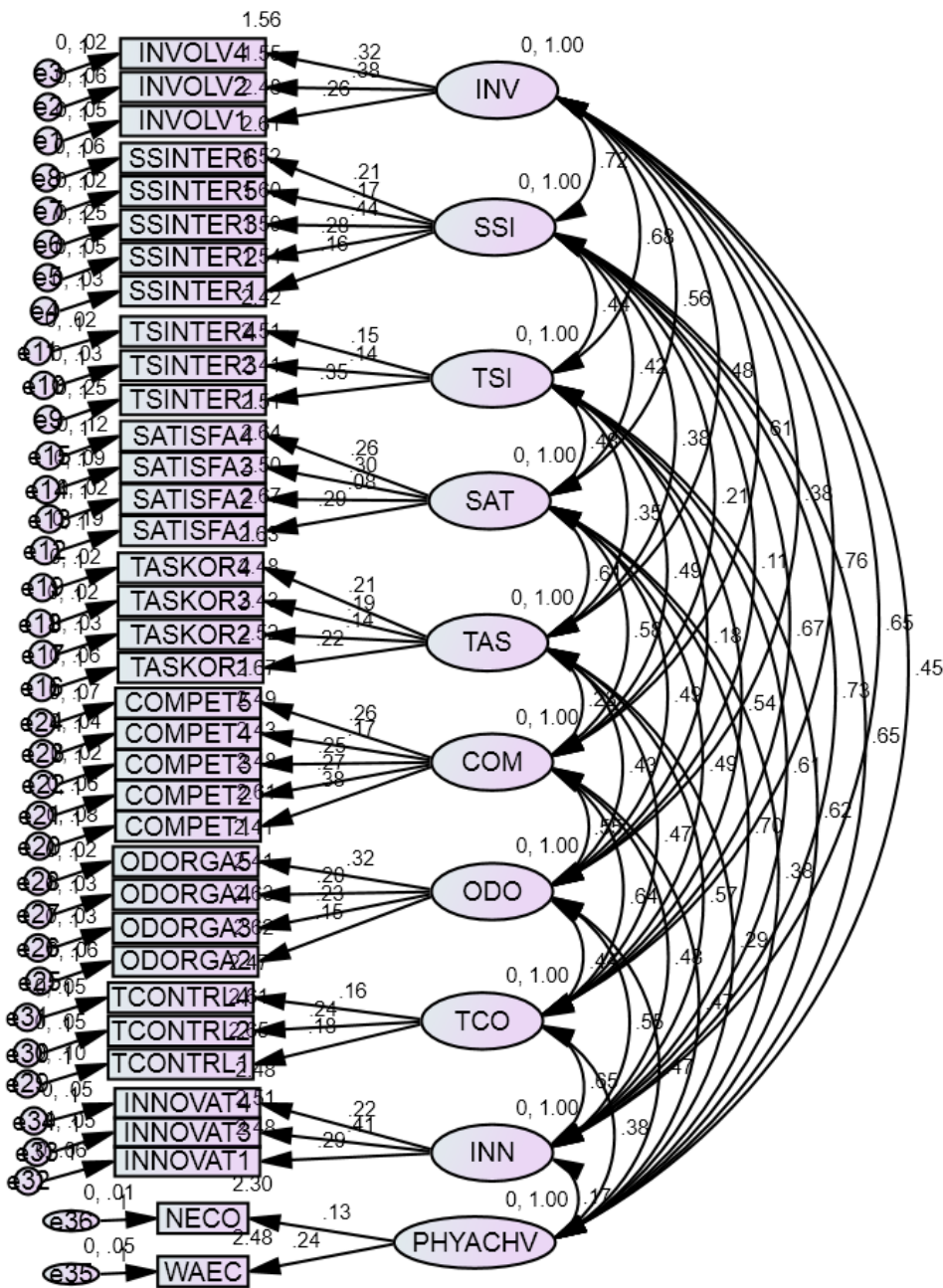
SAT<->INN	0.65	0.01	0.07	1.01	0.66	0.52	0.77	-0.29	-0.14	0.42	0.85
TAS<->COM	0.18	0.01	0.07	1.01	0.18	0.03	0.32	-0.19	0.19	-0.11	0.42
TAS<->ODO	0.39	0.01	0.08	1.00	0.39	0.23	0.55	-0.03	-0.07	0.12	0.62
TAS<->TCO	0.42	0.01	0.08	1.00	0.43	0.26	0.58	-0.15	-0.25	0.14	0.68
TAS<->INN	0.52	0.01	0.06	1.00	0.52	0.40	0.64	0.02	-0.30	0.33	0.71
COM<->ODO	0.51	0.01	0.06	1.01	0.51	0.38	0.63	-0.18	-0.13	0.30	0.68
COM<->TCO	0.59	0.01	0.07	1.01	0.59	0.44	0.72	-0.24	0.11	0.32	0.81
COM<->INN	0.43	0.01	0.07	1.01	0.43	0.27	0.55	-0.35	0.09	0.19	0.62
ODO<->TCO	0.40	0.01	0.08	1.01	0.40	0.25	0.54	-0.22	0.42	0.10	0.64
ODO<->INN	0.50	0.01	0.06	1.01	0.51	0.37	0.62	-0.40	0.59	0.24	0.70
TCO<->INN	0.59	0.01	0.07	1.01	0.59	0.45	0.72	-0.46	0.93	0.24	0.79
PHYACHV<->INV	0.40	0.01	0.08	1.01	0.41	0.23	0.55	-0.29	-0.25	0.16	0.62
PHYACHV<->SSI	0.61	0.01	0.06	1.01	0.62	0.47	0.72	-0.75	1.29	0.34	0.77
PHYACHV<->TSI	0.59	0.01	0.07	1.01	0.59	0.43	0.72	-0.35	0.11	0.33	0.81
PHYACHV<->SAT	0.34	0.01	0.09	1.01	0.34	0.16	0.51	-0.07	-0.01	0.04	0.67
PHYACHV<->TAS	0.26	0.01	0.08	1.01	0.25	0.10	0.42	0.10	-0.01	0.01	0.56
PHYACHV<->COM	0.44	0.01	0.07	1.01	0.44	0.30	0.58	-0.12	-0.24	0.20	0.65
PHYACHV<->ODO	0.45	0.01	0.07	1.01	0.45	0.31	0.59	-0.11	-0.14	0.20	0.67
PHYACHV<->TCO	0.32	0.01	0.10	1.01	0.33	0.11	0.51	-0.39	0.40	-0.09	0.62
PHYACHV<->INN	0.10	0.01	0.09	1.01	0.10	-0.08	0.26	-0.15	-0.22	-0.18	0.35

Variances

e1	0.05	0.00	0.01	1.01	0.05	0.04	0.06	0.28	-0.11	0.04	0.07
e2	0.06	0.00	0.01	1.00	0.06	0.05	0.08	0.55	0.79	0.04	0.09
e4	0.03	0.00	0.00	1.00	0.03	0.03	0.04	0.28	0.10	0.02	0.04
e6	0.26	0.00	0.02	1.00	0.25	0.22	0.30	0.19	-0.04	0.18	0.33
e7	0.03	0.00	0.00	1.00	0.03	0.02	0.03	0.32	-0.18	0.02	0.03
e8	0.06	0.00	0.01	1.01	0.06	0.05	0.07	0.39	0.08	0.04	0.08
e10	0.03	0.00	0.00	1.01	0.03	0.03	0.04	0.53	0.36	0.02	0.05
e11	0.03	0.00	0.00	1.01	0.03	0.02	0.03	0.19	-0.35	0.02	0.03
e12	0.19	0.00	0.02	1.01	0.19	0.15	0.24	0.42	0.06	0.13	0.27
e13	0.02	0.00	0.00	1.01	0.02	0.02	0.02	0.38	-0.37	0.01	0.03
e15	0.13	0.00	0.02	1.01	0.13	0.10	0.16	0.32	0.19	0.08	0.19
e16	0.06	0.00	0.01	1.01	0.06	0.05	0.08	0.40	-0.21	0.04	0.09
e17	0.03	0.00	0.00	1.01	0.03	0.02	0.04	0.34	-0.01	0.02	0.04
e18	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.32	0.13	0.01	0.03
e19	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.10	-0.04	0.01	0.04
e20	0.08	0.00	0.01	1.01	0.08	0.07	0.11	0.36	-0.09	0.06	0.11

e21	0.06	0.00	0.01	1.01	0.06	0.05	0.07	0.18	-0.01	0.04	0.08
e22	0.02	0.00	0.00	1.00	0.02	0.02	0.02	0.19	-0.09	0.01	0.03
e23	0.04	0.00	0.00	1.01	0.04	0.03	0.05	0.23	-0.30	0.03	0.05
e24	0.07	0.00	0.01	1.00	0.07	0.06	0.08	0.37	0.18	0.05	0.10
e25	0.06	0.00	0.01	1.01	0.06	0.05	0.07	0.33	0.47	0.04	0.08
e26	0.03	0.00	0.00	1.00	0.03	0.02	0.03	0.41	0.14	0.02	0.04
e27	0.03	0.00	0.00	1.01	0.03	0.02	0.04	0.47	0.19	0.02	0.04
e28	0.02	0.00	0.00	1.00	0.02	0.02	0.03	0.39	-0.09	0.01	0.03
e29	0.11	0.00	0.01	1.00	0.10	0.09	0.13	0.56	0.31	0.08	0.15
e30	0.05	0.00	0.01	1.00	0.05	0.04	0.07	0.22	-0.12	0.04	0.08
e31	0.05	0.00	0.01	1.01	0.05	0.04	0.06	1.15	2.68	0.04	0.08
e32	0.06	0.00	0.01	1.00	0.06	0.05	0.07	0.15	-0.15	0.05	0.08
e33	0.06	0.00	0.01	1.00	0.06	0.05	0.07	0.18	0.23	0.04	0.07
e34	0.05	0.00	0.00	1.00	0.05	0.05	0.06	0.32	0.12	0.04	0.07
e3	0.02	0.00	0.00	1.01	0.02	0.01	0.03	-0.09	0.00	0.01	0.03
e9	0.26	0.00	0.02	1.00	0.26	0.22	0.30	0.18	-0.11	0.20	0.33
e5	0.05	0.00	0.00	1.00	0.05	0.04	0.06	0.42	0.21	0.04	0.06
e14	0.10	0.00	0.01	1.00	0.10	0.07	0.13	0.28	0.31	0.06	0.14
e35	0.05	0.00	0.01	1.01	0.05	0.04	0.06	0.60	0.39	0.03	0.07
e36	0.01	0.00	0.00	1.01	0.01	0.01	0.01	0.54	0.58	0.01	0.01

Appendix B: Maximum Likelihood Estimation Trial Result



KEY: INV-involvement, SSI-student-student interaction, TSI-teacher-student interaction, SAT-satisfaction, TAS-task orientation, COM-competition, ODO-order and organization, TCO-teachers' control, INN-innovations and PHYACHV-physics achievement.