

UNSTEADY FREE MHD CONVECTION FLOW PAST A VERTICAL POROUS PLATE IN SLIP-FLOW REGIME UNDER FLUCTUATING THERMAL AND MASS DIFFUSION

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ABSTRACT

Free convection flow involving coupled heat and mass transfer occurs frequently in several areas of chemical engineering and manufacturing process areas. Thermal radiation has become a significant branch of engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical and solar power engineering. The flow of Newtonian electrically - conducting fluids is also of great interest in high speed aerodynamics, astronautically plasma flows, MHD boundary layer control, MHD accelerator technologies and the applications are many from the view of science and technology. Free convection flow involving heat transfer occurs frequently in an environment where difference between land and air temperature can give rise to complicated flow patterns. Many processes in engineering and manufacturing sectors, the product production occurs at high level of temperature and acknowledges radiation heat transfer for the design of pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are few such examples in the engineering areas. This paper mainly deals with unsteady MHD free convective flow past vertical porous plate in slip-flow regime under fluctuating thermal and mass diffusion.

Keywords- MHD convection flow, Vertical porous plate, slip-flow regime, thermal and mass diffusion

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Introduction-

The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Free convective flow past vertical plate has been studied extensively and more intensively by Ostrach [1], [2], [3]. The transient free convection from a vertical flat plate has been examined by Siegel [4] while, the free convective heat transfer on a vertical semi-infinite plate has been investigated by Berezovsky et al. [5]. The laminar free convection from a vertical plate has been studied by Martynenko *et al.* [6].

The natural convection flows adjacent to both vertical and horizontal surface, which result from the combined buoyancy effects of thermal and mass diffusion, was first investigated by Gebhart and Pera [7] and Pera and Gebhart [8] while, Soundalgekar [9] investigated the situation of unsteady free convective flows wherein the effects of viscous dissipation on the flow past an infinite vertical porous plate was highlighted. In the course of analysis, it was assumed that the plate temperature oscillates in such a way that its amplitude is small. Later, Chen *et al* [10] studied the combined effect of buoyancy forces from thermal and mass diffusion on forced convection. Subsequently, Ramanaiah and Malarvizhi [11] investigated the free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient. Vighnesam and Soundalgekar [12] investigated the combined free and forced convection flow of water from a vertical plate with variable temperature. The transient free convection flow past an infinite vertical plate with periodic temperature variation was studied by Das *et al* [13]. In recent times, Hossain *et al.* [14] studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate.

The present analysis discussed herein is based on the study as referred and suggested by Soundalgekar and Wavre [15], [16]. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particles at the surface have a finite tangential velocity; it "slips" along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Under the assumptions made by Sharma and Chaudhary [17] and Sharma [18] have also discussed the free convection flow past a vertical plate in slip-flow

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regime. They had quoted several applications that occur in several engineering applications wherein heat and mass transfer occurs at high degree of temperature differences. Therefore in the fitness of the industrial and scientific applications and to be more realistic the effect of periodic heat and mass transfer on unsteady free convection flow past a vertical flat porous plate under the influence of applied transverse magnetic field has been examined. The slip flow regime it is assumed that the suction velocity oscillates in time about a non-zero constant mean because in actual practice temperature, species concentration and suction velocity may not always be uniform.

Nomenclature-

Suction parameter (A), Dimensionless species concentration (C), Specific heat at constant pressure (C_p), Species concentration (C^*), Concentration at the wall (C^*_w), Concentration in free stream (C^*_∞), Molecular diffusivity of the species (D), Gravity (g), Modified Grashof number (G_c), Grashof number (G_r), Rarefaction parameter (h), Thermal conductivity (k), Dimensionless permeability parameter (K), Permeability parameter (K^*), Constant (L^*), Magnetic intensity (M), Prandtl number (P_r), Heat flux at the wall (q^*_w), Schmidt number, (S_c), Dimensionless time (t), Time (t^{*}), Temperature (T^{*}), Temperature of wall (T^*_w), Temperature of fluid in free stream (T^*_∞), Dimensionless velocity component (u), Velocity component (u^{*}), Suction velocity (V), Constant mean suction velocity (V^*_0)

Greek symbols

Thermal diffusivity (α), Coefficient of thermal expansion (β), Coefficient of thermal expansion with concentration (β_0), Amplitude (<<1) (ϵ), Viscosity (μ), Kinematic viscosity (ν), Dimensionless temperature (θ), Density (ρ), Stefan-Boltzmann constant (σ), Dimensionless shearing stress (τ), Shearing stress (τ^*), Dimensionless frequency (ω), Frequency (ω^*)

Mathematical formulation

Additionally, a periodic temperature and concentration when variable suction velocity distribution $[V^* = -V_0^* (1 + \epsilon A e^{i\omega^* t^*})]$ fluctuating with respect to time is considered. A rectangular Cartesian co-ordinate system with wall lying vertically in $x^* y^*$ - plane is employed.

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The x^* - axis is taken in vertically upward direction along the vertical porous plate and y^* - axis is taken normal to the plate.



Figure 1: Schematic representation of the problem

Since the plate is considered infinite in the x^* - direction, hence all physical quantities will be independent of x^* . Under these assumption, the physical variables are function of y^* and t^* only. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation) the problem can be governed by the following set of equations:

The boundary conditions of the problem are:

We now introduce the following non-dimensional quantities into equations (1) to (4)

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$$y = \frac{y^* V_0^*}{v}, \quad t = \frac{t^* V_0^{*2}}{4v}, \quad u = \frac{u^*}{V_0^*}, \qquad \omega = \frac{4v\omega^*}{V_0^{*2}}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*},$$
$$Gr = \frac{g\beta v(T_w^* - T_\infty^*)}{V_0^{*3}}, \quad Gc = \frac{g\beta^0 v(C_w^* - C_\infty^*)}{V_0^{*3}}, \quad P_r = \frac{\mu C_p}{k} = \frac{v\rho C_p}{k},$$
$$S_c = \frac{V}{D}, \qquad M = \frac{\sigma\beta_0^2 v}{\rho V_0^{*2}}, \qquad K = \frac{K^* V_0^{*2}}{v^2}, \quad h = \frac{V_0^* L^*}{v}$$

All physical variables are defined in nomenclature where (*) indicates the dimensional quantities. The subscript (∞) denotes the free stream condition. Then equations (1) to (3) reduce to the following non-dimensional form:

The boundary conditions to the problem in the dimensionless form are:

$$u = h\left(\frac{\partial u}{\partial y}\right), \theta = 1 + \varepsilon e^{iwt}, C = 1 + \varepsilon e^{iwt} \text{ at } y = 0 \text{ and } u \to 0, \theta \to 0, C \to o \text{ as } y \to \infty. (8)$$

Method of Solution:

Assuming the small amplitude oscillations ($\epsilon \ll 1$), we can represent the velocity *u*, temperature θ and concentration *C* near the plate as follows:

Substituting (9) to (11) in (5) to (7), equating the coefficients of harmonic and non harmonic terms, neglecting the coefficients of ε^2 , we get:

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The corresponding boundary conditions reduce to:

$$u_0 = h\left(\frac{\partial u_0}{\partial y}\right), u_1 = h\left(\frac{\partial u_1}{\partial y}\right), \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \text{ at } y = 0$$

 $u_0 = 0$, $u_1 = 0$ $\theta_0 = 0$, $\theta_1 = 0$, $C_0 = 0$, $C_1 = 0$ as $y \to \infty$(18) Where primes denote differentiation with respect to y. Solving the equations (12) to (17) under the boundary conditions (18) we get:

$$u_{1}(y) = m_{11}e^{-m_{12}y} - m_{13}e^{-m_{6}y} - m_{14}e^{-m_{9}y} + m_{15}e^{-m_{2}y} + m_{16}e^{-P_{r}y} + m_{17}e^{-S_{c}y} \dots (24)$$

Where $M_{1} = M + \frac{1}{\kappa}$, $m_{1} = -\frac{1}{1+h_{r}}[m_{3}(1+hP_{r}) + m_{4}(1+hS_{c}]]$,

$$m_{2} = \frac{1+\sqrt{1+4m_{1}}}{2}, \quad m_{3} = \frac{G_{r}}{p_{r}^{2} - P_{r} - M_{1}}, \quad m_{4} = \frac{G_{c}}{S_{c}^{2} - S_{c} - M_{1}},$$

$$m_{5} = 1 - \frac{4iAP_{r}}{w}, \quad m_{6} = \frac{P_{r} + \sqrt{P_{r}^{2} + iwP_{r}}}{2}, \quad m_{7} = \frac{4iAP_{r}}{w}, \quad m_{8} = 1 - \frac{4iAS_{c}}{w},$$

$$m_{9} = \frac{S_{c} + \sqrt{S_{c}^{2} + iwS_{c}}}{2}, \quad m_{10} = \frac{4iAS_{c}}{w}$$

$$m_{11} = \frac{1}{1 + hm_{12}} [m_{13}(1 + hm_{6}) + m_{14}(1 + hm_{9}) - m_{15}(1 + hm_{2}) - m_{16}(1 + hP_{r}) - m_{17}(1 + hS_{c})],$$

$$m_{12} = \frac{1 + \sqrt{1 + 4M_{1} + iw}}{2}, \qquad m_{13} = \frac{m_{5}G_{r}}{m_{6}^{2} - m_{6} - (M_{1} + iw/4)}, \qquad m_{14} = \frac{m_{8}G_{c}}{m_{9}^{2} - m_{9} - (M_{1} + iw/4)}, \qquad m_{15} = \frac{m_{1}m_{2}A}{m_{2}^{2} - m_{2} - (M_{1} + iw/4)},$$

$$m_{16} = \frac{m_3 A P_r - m_7 G_r}{P_r^2 - P_r - (M_1 + iw/4)}, \qquad m_{17} = \frac{m_4 A S_c - m_{10} G_c}{S_c^2 - S_c - (M_1 + iw/4)'},$$

The important characteristics of the problem are the skin-friction and heat transfer at the plate.

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Skin-friction: The dimensionless shearing stress on the surface of a body, due to a fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity.

Substituting equations (21) and (24) into (9) we can calculate the shearing stress component in dimensionless form as

Results and conclusions:

 The effect of Grashof number on velocity profiles is illustrated in figure 2. It is noticed that as Grashof number increases, the velocity of the fluid medium increases. Further, it is noticed in the boundary layer region the velocity increases and thereafter, it decreases. Also, far away from the plate, not much of significant effect of Grashof number is noticed.



Figure 2: Effect of Grashof number on velocity profiles

2. The variation of velocity with respect to modified Grashof number is noticed in figure 3. It is observed that as modified Grashof number increases, in general the velocity increases. As was seen in earlier case, the rise in velocity is noticed in the boundary layer region and thereafter, it decreases.

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Figure 3: Effect of modified Grashof number on velocity profiles

3. The contribution of Schmidt number on the velocity profiles is observed in figure 4. For a constant Schmidt number, the velocity increases initially in the boundary layer region and thereafter it decreases. In general, it can be seen that increase in the Schmidt number, causes the velocity of the fluid medium to decrease.



Figure 4: Effect of Schmidt number on velocity profiles

4. The influence of magnetic field on the velocity field is illustrated graphically in figure 5. It is observed that as the magnetic intensity is increasing, the fluid velocity is found to be decreasing. Therefore, it can be concluded that the magnetic field suppresses the motion of the fluid to a great extent.

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Figure 5: Effect of magnetic field on velocity

5. The contribution of the porosity of the fluid bed over the velocity is observed in figure 6. It is observed that, as the pore size of the fluid bed increases, the fluid velocity increases. Such a phenomenon is due to the percolation of fluid through such pores resulting in the increase of fluid velocity.



Figure 6: Effect of permeability parameter on velocity

6. Variation in the velocity of the fluid medium with respect to suction parameter is illustrated in figure 7. It is noticed that as the suction parameter is increased, the velocity of the fluid medium is found to be decreasing. Further, as we move far away from the plate, the contribution of such suction parameter is not that significant as expected.

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Figure 7: Effect of suction parameter on velocity

7. The influence of rarefaction parameter on the velocity field is illustrated in figure 8. It is seen that as the parameter is increased, the velocity of the fluid medium is found to be increasing. An interesting observation is that, on the boundary, the dispersion is found to be more distinguishable, while we move far away from the plate the effect seems to be consolidated and not much of variation is noticed.



Figure 8: Effect of rarefaction parameter on velocity

8. The influence of Prandtl number on the temperature field is illustrated in figure 9. It is observed that the prandtl number has significant contribution over the temperature field. From the illustrations it is seen that as the Prandtl number increases, the temperature decreases. Further, not much of significant contribution by the Prandtl number is noticed as we move far away from the plate.

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Figure 9: Effect of Prandtl number on temperature profiles

9. The influence of suction parameter on the temperature field is illustrated in figure 10. It is seen that as the suction parameter increases, the temperature field decreases. Further, far away from the plate, it is noticed that the parameter has no effect at all.



Figure 10: Effect of suction parameter on temperature

10. The influence of Schmidt number on concentration is illustrated in figure 11. It is noticed that as Schmidt number increases, the fluid concentration decreases. Such a decrease is found to be in the exponential order. It is observed that as Schmidt number increases, the concentration field profiles are found to be more parabolic in nature.

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Figure 11: Effect of Schmidt number on concentration

11. The influence of suction parameter on the concentration of the fluid medium is shown in figure 12. It is observed that as the suction parameter increases, the concentration of the fluid medium decreases. Further, as we move far away from the plate, it is observed that the effect of such suction parameter to be diminishing rapidly.



Figure 12: Effect of suction parameter on concentration

12. The combined influence of magnetic intensity and Grashof number over the skin-friction is noticed in figure 13. It is noticed that for a fixed magnetic intensity, as Grashof number increases, the skin-friction is found to be increasing. Further, when Grashof number is varied, it is seen that, the stir friction decreases as the magnetic intensity increases.

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Figure 13: Effect of Grashof number on skin-friction

13. The consolidated effected of Schmidt number with respect to the magnetic intensity on skin-friction is observed in figure 14. It is noticed that in general, as Schmidt number increases, the skin-friction decreases. Further, as the magnetic influence is increased over the system, the skin-friction decreases. Also, it is noticed that the influence of Schmidt number at higher values of the magnetic intensity reduces drastically.



Figure 14: Effect of Schmidt number on skin-friction

14. The combined influence of Schmidt number with respect to the rarefaction parameter is illustrated in figure 15. It is observed that as Schmidt number increases, the skin-friction reduces with respect to the rarefaction parameter. Also it is noticed that, both the parameters do not exhibit any significant influence on skin-friction at higher values.

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Figure 15: Effect of Schmidt number on skin-friction

15. The influence of Grashof number on skin-friction is noticed in figure 16. It is observed that as Grashof number increases, the skin-friction is found to be increasing with respect to the rarefaction parameter.



Figure 16: Effect of Grashof number on skin-friction

16. The combined influence of Grashof number and suction parameter on skinfriction is shown in figure 17. In general it is observed that for a constant suction parameter as Grashof number increases, the skin-friction is found to be increasing. Interestingly, the relation is found to be nearly linear.

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Figure 17: Effect of Grashof number on skin-friction

17. The influences of suction parameter and Schmidt number on skin-friction are noticed in figure 18. It is seen that as Schmidt number increases, for a constant suction parameter the skin-friction decreases. Further, the relation is found to inverse and is linear.



Figure 18: Effect of Schmidt number on skin-friction

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