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SEQUENTIAL TESTING PROCEDURE AND THEIR ROBUSTNESS STUDY FOR ERLANG DISTRIBUTION

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ABSTRACT

The sequential testing procedures are developed for testing the hypotheses regarding the shape and rate parameters of the Erlang distribution. Theoretical expression for the operating characteristics (OC) and average sample number (ASN) functions are derived for these parameters. The robustness of the SPRT'S in respect of OC and ASN functions are studied, when the distribution under study has undergone a change. The acceptance and rejection regions for H_0 *against* H_1 *are derived in case of rate parameter. The expressions of OC and ASN functions for the robustness of the SPRT in case of rate parameter, when the coefficient of variation is known are also derived and studied. Finally, the results are presented through Tables and Graphs, so that one can see the numerical evaluated departures in OC and ASN functions.*

Keywords: Erlang distribution, Sequential probability ratio test, Operating characteristics, Average sample number, Robustness, Coefficient of variation, Acceptance and rejection region.

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1. Introduction

The concept of sequential testing of statistical hypotheses for testing between two simple hypotheses is first developed by Wald (1947). This concept is heavily dominated by the sequential probability ratio test (SPRT). In order to study the performance of the SPRT'S, Wald (1947) derived the theoretical expressions for the operating characteristics (OC) and average sample number (ASN) functions. Sequential probability ratio test has been applied by several researchers, to tackle with different testing problems, for references one may referred to Oakland (1950), Epstein and Sobel (1955), Johnson (1966), Phatarford (1971), Bain and Engelhardt (1982), Chaturvedi et al. (2000), Sevil and Demirhan (2008).

The robustness of the SPRT in respect of OC and ASN functions has been studied by several authors, when the distribution under consideration has undergone a change, while dealing with various probabilistic models. For references, Harter and Moore (1976) gives sampling plans for reliability tests under the assumption of a constant failure rate and by using Monte Carlo techniques, the robustness of the exponential SPRT is studied, when the underlying distribution is a Weibull distribution. Montagne and Singpurwalla (1985) investigated the robustness of the sequential life-testing procedure with respect to the risks and the expected sample sizes for the exponential distribution when the life length is not exponential. Hubbard and Allen (1991) applied SPRT on the mean of the negative binomial distribution when the dispersion parameter is known and the robustness of the test to the misspecification of dispersion parameter is studied. Chaturvedi et al. (1998) considered a family of life-testing models and studied the robustness of the SPRT'S for various parameters involved in the model and also generalised the results of Montagne and Singpurwalla (1985).

Joshi and Shah (1990) developed SPRT for testing a simple hypothesis (against a simple alternative) for the mean of an inverse Gaussian distribution, assuming the coefficient of variation (CV) to be known. They obtained theoretical expressions for the OC and the ASN functions.

2. Set up a problem:

Let us consider a random variable (r.v.) X follows the Erlang distribution presented by the probability density function (pdf)

$$f(x;\lambda,k)=\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}; \quad 0<x<\infty,\ \lambda\geq 0,\ k\in N \tag{2.1}$$

where k is the shape parameter and λ is the rate parameter. The Erlang distribution is the sum of 'k' independent exponential random variables each with the same parameter. For a given sequence of observations $X_1, X_2, X_3 \ldots$ from (2.1), the problem of testing the simple null hypothesis $H_0:\lambda=\lambda_0$ against the simple alternative hypothesis $H_1:\lambda=\lambda_1(>\lambda_0)$ is considered.

In Section 3, 4, 5, 6 and 7, respectively, we develop the SPRT'S for the parameters involved in the model (2.1). The robustness of the SPRT'S in respect of OC and ASN functions, when the distribution under consideration has undergone a change is studied [see Remarks 3.1, 4.1, 5.1, 6.1]. Also, the robustness of the SPRT for a mis-specified coefficient of variation is studied in Section 7[see Remarks 7.1]. In Section 8, the acceptance and rejection regions for H_0 vs H_1 in case of λ are derived and plotted in Figure 8.1. Finally, in Section 9, the results and findings are presented through Tables and Graphs.

3. SPRT for testing the hypothesis regarding 'λ'

The SPRT for testing $H_0:\lambda=\lambda_0$ against $H_1:\lambda=\lambda_1(\lambda_1>\lambda_0)$ is defined as follows

$$Z_i=\ln\left[\frac{f(x_i;\lambda_1,k)}{f(x_i;\lambda_0,k)}\right] \tag{3.1}$$

$$Z_i=k\ln\left(\frac{\lambda_1}{\lambda_0}\right)+x_i(\lambda_0-\lambda_1) \tag{3.2}$$

We choose two numbers A and B such that $0<B<1<A$. At the nth stage of sampling accept H_0 if $\sum_{i=1}^{n} z_i \leq \ln B$, reject H_0 if $\sum_{i=1}^{n} z_i \geq \ln A$, otherwise continue sampling by taking the (n+1)th observation. If α $\in (0,1)$ and β $\in (0,1)$ are Type I and Type II errors respectively, then according to Wald (1947), A and B are approximately given by

$$A \approx \frac{1-\beta}{\alpha} \text{ and } B \approx \frac{\beta}{1-\alpha} \tag{3.3}$$

The Operating Characteristic (OC) Function $L(\theta)$ is given by

$$\mathrm{L}(\theta) \approx \frac{A^h - 1}{A^h - B^h} \tag{3.4}$$

where ‘h’ is the non-zero solution of

$$E\left[e^{Z_i}\right]^h = 1 \tag{3.5}$$

or,

$$\int_0^\infty \left[\frac{f(x_i;\lambda_1,k)}{f(x_i;\lambda_0,k)}\right]^{\mathrm{h}} f(x_i;\lambda,k)dx = 1 \tag{3.6}$$

From (2.1) and (3.2), we obtain

$$E[e^{Z_i}]^h = \frac{\left(\dfrac{\lambda_1}{\lambda_0}\right)^{kh}}{\left[1-\frac{h}{\lambda}(\lambda_0-\lambda_1)\right]^k} \tag{3.7}$$

Finally, from equation (3.5) we get

$$\lambda = \frac{h(\lambda_0 - \lambda_1)}{1-\left(\dfrac{\lambda_1}{\lambda_0}\right)^h} \tag{3.8}$$

This expression (3.8), is not useful for finding the values of OC and ASN functions, hence, in order to tackle this problem we take the logarithm of both sides and using the expansion $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}; -1 < x < 1$ and retaining the term up to third degree in ‘h’, we get

$$kh\ln\left(\frac{\lambda_1}{\lambda_0}\right) = k\ln\left[1-\left\{h\frac{(\lambda_0-\lambda_1)}{\lambda}\right\}\right]$$

$$kh\ln\left(\frac{\lambda_1}{\lambda_0}\right) = k\left[-h\left(\frac{\lambda_0-\lambda_1}{\lambda}\right) - \frac{h^2}{2}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^2 - \frac{h^3}{3}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^3\right]$$

or

$$\frac{h^2}{3}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^3+\frac{h}{2}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^2+\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)+\ln\left(\frac{\lambda_1}{\lambda_0}\right)=0 \tag{3.9}$$

which is a quadratic equation in 'h'. On solving (3.9), we get the real roots of 'h'. Finally, on substituting the values of 'h' in equation (3.4) the numerical values of OC function are obtained. The ASN function is approximately given by

$$E(N/\lambda)=\frac{L(\lambda)\ln B+\left[1-L(\lambda)\right]\ln A}{E(Z)} \tag{3.10}$$

provided that E(Z)≠0, where

$$E(Z)=k\left[\ln\left(\frac{\lambda_1}{\lambda_0}\right)+\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)\right] \tag{3.11}$$

From equation (3.11), the ASN function under H_0 and H_1 is given by

$$E_0(N)=\frac{(1-\alpha)\ln B+\alpha\ln A}{k\left[\ln\left(\frac{\lambda_1}{\lambda_0}\right)+\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)\right]} \tag{3.12}$$

and

$$E_1(N)\approx\frac{\beta\ln B+(1-\beta)\ln A}{k\left[\ln\left(\frac{\lambda_1}{\lambda_0}\right)+\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)\right]} \tag{3.13}$$

Remarks 3.1: Let us consider the problem of testing the simple null hypothesis $H_0:\lambda_0=13$ against the simple alternative hypothesis $H_1:\ \lambda_1=15$, for $\alpha=\beta=0.05$. The numerical values of the OC and ASN functions are derived and are presented in Table 3.1 and their curves are shown in Figures 3.1(a) and 3.1(b), respectively. The Table and Curves shows that the approximate method for obtaining the real roots of 'h' gives satisfactorily results, i.e. the values of L(λ) are 0.9518 and 0.0488 for $\lambda=13.0$ and $\lambda=15.0$, respectively which are very close to true values of the OC function.

4. Robustness of the SPRT for 'λ'when '*k*' has undergone a change

Let us suppose that the parameter '*k*' has undergone a change to $k^*$and then probability distribution in (2.1) becomes $f(x;\lambda,k^*)$. In order to study the robustness of SPRT developed in Section 3 with respect to OC function, the values of 'h'are obtained by solving the following

$$\int_0^\infty \left[\frac{f(x_i;\lambda_1,k)}{f(x_i;\lambda_{0,},k)}\right]^h f(x_i;\lambda,k^*)dx = 1 \tag{4.1}$$

$$\left(\frac{\lambda_1}{\lambda_0}\right)^{kh} \frac{\lambda^{k^*}}{\Gamma k^*}\int_0^\infty e^{-x[\lambda-h(\lambda_0-\lambda_1)]}x^{(k^*-1)}dx = 1$$

Finally, we get

$$\left(\frac{\lambda_1}{\lambda_0}\right)^{kh}\left[1-h\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)\right]^{-k^*} = 1 \tag{4.2}$$

taking logarithm on both sides of equation (4.2) and using the expression of $\ln(1\text{-}x), |x| \le 1.$ In order to obtain the roots of the given equation and retain the terms upto third degree in 'h' we get the following quadratic equation

$$\frac{h^2 p}{3}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^3 + \frac{hp}{2}\left(\frac{\lambda_0-\lambda_1}{\lambda}\right)^2 + p\left(\frac{\lambda_0-\lambda_1}{\lambda}\right) + \ln\left(\frac{\lambda_1}{\lambda_0}\right) = 0, \tag{4.3}$$

where $p = \frac{k^*}{k}$

which is quadratic equation in 'h'. On solving, we get the real roots of 'h'. The Robustness of the SPRT with respect to ASN is studied by replacing the denominator of (3.10) by

$$E(Z/\lambda) = \int_0^\infty z\ f(x;\lambda,k^*)dx$$

$$E(Z/\lambda) = k\ In\left(\frac{\lambda_1}{\lambda_0}\right) + (\lambda_0-\lambda_1)E(x)$$

$$E(Z/\lambda) = k\ In\left(\frac{\lambda_1}{\lambda_0}\right) + (\lambda_0-\lambda_1)\left(\frac{k^*}{\lambda}\right)$$

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$$E(Z/\lambda) = \ln\left(\frac{\lambda_1}{\lambda_0}\right) - p\left\{\frac{\lambda_0 - \lambda_1}{\lambda}\right\}, \tag{4.4}$$

Remarks 4.1: For testing the simple null hypothesis $H_0 : \lambda_0 = 13$ against the simple alternative hypothesis $H_1 : \lambda_1 = 15$ for $\alpha = \beta = 0.05$ for different values of $'k'$ the real roots of 'h' are obtained from equation (4.3). From Table 4.1(a) and Table 4.1(b), the values of OC and ASN curves are plotted in Figure 4.1 (a) and Figure 4.1 (b) for various values of $'p'$. The OC curve shifts to the left (right) and ASN curve shifts to the left downward (right upward) for $p < 1$(p >1). From both the curves, it is evident that the SPRT is highly sensitive for any change in 'k'.

5. SPRT for testing the hypothesis regarding $'k'$

Let the sequence of observation X_1, X_2, $X_3 \ldots$ from (2.1). Our goal is to test the simple null hypothesis $H_0 : k = k_0$ against the simple alternative hypothesis $H_0 : k = k_1 (k_1 > k_0)$ .The SPRT for testing H_0 is defined as follows:

Let

$$Z_i = \ln\left\{\frac{f(x_i;\lambda,k_1)}{f(x_i;\lambda,k_0)}\right\} \tag{5.1}$$

$$Z_i = \ln\left(\frac{\Gamma k_0}{\Gamma k_1}\right) + (k_1 - k_0)\ln\lambda + (k_1 - k_0)\ln x_i \tag{5.2}$$

For OC curve we have

$$\int_0^\infty \left[\frac{f(x_i;\lambda,k_1)}{f(x_i;\lambda,k_0)}\right]^h f(x_i;\lambda,k)dx = 1$$

(5.3)

From (2.1) and (5.2)

$$\left(\frac{\Gamma k_0}{\Gamma k_1}\right)^h \frac{1}{\Gamma k}\Gamma\{h(k_1 - k_0) + k\} = 1 \tag{5.4}$$

This expression (5.4) is not useful for finding the values of OC and ASN functions, hence we further calculate and taking the logarithm of both sides and using the approximation

$$\ln\Gamma x = \ln\sqrt{2\pi} - x + \left(x - \frac{1}{2}\right)\ln x \tag{5.5}$$

we have

$$\frac{h^2}{12}\left(\frac{k_1 - k_0}{k}\right)^3 (4k+1) - \frac{h}{4}\left(\frac{k_1 - k_0}{k}\right)^2 (2k+1) - \left(k_0 - \frac{1}{2}\right)\ln k_0 + \left(k_1 - \frac{1}{2}\right)\ln k_1 - \left(1 + \ln k - \frac{1}{2k}\right)(k_1 - k_0) = 0 \tag{5.6}$$

which is quadratic equation in ‘h’. On solving, we get the real roots of ‘h’. The numerical values of OC function is now obtain from equation (5.6). The Robustness of the SPRT with respect to ASN is studied by replacing the denominator of (3.10) by

$$E(Z_i/k) = \ln\Gamma \mathrm{k}_0 - \ln\Gamma \mathrm{k}_1 + (k_1 - k_0)\ln\lambda + (k_1 - k_0)\,E(\ln x_i)$$

and

$$E(In\ x_i) = \psi(k) - \ln\lambda$$

Using the result of Gradshteyn and Ryzhik (1965, p.576 § 4.352(1)) that

$$\Psi(x) = \ln x - \frac{1}{2x} \tag{5.7}$$

$$E(Z_i/k) = \left(k_0 - \frac{1}{2}\right)\ln k_0 + \left(k_1 - \frac{1}{2}\right)\ln k_1 + \left(1 + \ln k - \frac{1}{2k}\right)(k_1 - k_0) \tag{5.8}$$

Remarks 5.1: Let us consider the problem of testing the simple null hypothesis $H_0 : k_0 = 13$ against the simple alternative hypothesis $H_1 :\ k_1 = 15,$ for $\alpha = \beta = 0.05$. The numerical values of the OC and ASN functions are derived and are presented in Table 5.1 and their curves are shown in Figures 5.1(a) and 5.1(b), respectively. The Table and Curves shows that the approximate method for obtaining the real roots of ‘h’ gives satisfactorily results, i.e. the values of L(k) are 0.9583 and 0.0548 for $k = 13.0$ and $k = 15.0$, respectively which are very close to true values of the OC function.

6. Robustness of the SPRT for 'k' when 'λ' has undergone a change

Let us suppose that the parameter 'λ' has undergone a change to λ^* and then probability distribution in (2.1) becomes $f(x;\lambda^*,k)$. In order to study the robustness of SPRT developed in Section 5 with respect to OC function, the values of 'h'are obtained by solving the following

$$\int_0^\infty \left\{ \frac{f(x_i;\lambda,k_1)}{f(x_i;\lambda,k_0)} \right\}^h f(x_i;\lambda^*,k)dx = 1 \qquad (6.1)$$

$$\left(\frac{\Gamma k_0}{\Gamma k_1}\right)^h \frac{1}{\Gamma k} \lambda^{h(k_1-k_0)} \lambda^{*k} \int_0^\infty x^{h(k_1-k_0)+k-1} e^{-\lambda^* x} dx = 1$$

Finally, we get

$$\left(\frac{\Gamma k_0}{\Gamma k_1}\right)^h \frac{1}{\Gamma k} \phi^{h(k_1-k_0)} \Gamma\{h(k_1-k_0)+k\} = 1 \qquad (6.2)$$

where $\phi = \dfrac{\lambda}{\lambda^*}$

taking logarithm on both sides of equation (6.2) and using the approximation (5.5). In order to obtain the roots of the given equation and retain the terms upto third degree in 'h' we get the following quadratic equation

$$\left(\frac{h^2}{12}\right)\left(\frac{k_1-k_0}{k}\right)^3 (4k+1) - \frac{h}{4}\left(\frac{k_1-k_0}{k}\right)^2 (2k+1) - \left(k_0 - \frac{1}{2}\right)\ln k_0 + \left(k_1 - \frac{1}{2}\right)\ln k_1 + (k_1-k_0)\ln\phi -$$
$$\left(1 + \ln k - \frac{1}{2k}\right)(k_1-k_0) = 0 \qquad (6.3)$$

which is quadratic equation in 'h'. On solving, we get the real roots of 'h'. The numerical values of OC function is now obtain from equation (6.3). The Robustness of the SPRT with respect to ASN is studied by replacing the denominator of (3.10) by

$$E(Z_i/k) = In\,\Gamma \mathrm{k}_0 - In\,\Gamma \mathrm{k}_1 + (k_1-k_0)\,In\,\lambda + (k_1-k_0)\,E(\ln x_i)$$

where

$$E(\ln\, x_i) = \psi(k) - \ln\, \lambda^*$$

And using the result of (5.7) we get

$$E(Z_i/\lambda^*) = \left(k_0 - \frac{1}{2}\right)\ln k_0 + \left(k_1 - \frac{1}{2}\right)\ln k_1 + \left\{1 + \ln\phi + \ln k - \frac{1}{2k}\right\}(k_1 - k_0) \quad (6.4)$$

Remarks 6.1: For testing the simple null hypothesis $H_0 : k_0 = 13$ against the simple alternative hypothesis $H_1 : k_1 = 15$ for $\alpha = \beta = 0.05$ for different values of 'k' the real roots of 'h' are obtained from equation (6.3). From Table 6.1(a) and Table 6.1(b), the values of OC and ASN curves are plotted in Figure 6.1 (a) and Figure 6.1 (b) for various values of 'ϕ'. The OC curve shifts to the right (left) and ASN curve shifts to the right upward (left downward) for $\phi < 1$ ($\phi > 1$). From both the curves, it is evident that the SPRT is highly sensitive for any change in 'k'.

7. Robustness of the SPRT for 'λ' with known coefficient of variation (CV)

For Erlang distribution mean and variance are (k/λ) and (k/λ^2)**,** respectively then coefficient of variation (CV) $= (1/\sqrt{k})$. Let us suppose that coefficient of variation changes from a to a*, so that, the (pdf) of (2.1) shifts to $f(x_i;\lambda,a^*)$.

Then OC and ASN function is

$$\int_0^\infty \left[\frac{f(x_i;\lambda_1,a)}{f(x_i;\lambda_0,a)}\right]^h f(x_i;\lambda,a^*)dx = 1 \quad (7.1)$$

From (3.2) and (2.1)

$$\left(\frac{\lambda_1}{\lambda_0}\right)^{\frac{h}{a^2}}\left[\frac{\lambda}{\lambda - h(\lambda_0 - \lambda_1)}\right]^{\left(\frac{1}{a^*}\right)^2} = 1 \quad (7.2)$$

taking logarithm on both sides of equation (7.2) and using the expression of $\ln(1\text{-}x), |x| \le 1$. In order to obtain the roots of the given equation and retain the terms upto third degree in 'h' we get the following quadratic equation

$$\frac{h^2}{3}\left(\frac{\lambda_0 - \lambda_1}{\lambda}\right)^3 Q + \frac{h}{2}\left(\frac{\lambda_0 - \lambda_1}{\lambda}\right)^2 Q + \left(\frac{\lambda_0 - \lambda_1}{\lambda}\right)Q + \ln\left(\frac{\lambda_1}{\lambda_0}\right) = 0 \quad (7.3)$$

where $Q = \left(\frac{a}{a^*}\right)^2$

which is quadratic equation in 'h'. On solving, we get the real roots of 'h'. The numerical values of OC function is now obtain from equation (7.3). The Robustness of the SPRT with respect to ASN is studied by replacing the denominator of (3.10) by

$$E(Z/\lambda) = \ln\left(\frac{\lambda_1}{\lambda_0}\right) + Q\left(\frac{\lambda_0 - \lambda_1}{\lambda}\right) \tag{7.4}$$

Remarks 7.1: For testing the simple null hypothesis $H_0 : \lambda_0 = 13$ against the simple alternative hypothesis $H_1 : \lambda_1 = 15$ for $\alpha = \beta = 0.05$ for different values of $'\lambda'$ the real roots of 'h' are obtained from equation (7.3). From Table 7.1(a) and Table 7.1(b), the values of OC and ASN curves are plotted in Figure 7.1 (a) and Figure 7.1 (b) for various values of 'Q'. The OC curve shifts to the left (right) and ASN curve shifts to the left downward (right upward) for $Q < 1$($Q > 1$). From both the curves, it is evident that the SPRT is highly sensitive for any change in 'λ'.

8. Implementation of Erlang distribution

We wish to test the simple hypothesis $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (\lambda_1 > \lambda_0)$ having pre-assigned $0 < \alpha, \beta < 1$. Let $A \approx \frac{(1-\beta)}{\alpha}$ and $B \approx \frac{\beta}{(1-\alpha)}$ and is defined as

$$Z_i = k \text{ In}\left(\frac{\lambda_1}{\lambda_0}\right) + x_i \, (\lambda_0 - \lambda_1)$$

Let us defined, $Y(n) = \sum_{i=1}^{n} X_i$ and N= first integer n (≥ 1) for which the inequality $Y(n) \leq c_1 + dn$ or $Y(n) \geq c_2 + dn$ holds with the constants

$$c_1 = \frac{\ln B}{(\lambda_0 - \lambda_1)}, c_2 = \frac{\ln A}{(\lambda_0 - \lambda_1)}, d = \frac{k \ln\left(\frac{\lambda_1}{\lambda_0}\right)}{(\lambda_0 - \lambda_1)} \tag{8.1}$$

Remarks 8.1: The figure (8.1) shows the acceptance and rejection regions for H_0 under the case when $H_0 : \lambda_0 = 13$ vs $H_1 : \lambda_1 = 15$, for $\alpha = \beta = 0.05$ and $k = 2$ and The values of constants are

$c_1 = 1.472$, $c_2 = -1.472$ and $d = 0.1431$, respectively. Thus, if $Y(N) \leq 0.1434\text{N} - 1.472$, we accept H_0 and if $Y(N) \geq 0.1434\text{N} + 1.472$, we accept H_1. At the intermediate stages, we continue sampling.

9. Tables and Figures

TABLE 3.1: OC and ASN Function

$(H_0 : \lambda_0 = 13, \text{H}_1 : \lambda_1 = 15, \alpha = \beta = 0.05)$

λ	**L(λ)**	**E(N)**	λ	**L(λ)**	**E(N)**
12.2	0.9971	140.4986	14.2	0.3417	413.3708
12.4	0.9936	159.8104	14.4	0.2248	384.7040
12.6	0.9869	183.4731	14.6	0.1399	346.8554
12.8	0.9744	212.4760	14.8	0.0837	307.7820
13.0	0.9518	247.6194	15.0	0.0488	272.0115
13.2	0.9127	288.8578	15.2	0.0281	241.2071
13.4	0.8491	334.0960	15.4	0.0160	215.4333
13.6	0.7539	377.7574	15.6	0.0090	194.0947
13.8	0.6274	410.6952	15.8	0.0051	176.4329
14.0	0.4825	423.6839			

TABLE 4.1(a): OC and ASN Function

$(H_0 : \lambda_0 = 13, \text{H}_1 : \lambda_1 = 15, \alpha = \beta = 0.05)$

λ	$p = 0.96$	$p = 0.98$	$p = 1.00$	$p = 1.02$	$p = 1.04$
12.2	0.9780	0.9916	0.9971	0.9991	0.9998
12.4	0.9579	0.9830	0.9936	0.9978	0.9993
12.6	0.9228	0.9671	0.9869	0.9952	0.9984
12.8	0.8647	0.9388	0.9744	0.9900	0.9964
13.0	0.7762	0.8910	0.9518	0.9802	0.9924
13.2	0.6555	0.8153	0.9127	0.9622	0.9847
13.4	0.5126	0.7070	0.8491	0.9306	0.9705
13.6	0.3691	0.5708	0.7539	0.8777	0.9451
13.8	0.2463	0.4247	0.6274	0.7957	0.9017
14.0	0.1548	0.2916	0.4825	0.6809	0.8321
14.2	0.0932	0.1873	0.3417	0.5410	0.7302
14.4	0.0547	0.1145	0.2248	0.3957	0.5985
14.6	0.0315	0.0677	0.1399	0.2676	0.4525
14.8	0.0180	0.0393	0.0837	0.1698	0.3153
15.0	0.0102	0.0225	0.0488	0.1030	0.2048
15.2	0.0057	0.0128	0.0281	0.0606	0.1262
15.4	0.0032	0.0072	0.0160	0.0350	0.0750
15.6	0.0018	0.0041	0.0090	0.0200	0.0436
15.8	0.0010	0.0023	0.0051	0.0113	0.0250

TABLE 4.1(b): OC and ASN Function

$(H_0 : \lambda_0 = 13, \mathrm{H}_1 : \lambda_1 = 15, \alpha = \beta = 0.05)$

λ	$p = 0.96$	$p = 0.98$	$p = 1.00$	$p = 1.02$	$p = 1.04$
12.2	197.1554	164.9055	140.4986	121.8907	107.4472
12.4	229.7191	190.0843	159.8104	136.8897	119.3350
12.6	268.2753	220.8636	183.4731	155.0764	133.5339
12.8	311.3303	257.8043	212.4760	177.3108	150.6794
13.0	354.2941	300.2334	247.6194	204.5853	171.5784
13.2	388.9491	344.8844	288.8578	237.8447	197.1987
13.4	406.0037	384.7538	334.0960	277.4993	228.5586
13.6	400.7087	410.2143	377.7574	322.4249	266.3805
13.8	376.3043	413.8602	410.6952	368.4888	310.3017
14.0	341.2197	395.7450	423.6839	407.6247	357.5151
14.2	303.6089	363.0557	413.3708	429.7627	401.3790
14.4	268.5015	324.8010	384.7040	428.4458	431.9220
14.6	237.9659	287.4981	346.8554	405.5380	440.2997
14.8	212.2997	254.3281	307.7820	369.4382	424.8342
15.0	191.0183	226.1628	272.0115	329.2599	392.0868
15.2	173.4081	202.7294	241.2071	291.0522	351.7520
15.4	158.7723	183.3453	215.4333	257.5169	311.4459
15.6	146.5152	167.2734	194.0947	229.2211	275.1647
15.8	136.1561	153.8588	176.4329	205.7377	244.1852

Table 5.1: OC and ASN Function

$(H_0 : k_0 = 13, \mathrm{H}_1 : k_1 = 15, \alpha = \beta = 0.05)$

k	$L(k)$	$E(k)$	k	$L(k)$	$E(k)$
12.0	0.9994	9.2375	14.0	0.4909	29.2605
12.2	0.9983	10.3343	14.2	0.3502	28.2711
12.4	0.9957	11.6721	14.4	0.2335	26.0919
12.6	0.9903	13.3148	14.6	0.1481	23.3677
12.8	0.9793	15.3305	14.8	0.0909	20.6218
13.0	0.9583	17.7702	15.0	0.0548	18.1363
13.2	0.9208	20.6176	15.2	0.0327	16.0063
13.4	0.8581	23.7009	15.4	0.0194	14.2266
13.6	0.7631	26.5972	15.6	0.0115	12.7524
13.8	0.6362	28.6525	15.8	0.0068	11.5307

Table 6.1(a): OC and ASN Function

$(H_0 : k_0 = 13, \mathrm{H}_1 : k_1 = 15, \alpha = \beta = 0.05)$

k	$\phi = .096$	$\phi = 0.98$	$\phi = 1.00$	$\phi = 1.02$	$\phi = 1.04$
12.2	0.999945	0.9996	0.998309	0.994513	0.984968
12.4	0.999745	0.998784	0.995741	0.987647	0.968782
12.6	0.99918	0.996855	0.99026	0.974054	0.938909
12.8	0.997805	0.992664	0.979255	0.948609	0.887357
13.0	0.994749	0.984108	0.958327	0.903873	0.805774
13.2	0.988391	0.967552	0.920781	0.831233	0.690689
13.4	0.975849	0.93721	0.858111	0.725164	0.550266
13.6	0.95235	0.885095	0.763090	0.590112	0.404904
13.8	0.910806	0.802902	0.636174	0.443707	0.276973
14.0	0.842629	0.687288	0.490931	0.309185	0.17894
14.2	0.741441	0.54661	0.350214	0.202464	0.111184
14.4	0.609813	0.401403	0.233488	0.12688	0.067465
14.6	0.463623	0.273985	0.148072	0.077353	0.040411
14.8	0.326145	0.176626	0.090898	0.046434	0.024058
15.0	0.215002	0.109515	0.054752	0.027661	0.014292
15.2	0.135245	0.066319	0.032657	0.016428	0.008491
15.4	0.082572	0.039648	0.019396	0.009753	0.00505
15.6	0.049562	0.023561	0.011508	0.005796	0.003009
15.8	0.029493	0.013972	0.006832	0.00345	0.001796

Table 6.1(b): OC and ASN Function

$(H_0 : k_0 = 13, \mathrm{H}_1 : k_1 = 15, \alpha = \beta = 0.05)$

k	$\phi = 0.96$	$\phi = 0.98$	$\phi = 1.00$	$\phi = 1.02$	$\phi = 1.04$
12.2	8.052826	9.070382	10.33428	11.9178	13.89641
12.4	8.870748	10.11046	11.67214	13.64174	16.08068
12.6	9.848712	11.37437	13.31481	15.75179	18.67666
12.8	11.03152	12.92305	15.33047	18.28361	21.59022
13.0	12.47603	14.82574	17.77018	21.18239	24.51754
13.2	14.25006	17.14521	20.61763	24.20702	26.90107
13.4	16.42349	19.89954	23.70091	26.85593	28.09234
13.6	19.04171	22.98667	26.59718	28.46045	27.73625
13.8	22.06654	26.08093	28.65254	28.53432	26.03132
14.0	25.27845	28.58762	29.26052	27.12234	23.55508

14.2	28.18622	29.81917	28.27114	24.74592	20.88337
14.4	30.08768	29.40520	26.0919	22.02627	18.37868
14.6	30.40746	27.56573	23.36773	19.40007	16.19373
14.8	29.10050	24.91887	20.62176	17.07458	14.35348
15.0	26.67011	22.07522	18.13627	15.10250	12.82553
15.2	23.78829	19.41609	16.00627	13.46138	11.56009
15.4	20.95726	17.10020	14.22656	12.10260	10.50794
15.6	18.42908	15.15175	12.75243	10.97470	9.626742
15.8	16.27716	13.53507	11.53068	10.03223	8.882170

TABLE 7.1(a): OC and ASN Function

$(H_0 : \lambda_0 = 13, \mathrm{H}_1 : \lambda_1 = 15, \alpha = \beta = 0.05)$

λ	$Q = 0.98$	$Q = 1.00$	$Q = 1.02$
12.4	0.9586	0.9936	0.9994
12.6	0.9240	0.9869	0.9984
12.8	0.8667	0.9744	0.9964
13.0	0.7792	0.9518	0.9925
13.2	0.6592	0.9127	0.9850
13.4	0.5167	0.8491	0.9710
13.6	0.3729	0.7539	0.9460
13.8	0.2493	0.6274	0.9032
14.0	0.1569	0.4825	0.8345
14.2	0.0946	0.3417	0.7335
14.4	0.0555	0.2248	0.6025
14.6	0.0320	0.1399	0.4566
14.8	0.0182	0.0837	0.3188
15.0	0.0103	0.0488	0.2074
15.2	0.0058	0.0281	0.1279
15.4	0.0033	0.0160	0.0761
15.6	0.0018	0.0090	0.0443
15.8	0.0010	0.0051	0.0254

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TABLE 7.1(b): OC and ASN Function

$(H_0 : \lambda_0 = 13, \mathrm{H}_1 : \lambda_1 = 15, \alpha = \beta = 0.05)$

λ	$Q = 0.98$	$Q = 1.00$	$Q = 1.02$
12.4	228.8271	159.8104	119.0274
12.6	267.2400	183.4731	133.1591
12.8	310.2311	212.476	150.2185
13.0	353.3110	247.6194	171.0078
13.2	388.3445	288.8578	196.4914
13.4	405.9958	334.096	227.6898
13.6	401.3142	377.7574	265.3421
13.8	377.3283	410.6952	309.1318
14.0	342.4041	423.6839	356.3408
14.2	304.7624	413.3708	400.4402
14.4	269.5302	384.7040	431.5077
14.6	238.8440	346.8554	440.5719
14.8	213.0351	307.7820	425.7066
15.0	191.6314	272.0115	393.2965
15.2	173.9209	241.2071	353.0365
15.4	159.2045	215.4333	312.6427
15.6	146.8827	194.0947	276.2077
15.8	136.4718	176.4329	245.0655

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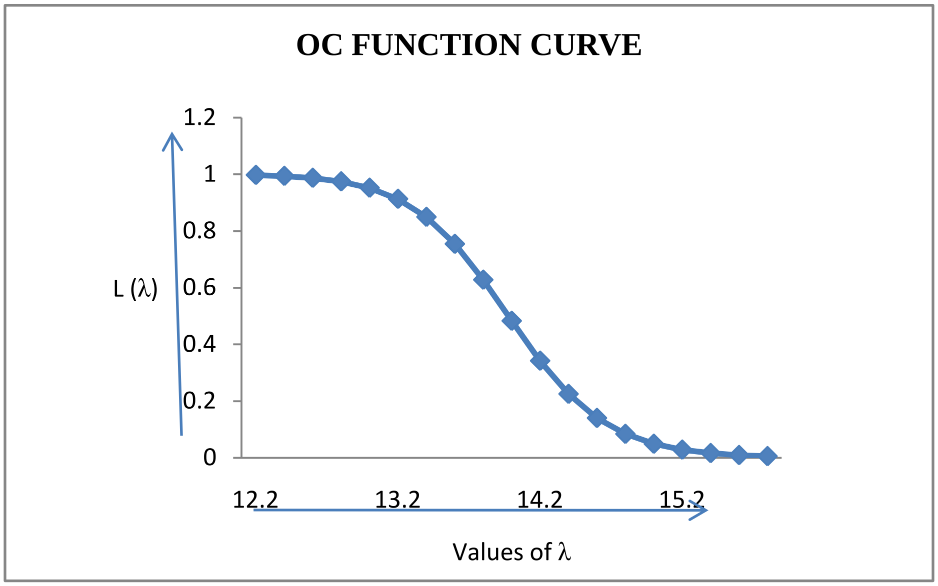

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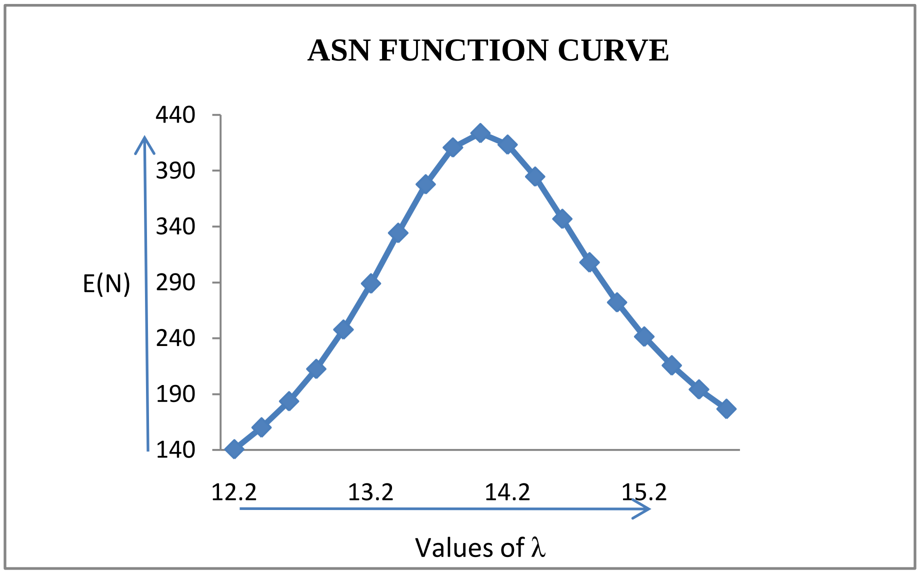
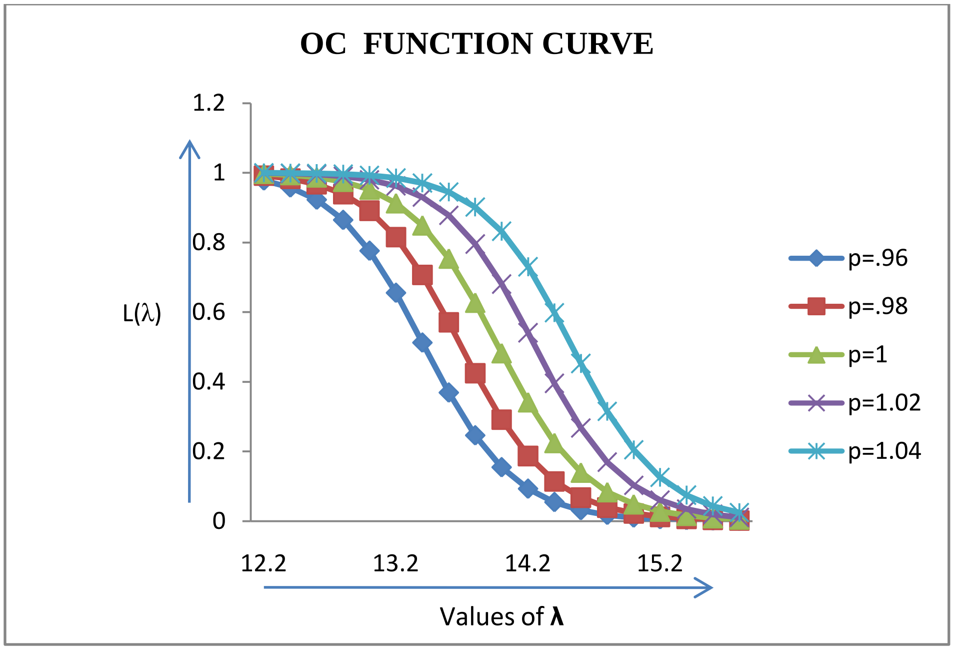


Figure 3.1(b)



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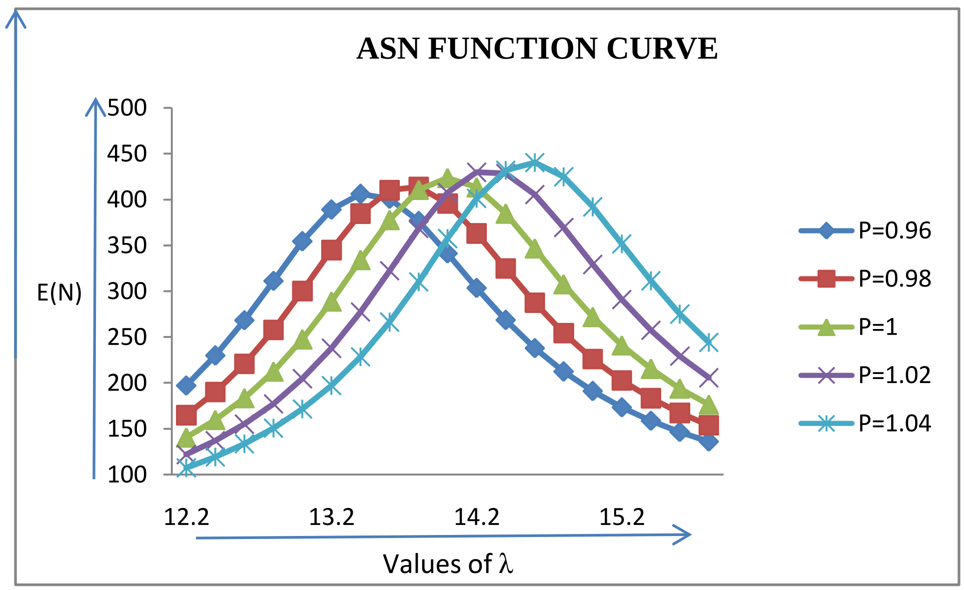
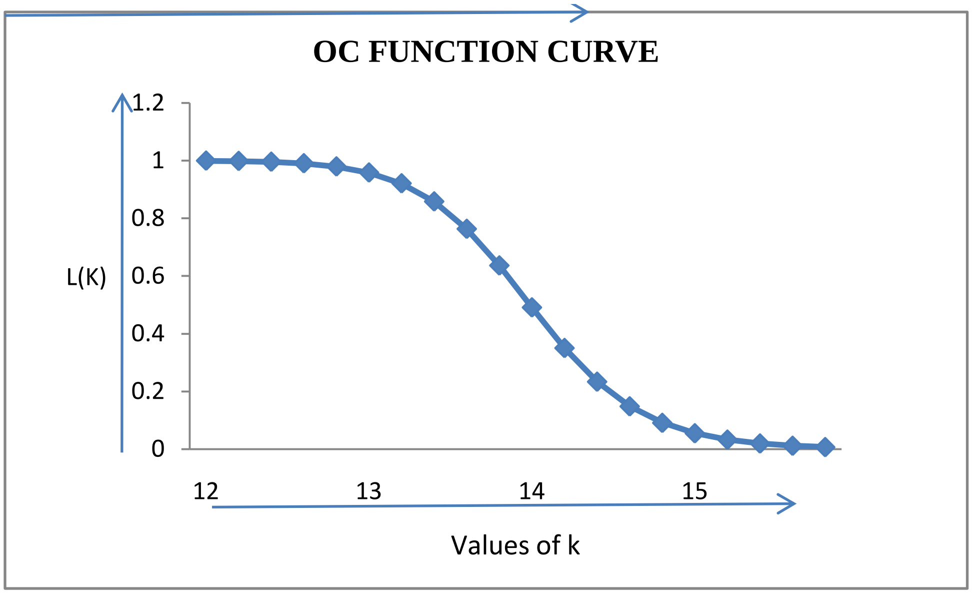


Figure

4.1(b)



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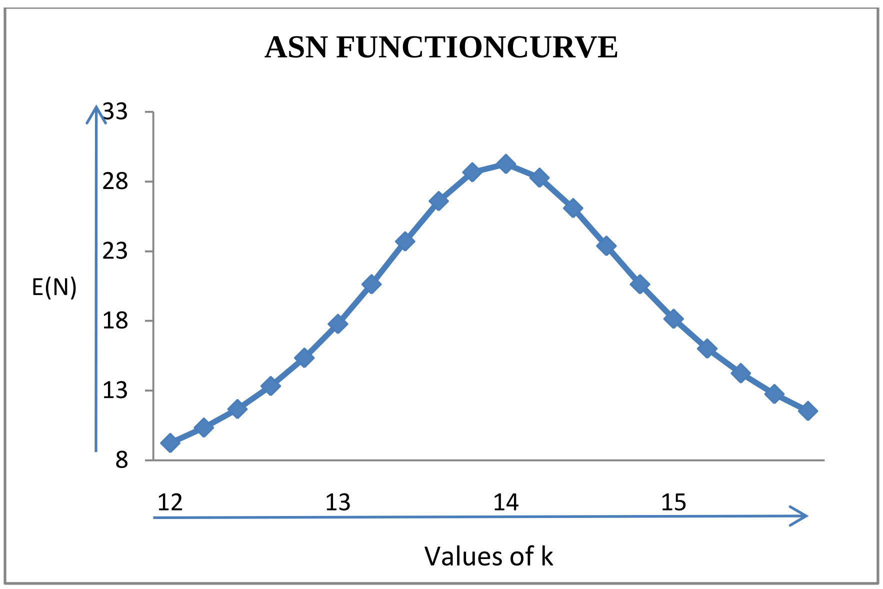

Figure 5.1(b)

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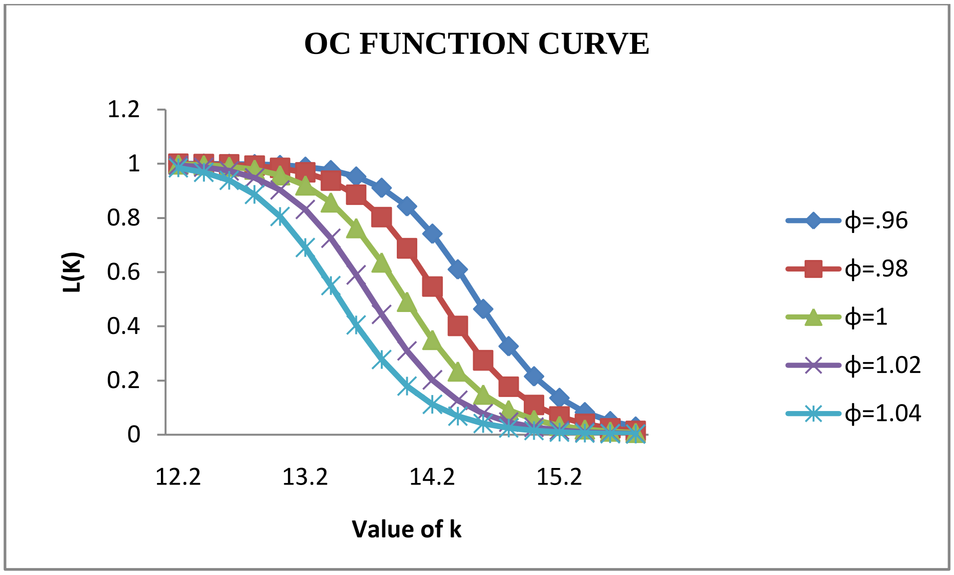

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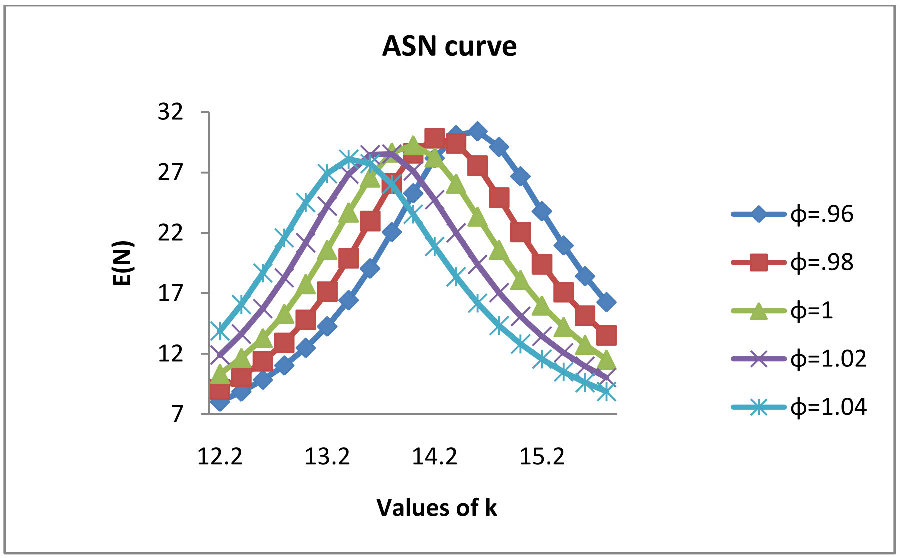

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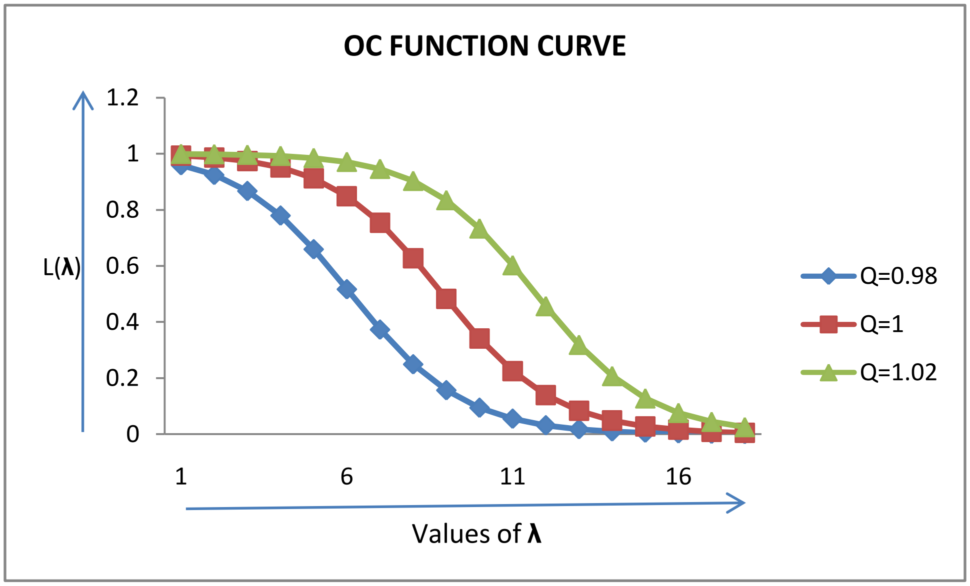

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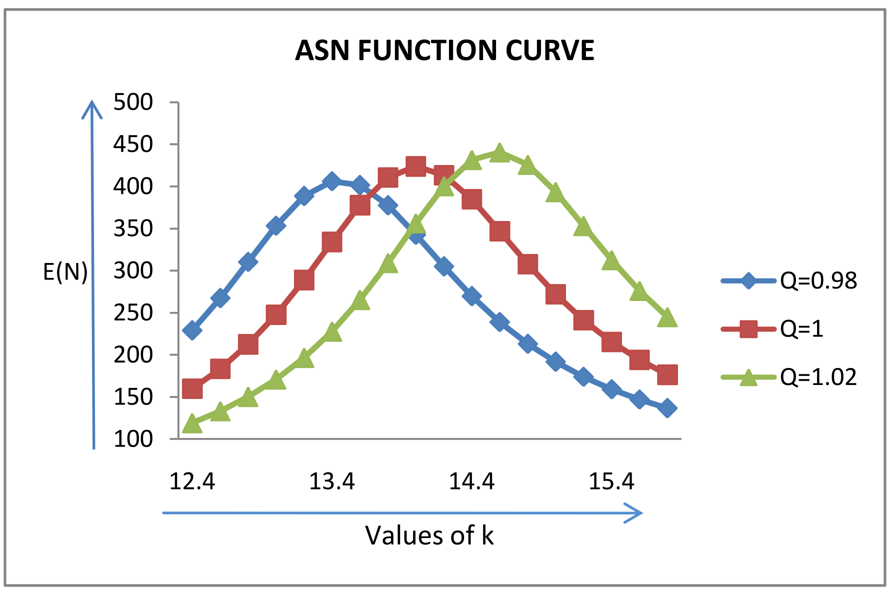

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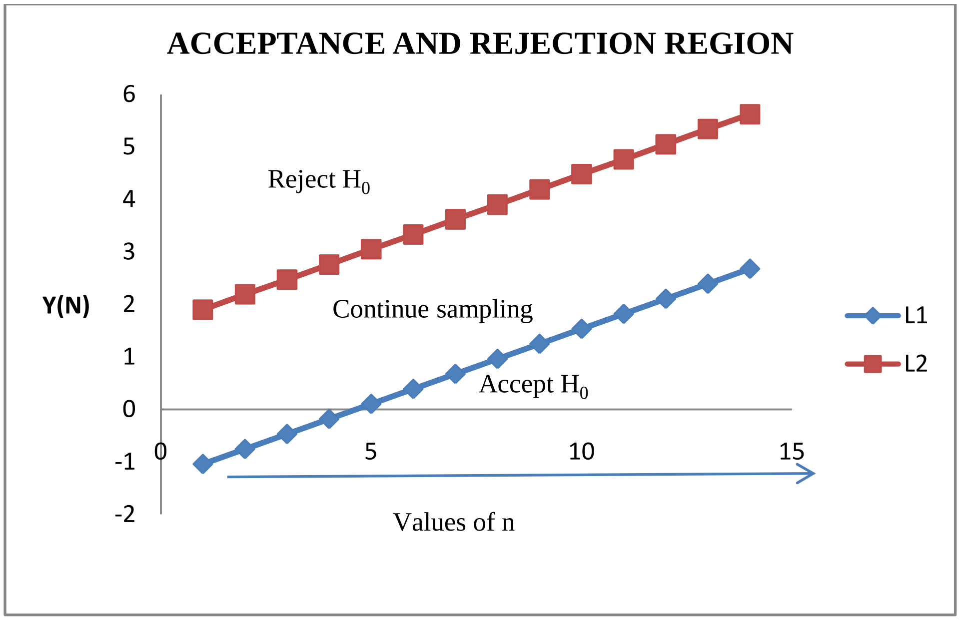

Figure 8.1

REFERENCES

1 Bacanli, S. and Demirhan, Y. P. (2008): A group sequential test for the Inverse Gaussian Mean *Statistical Papers*, 49, 337-386.

2 Bain, L. J. and Engelhardt, M. (1982): Sequential Probability Ratio Tests for the shape parameter of NHPP, *IEEE Transaction on Reliability*, R-31, 79-83.

3 Chaturvedi, A., Kumar, A. and Surinder, K. (1998): Robustness of the sequential procedures for a family of life-testing models. *Metron*, 56, 117-137.

4 Chaturvedi, A., Kumar, A. and Surinder, K. (2000): Sequential testing procedures for a class of distributions representing various life testing models. *Statistical papers*, 41, 65-84.

5 Chaturvedi, A., Tiwari, N., and Tomer, S. (2002): Robustness of the sequential testing procedure for the generalized life distribution. *Brazilian Journal of Probability and Statistics* , 16, 7-24.

6 Epstein, B. and Sobel, M. (1955): Sequential life tests in exponential case. *Ann. Math. Statist.*, 26, 82-93.

7 Gradshteyn, I.S and Ryzhik, I.M (1980): Tables of integrals, Series and Products Academic Press, London.

8 Harter, L. and Moore, A. H. (1976): An evaluation of the exponential and Weibull test plans. *IEEE Transaction on Reliability,* 25**,** 100-104.

9 Hubbard, D. J. and Allen, O. B. (1991): Robustness of the SPRT for a negative binomial to misspecification of the dispersion parameter. *Biometrics*, 47, 419-427.

10 Johnson, N. L. (1966): Cumulative sum control chart and the Weibull Distribution. *Technometrics*, 8, 481-491.

11 Joshi, S. and Shah, M. (1990): Sequential Analysis applied to testing the mean of an inverse Gaussian distribution with known coefficient of variation. *Commun. Statist.-Theor. Meth*., 19(4), 1457-1466.

12 Montagne, E. R. and Singpurwalla, N. D. (1985): Robustness of sequential exponential life-testingprocedures. *Jour. American Statist. Assoc*., 391, 715-719.

13 Oakland, G. B. (1950): An application of sequential analysis to whitefish sampling. *Biometrics*, 6, 59-67.

14 Phatarfod, R. M. (1971): A sequential test for Gamma distribution. *Jour. American Statist. Assoc.*, 66, 876-878.

15 Wald, A. (1947): Sequential Analysis. *John Wiley and Sons. New York.*

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