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THERMAL ENERGY TRANSMISSION IN THE UNSTEADY COUETTE FLOW OF OLDROYD LIQUID BETWEEN TWO HORIZONTAL PARALLEL POROUS PLATES WITH HEAT GENERATING SOURCES

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ABSTRACT

This paper deals with thermal energy transmission in the unsteady Couette flow of Oldroyd liquid between two horizontal parallel porous plates with heat generating sources. The problem is formulated with the development of the constitutive equations of continuity, momentum and energy and these equations are solved by Galerkin technique. After necessary computation with numerical values of fluid parameters, profiles of velocity of flow and the temperature of the fluid are plotted. The values of shear stresses and the rates of heat transfer are entered in the tables. It is observed that the velocity of flow decreases with the rise of the value of the Reynolds number as well as the elastic parameter Rc. The rates of heat transfer at both the plates of the channel increases with the increase of Rc. Cooling effect is produced while the visco-elastic fluid exhibits Couette flow under the action of an external transverse magnetic field. These results are in good agreement with earlier findings.

KEYWORDS: COUETTE FLOW, GALERKIN TECHNIQUE, POROUS PLATE, THERMAL ENERGY, VISCO-ELASTIC FLUID.

1. Introduction

Heat transfer is the science of spontaneous irreversible process of heat propagation in space. The process of heat propagation is the exchange of internal energy between individual elements and regions of the medium considered. Heat transfer problems are becoming increasingly important in view of their application in many fields of recent origin such as high speed aircrafts, extraction of heat energy from atomic pipes, electronic components, rocket nozzles, cooling or nuclear reactors, atmospheric re-entry vehicles and providing heat links in turbine blades. A good knowledge of heat transfer characteristics is necessary for the successful design of heat exchangers. The operation of electronic devices, transformers and bearings needs the study of heat transfer for saving the equipment from damage due to overheating. Hence, a number of researchers are carrying out investigations on heat transfer.

The problem of unsteady Couette flow of an incompressible viscous liquid between two plates occurring due to the sudden motion of one of the plates has already been studied by Pai[1]. The same flow through a porous channel has been investigated by Nanda[2], while Katagiri[3] and Muhuri[4] have analyzed the same problem independently, taking into consideration the imposition of magnetic field on the field of flow. Rath et. al.[5] have discussed the heat transfer problem in case of unsteady Couette flow between two parallel walls maintained at different temperatures. Mishra[6] has extended the problem studied by Dutta[7] and Kaloni[8] so as to study the plane Couette flow of an Oldroyd liquid with equal rates of injection at the lower wall and suction at the upper wall with the application of uniform transverse magnetic field. Dash and Biswal[9] have investigated the problem of commencement of Couette flow in Oldroyd liquid through a channel in the presence of heat sources.

From technological view point, the study of both Newtonian and Non-Newtonian Couette flow problems in the presence of porous media is very important. Consequently, the literature is replete with copious instances of such investigations on Couette flows, through porous channel.

In the present problem, our aim is to study the commencement of Couette flow in Oldroyd liquid between two parallel porous plates, with heat sources under the following physical situation i.e.

(i) When the lower wall suddenly starts moving with time varying velocity At'^{n} where n is positive.

2. Formulation of the Problem

Let X'-axis be chosen along the lower wall and Y'-axis be normal to it. The upper plane be specified by the equation y'=L, where the number L will be defined later. It is also supposed that the walls extend to infinity in both sides of the X'-axis and the walls are porous. The suction and injection velocity V' at the walls is considered to be a constant. Now, the velocity components u' and v' at any point (X', Y') in the flow field compatible with the equation of continuity can be given by

$$U = U'\ (y,t) \qquad \text{--} \ (2.1)$$

Following the stress-strain rate relation, the stress components are given by

$$P^{x'x'} = 2K_0\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad \text{--} \ (2.2)$$

$$P^{x'y'} = \eta_0\left(\frac{\partial u'}{\partial y'}\right) - K_0\left(V\frac{\partial^2 u'}{\partial y'\partial t'}\right) \text{---} \ (2.3)$$

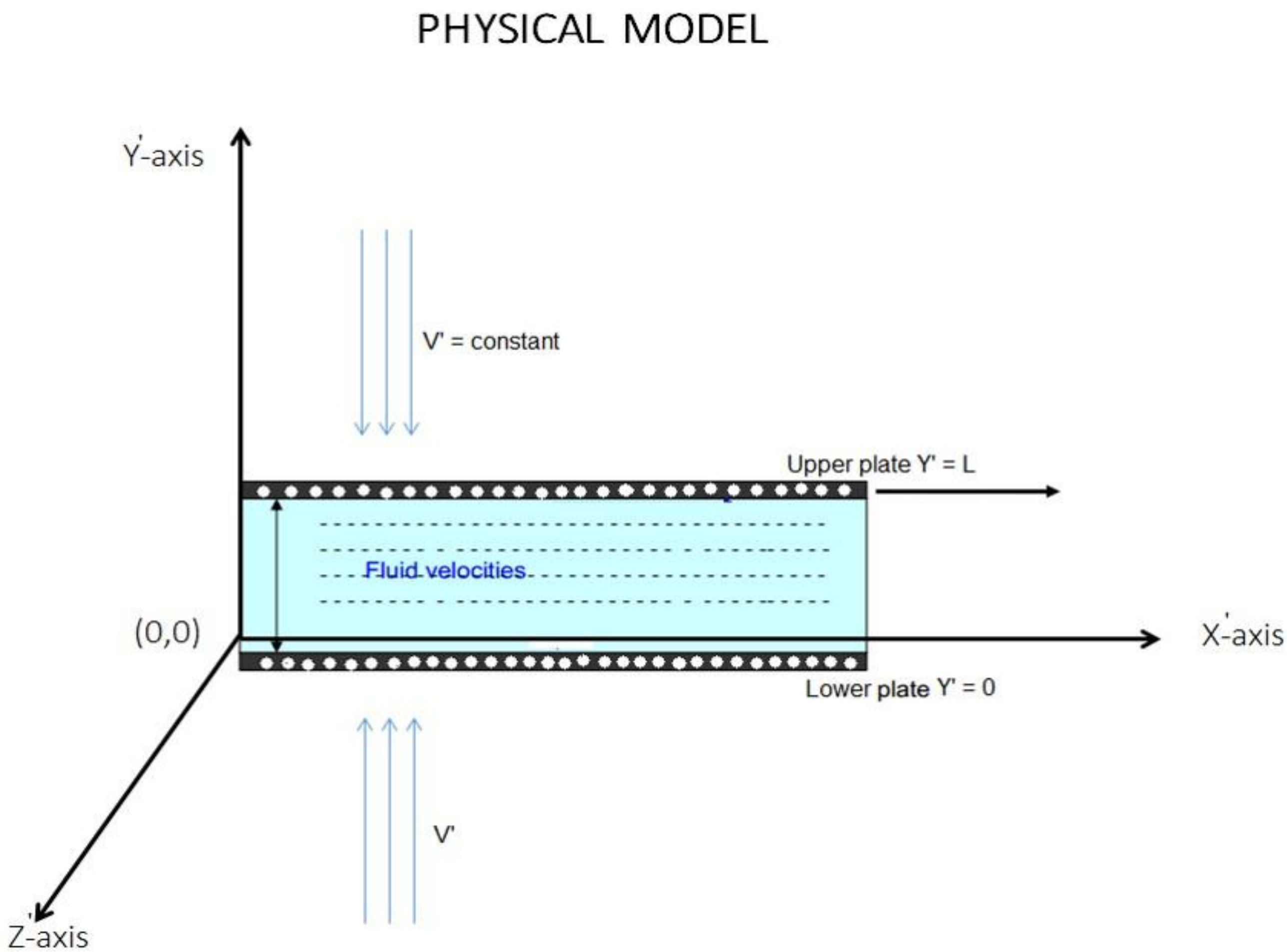
$$P^{y'y'} = 0 \qquad \text{--------------------------------} \ (2.4)$$

Where $K_0 = \eta_0 (\lambda_1 - \lambda_2)$

Since the motion in both the cases is due to the shearing action of the fluid layers

$$\frac{\partial P'}{\partial y'} = 0 \qquad \text{--} \ (2.5)$$

Thus the equations of motion and energy including viscous dissipation and heat sources are given below following the visco-elastic fluid model of Oldroyd's B' liquid.



EQUATION OF MOTION:

$$\rho\left(\frac{\partial u'}{\partial t'}+V\frac{\partial u'}{\partial y'}\right)=\eta_0\frac{\partial^2 u'}{\partial y'^2}-K_0\left(\frac{\partial^3 u'}{\partial y'^2\,\partial t'}+V\frac{\partial^3 u'}{\partial y'^3}\right)-\frac{\eta_0}{K'}u' \text{--------------------------} \quad (2.6)$$

EQUATION OF ENERGY:

$$\left(\frac{\partial\theta'}{\partial t'}+V\frac{\partial\theta'}{\partial y'}\right)=\frac{K}{\rho C}\frac{\partial^2\theta'}{\partial y'^2}+\frac{\eta_0}{\rho C}\left(\frac{\partial u'}{\partial y'}\right)^2-\frac{K_0}{\rho C}\left(\frac{\partial^2 u'}{\partial y'\partial t'}\frac{\partial u'}{\partial y'}+V\frac{\partial u'}{\partial y'}\frac{\partial u'}{\partial y'^2}\right)-S'(\theta'-\theta_L) \text{ ------- } (2.7)$$

Where ρ, C, η_0, K, S', and K' stand for the density, specific heat, co-efficient of viscosity, co-efficient of elasticity, temperature at any point, thermal conductivity, source-sink term and permeability factor of the fluid respectively.

3. Formulation of the equations:

The relevant boundary conditions to which equation (2.6) is subjected to are

$t'=0:\theta'=0$ for all y'

$t'>0:u'=At^n$ for $y'=0$

$\theta'=\theta_L$ for $y'=$L --(3.1)

We introduce here the following non-dimensional parameters:

$$Y=\frac{y'}{\sqrt{\nu_1},\sqrt{T'}}=\frac{t'}{T'},\quad u=\frac{u'}{AT''}$$

$R=\frac{V\sqrt{T}}{\sqrt{\nu_1}}$, the suction parameter,

$R_c=\frac{\lambda_1-\lambda_2}{T}$, the elastic parameter,

$\Pr=\frac{\nu_1\rho C}{K}$, the Prandtl number,

$E=\frac{A_2T^{2n}}{C\theta_L}$, the Eckert number,

$\theta = \dfrac{\theta' - \theta_L}{\theta_L}, \theta_L$ being the temperature of the upper plate,

$S = \dfrac{4S'\nu_1}{V^2}$, the source parameter,

$K^* = \dfrac{K'u^2}{\nu^2}$, the non-dimensional permeability factor of the porous medium, where T is some reference time.

$\nu_1 = \dfrac{\eta_0}{\rho}$, the kinematic viscosity,

$L = \sqrt{\nu_1 T}$, the distance between the two walls of the channel

And $K_0 = \eta_0(\lambda_1 - \lambda_2)$ is the volume coefficient of elasticity of the fluid. With the help of the above non-dimensional parameters, the equations (2.6) and (2.7) are now reduced to their dimensionless forms as follows:

$$\frac{\partial u}{\partial t} + R\frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} + RRc\frac{\partial^3 u}{\partial y^3} + Rc\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{1}{K^*}u = 0 \text{ -------------------------------------- (3.2)}$$

And

$$\frac{\partial \theta}{\partial t} + R\frac{\partial \theta}{\partial y} - \frac{1}{\mathrm{P_r}}\frac{\partial^2 \theta}{\partial y^2} + R_c E\left[\frac{\partial^2 u \partial u}{\partial y^2 \partial y} + \frac{\partial u \partial^2 u}{\partial y \partial y \partial t}\right] - E\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{4}R^2 S\theta = 0 \text{ ---------------------- (3.3)}$$

The modified boundary conditions are

t=0 : u=0 for all y

t>0 : $u=t^n$ for y=0

u=0 for y=1 --- (3.4)

and

t=0 : $\theta = 0$ for all y

$$t > 0 : \frac{\partial \theta}{\partial y} = 0 \text{ for all y=0}$$

$$\theta = 0 \quad \text{for y=1} \qquad \text{---(3.5)}$$

4. Solution of the Equations:

Now equation (3.2) is a third order differential equation, which requires three boundary conditions for its solution. But the present problem provides only two boundary conditions. To overcome this difficulty, we follow small perturbation technique to obtain the approximate solution of equation (3.2) and finally we get the skin friction at the lower and upper plate are calculated taking y=0 and y=1 respectively.

$$\tau_0 = \tau_{xy} |_{y=0}$$

$$= -t^n + t(a_1 + R_c b_1) - R_c \left\{ -2Rta_1 + 2Ra_2 t^2 - nt^{n-1} + a_1 \right\} \qquad \text{--------------------------------------- (4.1)}$$

And

$$\tau_1 = \tau_{xy} |_{y=1}$$

$$= -t^n + t(a_1 + R_c b_1) - R_c \left\{ -2Rta_1 + 2Ra_2 t^2 - nt^{n-1} - a_1 \right\} \qquad \text{-------------------------------------- (4.2)}$$

Further, the rate of heat transfer at the lower plate is given by

$$NU_0 = -\frac{\partial \theta}{\partial y} |_{y=0} = -c_2 t^2 \qquad \text{-------------------------------------- (4.3)}$$

And that at the upper plate is

$$NU_1 = -\frac{\partial \theta}{\partial y} |_{y=1} = 2c_1 t \qquad \text{-------------------------------------- (4.4)}$$

5. Results and Discussions:

In this case, the effects of various fluid parameters on the flow behaviour of viscoelastic fluid flowing through a medium have been studied with the help of graphs and tables. The study is carried out for two positive values of n i.e.

(i) n=1, constant acceleration

(ii) n=1/2, variable acceleration

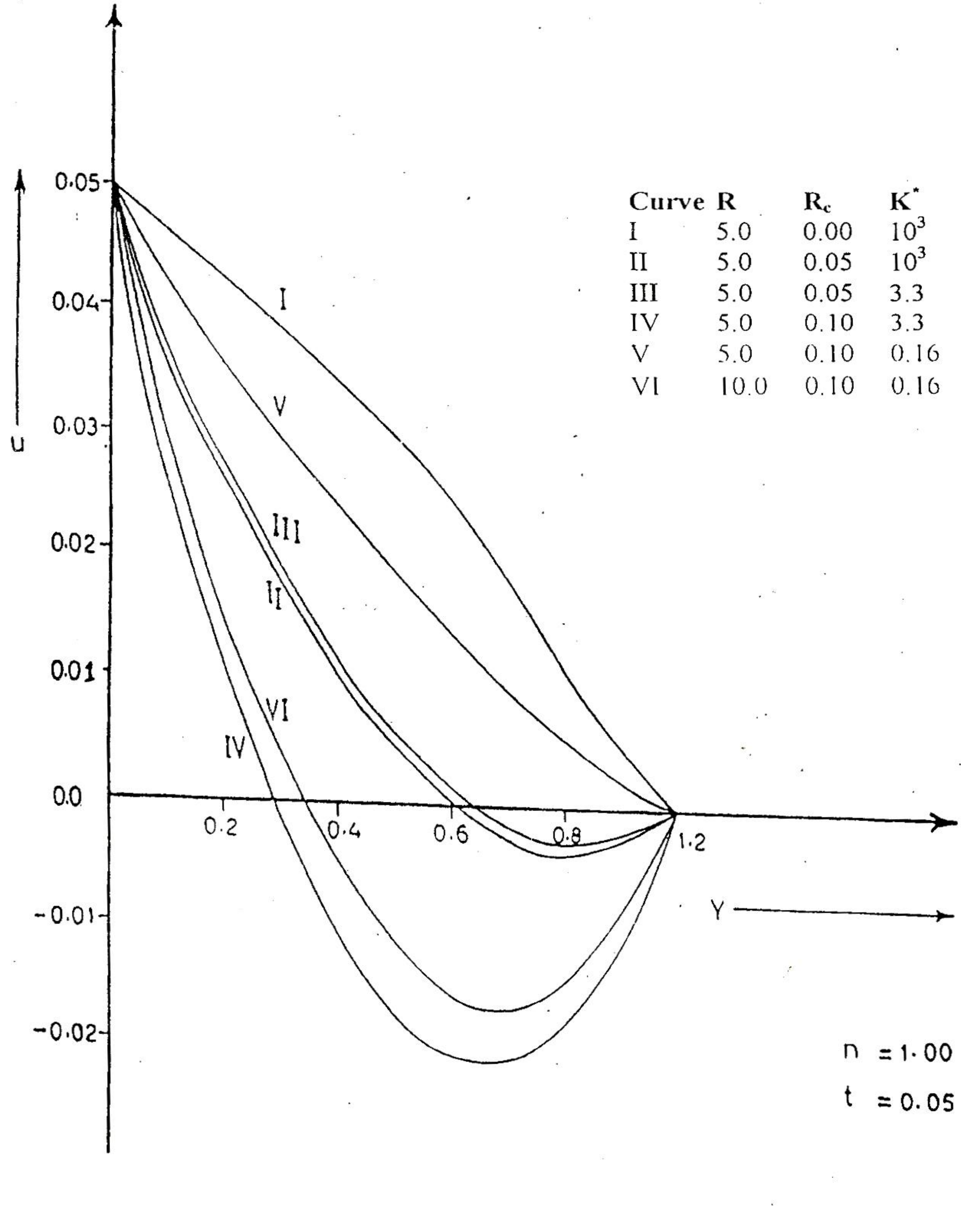


Fig. 1 : Effect of R, R_c and K^* on velocity field.

The effects of R, Rc and K^* velocity field are exhibited by the curves of fig. 1. In the absence of magnetic field, it is observed that the velocity decreases with Rc attaining negative values between y=0.6 and y=1.0 (curves I and II). The velocity of the fluid decreases rapidly with Rc (curves III and IV), the velocity increases having all positive values. This behaviour is ascertained from the curves IV and V. The effects of Reynolds number (R) on the velocity profiles are shown in the curves V and VI, which reveal that the velocity decreases as R increases keeping the permeability of the porous medium constant.

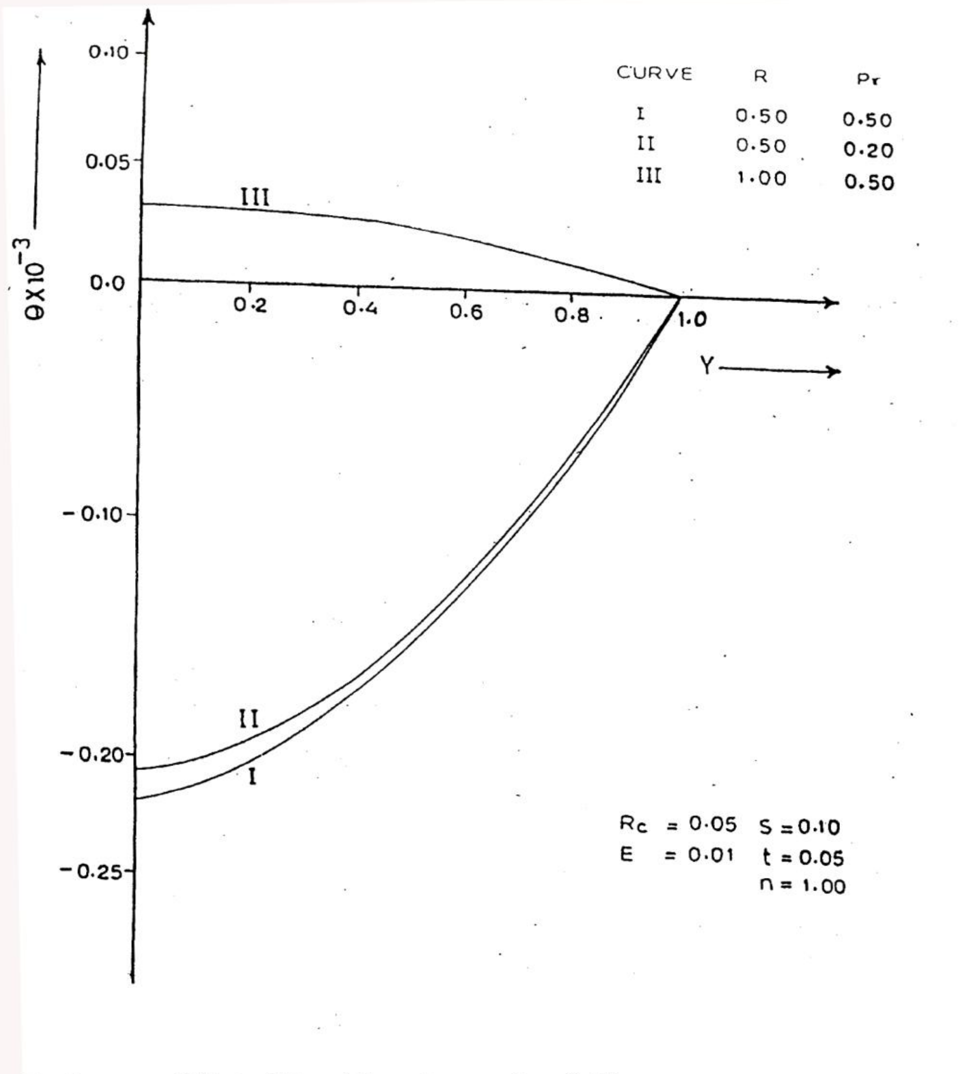


Fig. 2 : Effect of R and P_r on temperature field

Fig. 2 depicts the influence of R and Pr on the temperature field. The temperature which is negative at the lower plate increases with decrease in Pr. Further, the rise in R increases the temperature at a high rate near the lower plate.

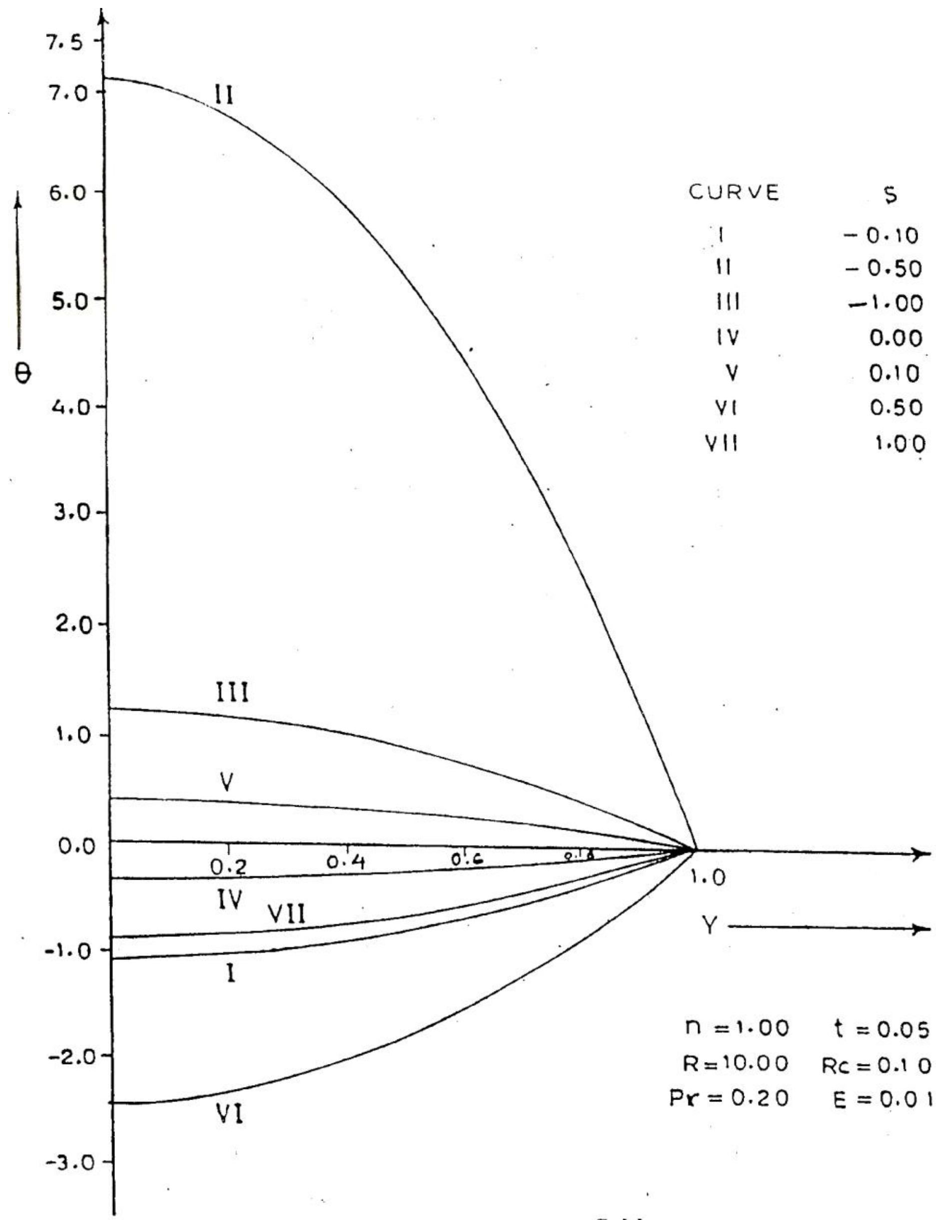


Fig. 3 Effect of S on temperature field

The effect of source parameter S on temperature field is illustrated in Fig. 3, where the negative values of S are meant for sink strength and positive values of S for source strength. As the sink strength falls from -0.10 to -0.50, the temperature of the fluid rises sharply (curves I and II). But, the temperature decreases with the further increase of sink strength which is marked from curves II and III. In the absence of source or sink, the temperature of the fluid is low having negative values. This clearly indicates that cooling effect is produced while the visco-elastic fluid exhibits Couette flow under the influence of an external uniform transverse magnetic field. In the presence of an internal heat generating source in the field, the temperature rises attaining positive values, when the source strength is low (S=0.1). Further increase in S decreases the temperature (Curve VI). However, a deviation is marked in case of S=1.0.

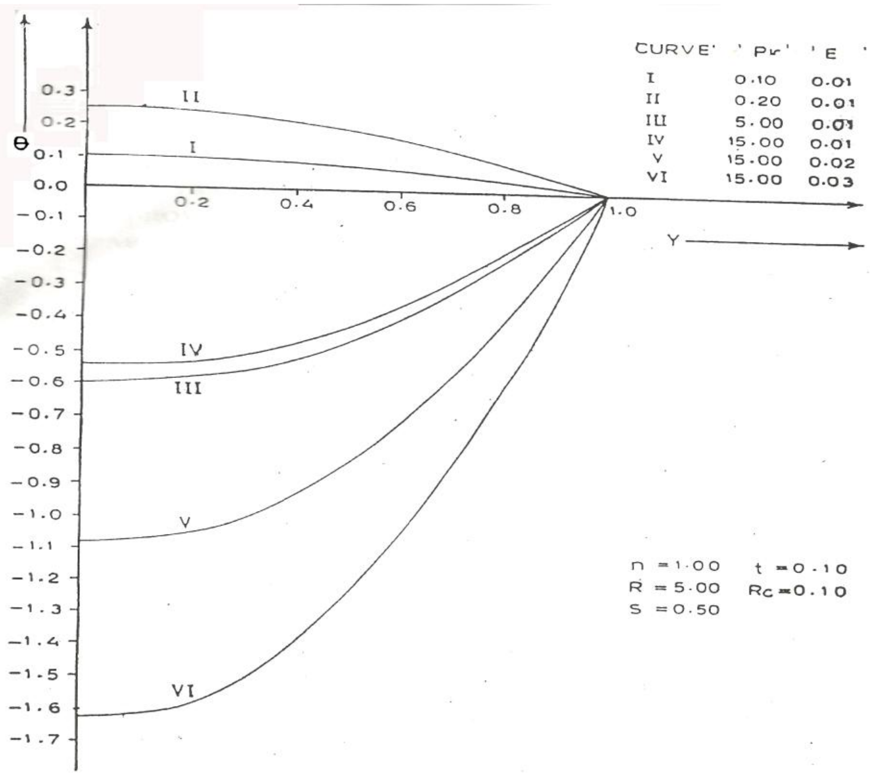


Fig. 4 : Effect of P_r and E on temperature field,

Fig. 4 explains the influence of Prandtl number and Eckert number on the temperature profiles. Curves I and II exhibit the effects of low Prandtl number rise and reveal the fact that the temperature rises with the rise of Pr from 0.1 to 0.2 within the boundary conditions imposed. Further, it is observed that the temperature falls with the rise of Prandtl number (Pr=5.0). For higher values of Prandtl number, the fall in temperature is somewhat slower (Curve IV). The effect of Eckert number on the temperature field shown in the curves IV, V and VI unveil the fact that the rise in E produces a sharp fall in temperature.

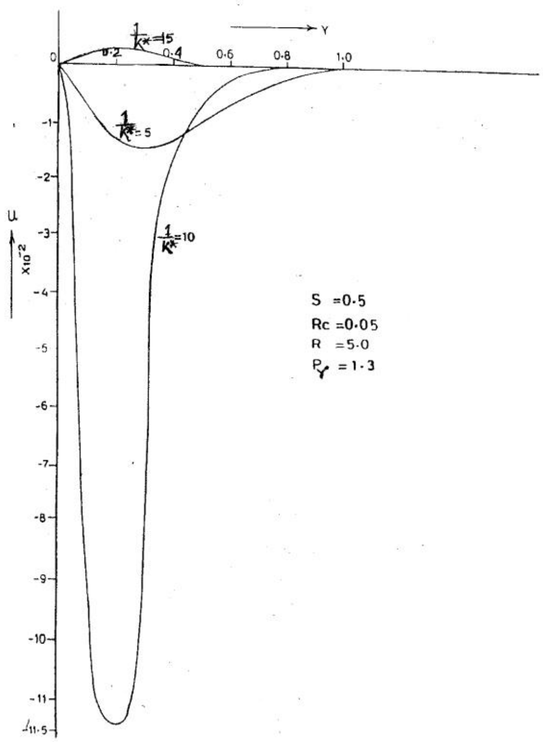


Fig. 5 : Effect of $\frac{1}{K^*}$ on Velocity Field

Fig.5. depicts the influence of $\frac{1}{K^*}$ on velocity field. The velocity which is negative at the lower

plate decreases with increase in value of $\frac{1}{K^*}$. Further, the rise in value of $\frac{1}{K^*}$ increases the velocity at a high rate near the lower plate and attains zero value between 0.4 and 0.6

TABLE

Effects of R, R_c and K^* on skin friction for n = 1.0, t = 0.05, S = 0.5, P_r = 5.0 and E = 0.02

K* \ Rc		0.0		0.05		0.10	
	R	SKFI	SKF2	SKF1	SKF2	SKF1	SKF2
3.3	5.0	$-.482503\times10^{-1}$	$-.517497\times10^{-1}$	.-133580	0.648524×10^{-1}	-.218910	0.181455
	10.0	$-.459776\times10^{-1}$	$-.540224\times10^{-1}$	-.378443	0.682500×10^{-1}	-.710909	0.190522
	15.0	$-.437049\times10^{-1}$	$-.562951\times10^{-1}$	-.709676	$-.147212\times10^{-1}$	$-.137565\times10^{-1}$	0.268527×10^{-1}
0.16	5.0	$-.496650\times10^{-1}$	$-.503350\times10^{-1}$	$-.227148\times10^{-1}$	0.968847x10-2	0.423549×10^{-1}	0.697119×10^{-1}
	10.0	$-.478722\times10^{-1}$	$-.521278\times10^{-1}$	-.160166	0.160444×10^{-1}	-.272459	0.842167×10^{-1}
	15.0	$-.460794\times10^{-1}$	$-.539206\times10^{-1}$	-.356650	-.667221	$-.193478\times10^{-1}$	-0.193478×10^{-1}

The values of shear stresses for different values of R, Rc and K* are entered in the above table keeping all other parameters fixed.

It is observed that the increase in Reynolds number (R) causes the value of shear stress at the lower plate to be more and more negative for visco-elastic fluid while for viscous fluid the negativeness of the shear stress at the lower plate decreases. It is further noticed that in case of viscous fluid (Rc=0), the value of shear stress at the upper plate decreases as R increases. Interestingly, the skin friction at the upper plate first increases and then decreases with the continuous rise of R for non-Newtonian fluid (Rc>0). The increase in the elasticity of the fluid decreases the value of skin friction at the lower plate and this effect is reversed for the upper plate. From the reading of this table, it is observed that the value of skin friction at the lower plate increases for visco-elastic fluid and decreases for viscous fluid, with the rise of permeability factor (K*), while this effect is reversed for upper plate.

TABLE

Effects of t, n, R, R_c on the rates of heat transfer for S = 0.1, K^* = 3.3, Pr = 0.1 and E = 0.01.

R	t	n/R_c	05		1.0	
			NU_0	NU_1	NU_0	NU_1
0.5	0.05	0.0	$-.571016 \times 10^{-6}$	$.297534 \times 10^{-4}$	$-.238178 \times 10^{-4}$	$-.639748 \times 10^{-4}$
		0.05	$.101282 \times 10^{-5}$	$.180371 \times 10^{-3}$	$-.230699 \times 10^{-4}$	$-.477565 \times 10^{-4}$
		0.10	$-.59665 \times 10^{5}$	$.170988 \times 10^{-3}$	$-.223221 \times 10^{-4}$	$-.315381 \times 10^{-4}$
	0.10	0.0	$-.228406 \times 10^{5}$	$.595068 \times 10^{-4}$	$-.952711 \times 10^{-4}$	$-.127950 \times 10^{-4}$
		0.05	$.405128 \times 10^{5}$	$.200741 \times 10^{-4}$	$-.922797 \times 10^{4}$	$-.955130 \times 10^{-4}$
		0.10	$.103866 \times 10^{-1}$	$.341976 \times 10^{-4}$	$-.892884 \times 10^{-4}$	$-.630762 \times 10^{-4}$
5.0	0.05	0.0	$-.117952 \times 10^{-5}$	$-.648397 \times 10^{-4}$	$-.494657 \times 10^{-4}$	$-.284936 \times 10^{-4}$
		0.05	$-.269204 \times 10^{-2}$	$.753349 \times 10^{-4}$	$-.124071 \times 10^{-4}$	$.351382 \times 10^{-4}$
		0.10	$-.526613 \times 10^{-2}$	$.157154 \times 10^{0}$	$-.243195 \times 10^{-2}$	$.73125 \times 10^{-4}$

This Table shows the dependence of rates of heat transfer at both the plates on t, n, R and Rc all other variables remaining fixed. It is noticed that when t increases, the rates of heat transfer also increases at both the plates for n=0.5, but decreases for n=1.0 in case of Newtonian and non-Newtonian fluids. The increase in the value of ‘n’ results in the reduction of rates of heat transfer at both the plates and for both viscous and visco-elastic fluids. It is observed that the value of Nusselts number at the lower plate decreases while that at the upper plate increases with the rise of Reynolds number (R) in case of Rc>0. When Rc=0 the value of Nusselts number falls at both lower and upper plates with the rise of R. Finally, as the elastic property of the fliid increases, the rates of heat transfer at both the plates also increases. All the above conclusions are drawn in the presence of a porous medium i.e. 1/K*>0.

CONCLUSIONS:

(i) The velocity of flow decreases with the rise of the value of the Reynolds number as well as the elastic parameter Rc.

(ii) The rates of heat transfer at both the plates of the channel increases with the increase of Rc.

(iii) Cooling effect is produced while the visco-elastic fluid exhibits Couette flow under the action of an external transverse magnetic field.

(iv) In the absence of porous media, the results obtained by the present investigation match with the results arrived at by Dash and Biswal.

This work can be applied in electric power generator, extrusion of plastics in the manufacture of Rayon and Nylon etc.

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