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AUTOMATION OF OPTIMAL PRODUCTION SCHEDULES FOR A CERTAIN CLASS OF DETERMINISTIC INVENTORY PROBLEMS USING DYNAMIC PROGRAMMING

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ABSTRACT

This research article conceptualized, formulated and designed Excel solution templates and corresponding exposition for optimal production schedules for a certain class of deterministic inventory problems. The article also provided illustrative examples which demonstrated the efficiency, utility and processing power of the solution templates. The deployment of the solution templates circumvents the inherent tedious, and prohibitive manual computations associated with dynamic programming formulations and recursions and may be optimally appropriated for sensitivity analyses on each model.

Keywords: *Automation, Deterministic, Dynamic programming recursions, Excel, Inventory, Optimal production schedules, Solution Templates.*

1. INTRODUCTION

The need for automation of processesis imperative; this is especially so of optimal policy prescriptionsof dynamic programming-based outputs with resulting tremendous savings in time, cost and energy. Consequently, any desired levels of sensitivity analyses can be easily undertaken and accomplished with great rapidity; needless to say that long horizon lengths can be assigned henceforth in such problems that hitherto could hardly be contemplated due to the 'curse of dimensionality' of dynamic programming recursions.

Ukwu [1,2] designed and implemented Excel solution templates for optimal investment strategies and the corresponding optimal rewards, for the largest class of certain probabilistic dynamic investment problems for practical and realistic consideration, using backward recursive dynamic programming. It went further to optimally deploy the templates for sensitivity analysis of the problem, in just a matter of minutes. These activities could hardly be contemplated in manual computations. The templates reflected and demonstrated consistency with the base results. In the sequel, Ukwu [3] designed and fully automated the solution templates for the determination of the optimal time replacement policies for a time perspective class of machine replacement problems with pertinent dynamic data given as functions of new machine purchase year and machine age. Ukwu [4,5] went on to design and automate prototypical solution templates for batch optimal policy prescriptions for a certain stationary class of equipment replacement problems, with any set of feasible starting ages, complete with an algorithmic exposition on the interface and solution process.

Ukwu et al. [6] investigated the problem of fund allocations from certain investment portfolio and obtained the optimal investment strategies using backward dynamic programming recursive approach. In what followed, Ukwu et al. [7] conceptualized, formulated and designed Excel solution templates and corresponding algorithm for the optimal allocation of funds from the investment portfolio. The work also provided illustrative examples which demonstrated the efficiency, utility and processing power of the solutions templates. A careful study of above works among others reveals that they can be leveraged and exploited for optimal production strategies of a certain class of inventory problems.

The optimal production schedule for a certain class of deterministic inventory problems using backward dynamic programming recursions has been investigated by Taha [8] and Winston [9], among other authors. Unfortunately the related issue of computational feasibility is yet to be addressed. Iterations in dynamic programming recursions are computationally intractable and doomed to failure for practical purposes, especially for large scale applications; thus, the need for electronic implementation of optimal production schedules cannot be overemphasized.

This paper fills the above need by formulating and designing prototypical Excel automated solution templates based on dynamic programming recursions with a view to minimizing production costs in inventory problems. These templates are deployed to eliminate the computational tedium and other constraints, as well as pave the way for instant generation of optimal strategies, as soon as the pertinent data are keyed in. The optimality results will be facilitated by a robust investigation of the solution templates in the afore-mentioned works of Ukwu and others, and by sound reasoning regarding the implicit dependence of the dynamic programming recursions on stage numbers. Finally, this study will deploy the templates to obtain optimal production schedules for problem instances with specified entering inventories.

2. THEORETICAL UNDERPINNING

The production scheduling problem is the determination of what quantity to produce during each period to minimize the over-all production cost, given the following parameters:

(i) Problem horizon length

(ii) Entering (Incoming) inventory during each period

(iii) Demand specifications for the various periods

(iv) Set up costs associated with positive production quantities

(v) Holding (Storage) cost per period per unit inventory

(vi) Capacity constraints on production/demand

(vii) Production function specification

The model for the paper is backward dynamic programming recursions for the determination of optimal production levels with respect to a stated class of deterministic inventory problems.

2.1 Notations for the model parameters:

T : Horizon length of the problem; t : Stage identifier; $t \in \{T, T-1, \cdots 2, 1\}$

i_t : Entering inventory during period *t*; d_t : Demand during period *t*

I_t : The set of feasible entering inventory in stage $t, t \in \{T, T-1, \cdots, 2, 1\}$.

h: Holding cost per unit inventory per period

$x_t(i_t)$: Production quantity during period *t*, given that the entering inventory is i_t

$c(x) = k_0 + k_1 x + k_2 x^2 + k_3 x^3 + k_4 x^4$: Production function

$f_t(i_t)$: Minimum production cost for period $t, t+1, \cdots,$ and T given that the entering inventory during period t is i_t.

The relevant dynamic programming recursions are the following:

$$f_t\left(i_t\right)=\min_{t\in I_t}\left\{c\left(x_t(i_t\right)+f_{t+1}\left(i_{t+1}\right)\right\},t\in\{T,T-1,\cdots,2,1\};\,f_{T+1}\left(.\right)=0\Rightarrow f_T\left(x_T(i_T)\right)=c\left(x_T(i_T)\right)$$

$i_{t+1}=i_t+x_t(i_t)-d_t\geq 0\Rightarrow$ demand must be satisfied; $x_t(i_t)\geq 0\Rightarrow$ production must be at nonnegative levels

$i_t,x_t(i_t)\in\mathbf{N},d_t\in\mathbf{N},\sum_{t=1}^{T}d_t\in\mathbf{N},h\in\mathbf{R}^+,T\in\mathbf{N},\,f_t\left(i_t\right)\in\mathbf{R}^+;\,f_t\left(i_t\right)=0\ \Leftrightarrow$ there is no production activity.

$\sum_{t=1}^{T}d_t\in\mathbf{N}\Rightarrow$ there must be at least one positive demand during the horizon, otherwise there is nothing to solve.

$$c\left(x_t\left(i_t\right)\right)=\left(k_0+\sum_{j=1}^{4}k_jx_t^j\left(i_t\right)\right)$$

The development of appropriate solution templatesisdetailed below and it is based on the above backward dynamic programming model.

3. METHODS, RESULTS AND DISCUSSION

3.1 Excel Implementation Template Design and interface For Optimal Production Schedules

Terminal Stage, *T*							
	A	B	C	D	E	F	G
1	Excel Solution…						
2	Recursions						
3							
4	No. of periods:	*T*	Max Dem =	5			
5	$c(x)=\ k_0\,\text{sgn}\left(x\right)+k_1x+k_2x^2+k_3x^3+k_4x^4$						
6							
7		k_0	k_1	k_2	k_3	k_4	
8	Values of k_i:						
9	Demand:	d_T					
10		Stage	= [1]				
11	*i*	$\boldsymbol{x}(i)$	$f(i)=c(x)$				
12	0	= [3]	= [5v]				
13	= [2v]	=[4v]					
…							
17							

19	Penultimate Stage, *T*–1						
20	Demand:	$d_{T\text{-}1}$					
21		Stage	= [6]	Computations			
22	*I*	$\boldsymbol{x}(i)$	$h(i+x-d)+c(x)$	$f(i+x-d)$	C+D	$x*$	$f(i\ )$
23	0	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
24	= [7v]	= [9v]					
…							
31							
33	1	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
34	= [7v]	= [9v]					
…							
42							
44	2	= [8]	= [10]	= [11]	= [12]	= [13]	=[14]
45	= [7v]	= [9v]					
…							
53							
55	3	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
56	= [7v]	= [9v]					
…							
6							

4							
66	4	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
67	= [7v]	= [9v]					
…							
75							
77	5	= [8]	=[10v]	=[11v]	= [12v]	=[13v]	= [14v]
78	= [7v]	= [9v]					
…							
89							

Stage (*T*-2)							
91	Demand	$d_{T\text{-}2}$					
92		Stage	= [15]	Computations			
93	*i*	$\boldsymbol{x}(i)$	$h(i+x-d)+c(x)$	$f(i+x-d)$	C+D	x^*	$f(i)$
94	0	= [17]	=[19v]	=[20v]	= [21v]	=[22v]	= [23v]
95	= [16v]	= [18v]					
…							
100							

The tabular process continues down to stage 1. Any stage number less than one indicates infeasibility. Blank cells also indicate forbidden choices.

3.2 Exposition on the Solution Template

“= [m] “indicates code segment m to be typed in the resident cell location.

“= [mv] “with letter v fixed indicates code segment m to be typed in the resident cell location followed by vertical crosshair-dragging activity.

Multiple “= [m] “ or“= [mv] “ indicates copying activity of the code segment from the initial cell locations.

Type the titles of the template in excel rows 1 and 2 as indicated above. Type the production cost function $c(x)$ and its parameters in the indicated cell locations. An identifier succeeded by a colon indicates desired user input subject to the imposed restrictions.

3.3 Initialization of Stage numbering:

= [1]: Type the following code segment =B5, in cell location C10, <Enter> to initialize the stage number at integral input value m.

3.3.1 Stage *T*Implementations

Incoming inventory, *i* :

Step 1: Input the number 0 in cell location A12.

Step 2: = [2v]: Type the following code segment =IF($A12<$B$9,1+$A12,""), in cell location A13,<Enter> to secure the next feasible higher contiguous integer. Click the cursor back on A13, position the cursor at the bottom right edge of the cell until a crosshair appears. Then drag the crosshair vertically down to cell location A17, to secure all feasible values of *i*.

Henceforth the act of clicking the cursor back on an indicated cell location, positioning the cursor at the bottom right edge of the cell until a crosshair appears and then dragging the crosshair vertically down to an indicated cell location will be described as clerical routine/duty.

Production Quantity, $x_T(i)$:

Step 1: =[3]: “=$B$9-$A12”, in B12,<Enter>

Step 2.=[4v]: “=IF($A13 ="","",$B$9-$A13)”, in B12, followed by the clerical duty, down to B17.

Minimum Costs, $f_T(i), f(i)$, for Entering Inventory

= [5v]: “=($B$8+$C$8*$B12+$D$8*$B12^2+$E$8*$B12^3+$F$8*$B12^4)*SIGN($B12)”, inC12, followed by the clerical duty, down to C17.

3.3.2 Stage (*T*-1) Computations.

Incoming inventory, *i* = 0:

Step 1: Input the number 0 in cell location A22.

Step 2: = [7v]: Type the following code segment “=IF(COUNT(A$22:A22)<$B$9+$B$19,0,"")”, in cell location A23,followed by the clerical duty down to A31.

Production Quantity, $x_{T-1}(i)$:

Step 1: =[8]: “=MAX($B$19-$A22,0)<Enter>

Step 2. =[9v]: “=IF(OR($A23="",$B22>=$B$9+$B$19),"",1+$B22)”, in B23, followed by the clerical duty, down to B31.

Implementation of C+D

=[12v]: “ =IF(OR($C22="",$D22=""),"",$C22+$D22)”,in E22, followed by the clerical duty, down to E31.

Implementation of x*

=[13v]: “ = IF($E22=MIN($E$22:$E$31),$B22,"")”, in F22, followed by the clerical duty, down to F31.

Minimum Costs, $f_{T-1}(i)$, for Entering Inventory

= [14v]: “=IF($F22="","",$E22)”, inG22, followed by the clerical duty, down to G31.

Incoming inventory, *i* = 1:

Step 1: Input the number 1 in cell location A33

Step 2: = [7v]: Type the following code segment “=IF(COUNT(A$33:A33)<$B$9+$B$19,0,"")”, in cell location A34,followed by the clerical duty down to A42.

Production Quantity, $x_{T-1}(i)$:

Step 1: =[8]: “=MAX($B$19-$A33,0)<Enter>

Step 2: = [9v]: “=IF(OR($A34="",$B33>=$B$9+$B$19),"",1+$B33)”, in B34, followed by the clerical duty, down to B42.

Implementation of *h*(*i*+*x*-*d*)+*c*(*x*)

=[10v]: “=IF(OR($A33 = "", $B33 = ""), "",0.5*($A33+$B33-$B$19)+($B$8+$C$8*$B33+$D$8*$B33^2+$E$8*$B33^3+$F$8*$B33^4)*SIGN($B33)), in C33, followed by the clerical duty, down to C42.

Implementation of *f* (*i*+*x*-*d*)

=[11v]: “=IF(OR($A33 = "", $B33 = ""), "",IF($A33+MAX($B33,0)− MAX($B$19,0)=0,MAX($C$12,0),IF($A33+MAX($B33,0)− MAX($B$19,0)=1,MAX($C$13,0),IF($A33+MAX($B33,0)− MAX($B$19,0)=2,MAX($C$14,0),IF($A33+MAX($B33,0)− MAX($B$19,0)=3,MAX($C$15,0),IF($A33+MAX($B33,0)− MAX($B$19,0)=4,MAX($C$16,0),IF($A33+MAX($B33,0)− MAX($B$19,0)=5,MAX($C$17,0),"")))))))”, in D33, followed by the clerical duty, down to D42.

Implementation of C+D

=[12v]: “=IF(OR($C33="",$D33=""),"",$C33+$D33)”,in E33, followed by the clerical duty, down to E42.

Implementation of *x**

=[13v]: “=IF($E33=MIN($E$33:$E$42),$B33,"")”, in F33, followed by the clerical duty, down to F42.

Minimum Costs, $f_{T-1}(i)$ **, for Entering Inventory**

= [14v]: “=IF($F33="","",$E33)”, in G33, followed by the clerical duty, down to G42.

Incoming inventory $i \in \{2,3,4,5\}$

Step 1: Input the number *i* in cell location A(33+(*i* -1)*11)

Step 2: Replace the “ 33” in the code segment “ = [7v] :“ with(33+11(*i*-1)) 1n cell location A(1+(33+11(*i*-1))),followed by the clerical duty down to A(9+(1+(33+11(*i*-1)))).

Production Quantity, $x_{T-1}(i)$ **:** $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in steps 1 and 2 corresponding to i =1,incorporating “=[8]:” and “ =[9v]:”.

Implementation of *h(i+x-d)+c(x)*: $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in “ =[10v]:” and the ensuing clerical duty.

Implementation of *f (i+x-d)*: $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in “ =[11v]:” and the ensuing clerical duty.

Implementation of *C+D*: $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in “ =[12v]:” and the ensuing clerical duty.

Implementation of *x**: $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in “ =[13v]:” and the ensuing clerical duty.

Minimum Costs, $f_{T-1}(i)$ **:** $i \in \{2,3,4,5\}$

Add 11(i-1) to all relative row references in “ =[14v]:” and the ensuing clerical duty.

Stage (*T*-2).

Implementations of *i, xi,h(i+x-d)+c(x)*:

Add 69 to all row references of cells in stage T-1, for corresponding *i*values, excluding the $B$8:$F$8, with global scope.

Implementations of *f (i+x-d)*:

Replace the code segment in $D22 (for i = 0, in stage T-1) with the code segment below:

“ =IF(OR($A91 = "", $B91 = ""), "",IF($A91+MAX($B91,0)−MAX($B$88,0)=0,MAX($G$22:$G$31,0),IF($A91+MAX($B91,0)−MAX($B$88,0)=1,MAX($G$33:$G$42,0),IF($A91+MAX($B91,0)−MAX($B$88,0)=2,MAX($G$44:$G$53,0),IF($A91+MAX($B91,0)−MAX($B$88,0)=3,MAX($G$55:$G$64,0),IF($A91+MAX($B91,0)−MAX($B$88,0)=4,MAX($G$66:G$75,0),IF($A91+MAX($B91,0)−MAX($B$88,0)=5,MAX($G$77:$G$86,0),"")))))))”, without the quotes.

Then apply the clerical duty from the input cell $D91down to $D100.

For $i \in \{1,2,3,4,5\}$, add 11i to all relative row references corresponding to *i* = 0.

Implementations C+D,x* and f (i):

The code segments for **C+D,x* and f (i)** are invariant and hence may be copy from stage T-1 down to stage 1.

Stage $t\,, t \in \{T-3,\cdots,1\}$.

Implementations of *i, x(i),h(i+x-d)+c(x)*:

Add $69\,(T-2-t)$ to all row references of cells in stage (T-2), for corresponding *i* values, excluding the $B$8:$F$8, with global scope, bringing the exposition to an end.

By an appeal to Ukwu[3]the optimal production strategies can be obtained in respect of sub-horizon lengths $n \in \{1,\ 2,\cdots,T-1\}$ from the same solution template using a top-down horizon length count in which stage $(T+t\ -n)$ in the T - horizon problem corresponds to stage $t;\, t \in \{1,2,\cdots,n\}$ in the *n*-horizon problem.

3.4 EXPOSITION

3.4.1 General Application problem

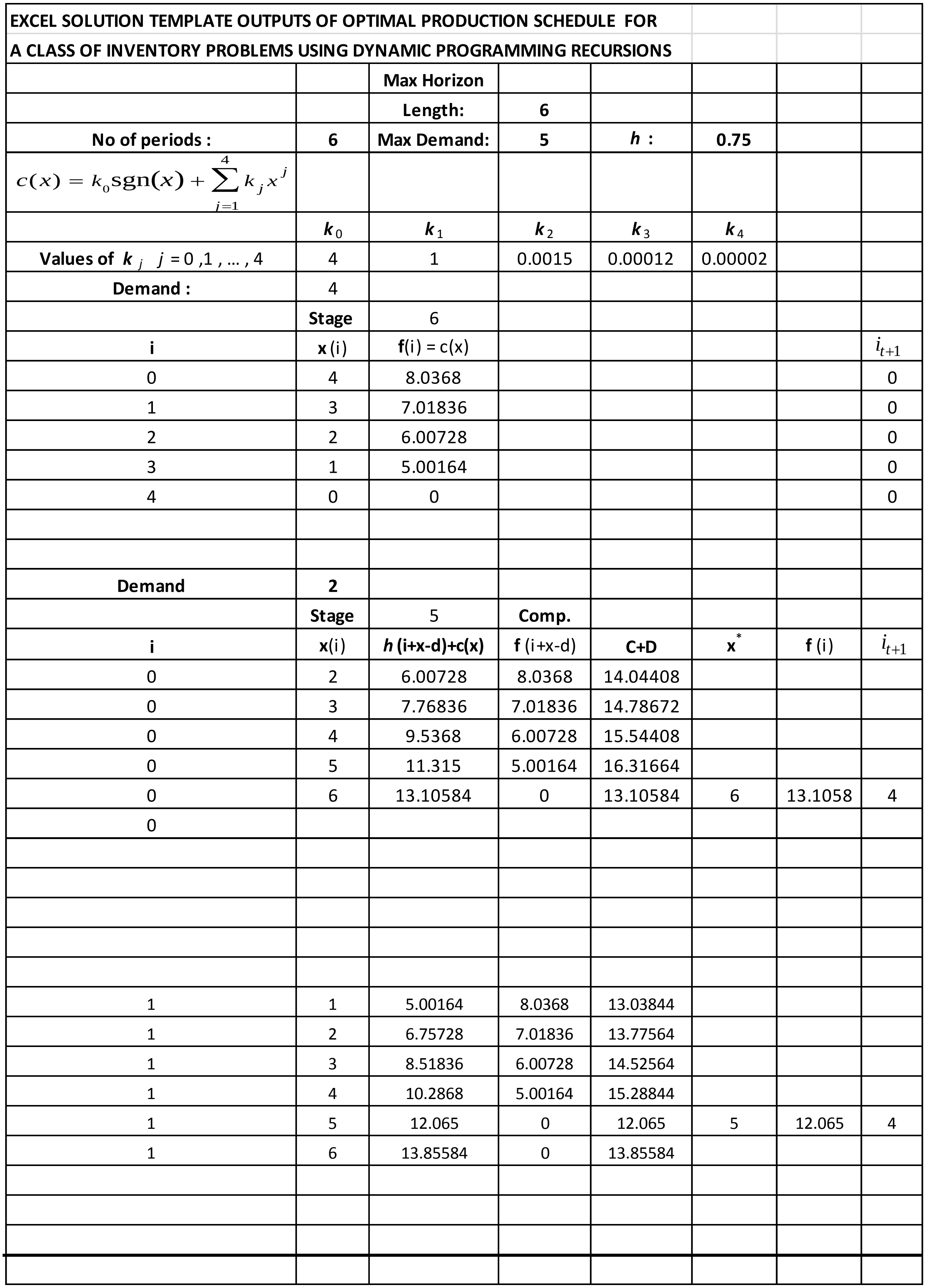
A company knows that the demand for its product during each of the next T months will be as follows: month$t$, $d_t$units; $t \in \{1, 2, \cdots, T\}$. At the beginning of each month the company must determine how many units should be produced during the current month. During a month in which any units are produced, a setup cost of $\$\, k_0$ is incurred. In addition, there is a variable quartic cost function:

$c(x) = k_1x + k_2x^2 + k_3x^3 + k_4x^4,$ for every x units produced.

At the end of each month a holding cost of $\$h$ per unit on-hand is incurred. Historical data reveal that demand for any period does not exceed D units during each month. The company wants to determine a production schedule that will meet all demands on time and will minimize the sum of production and holding costs during the T months.

Solve the above problem with the following pertinent data: $D = 5, T = 6$

Figure 1: Excel solution template outputs of optimal production schedule for a class of inventory problems using dynamic programming recursions



EXCEL SOLUTION TEMPLATE OUTPUTS OF OPTIMAL PRODUCTION SCHEDULE FOR							
A CLASS OF INVENTORY PROBLEMS USING DYNAMIC PROGRAMMING RECURSIONS							
		Max Horizon					
		Length:	**6**				
No of periods :	**6**	**Max Demand:**	**5**	***h*** **:**	**0.75**		
$c(x) = k_0 \text{sgn}(x) + \sum_{j=1}^{4} k_j x^j$							
	$\boldsymbol{k}_0$	$\boldsymbol{k}_1$	$\boldsymbol{k}_2$	$\boldsymbol{k}_3$	$\boldsymbol{k}_4$		
Values of $\boldsymbol{k}_j$ $j = 0, 1, \ldots, 4$	4	1	0.0015	0.00012	0.00002		
Demand :	4						
	Stage	6					
i	**x** (i)	**f**(i) = c(x)					i_{t+1}
0	4	8.0368					0
1	3	7.01836					0
2	2	6.00728					0
3	1	5.00164					0
4	0	0					0
Demand	**2**						
	Stage	5	**Comp.**				
i	**x**(i)	***h*** **(i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	$\mathbf{x}^*$	**f** (i)	i_{t+1}
0	2	6.00728	8.0368	14.04408			
0	3	7.76836	7.01836	14.78672			
0	4	9.5368	6.00728	15.54408			
0	5	11.315	5.00164	16.31664			
0	6	13.10584	0	13.10584	6	13.1058	4
0							
1	1	5.00164	8.0368	13.03844			
1	2	6.75728	7.01836	13.77564			
1	3	8.51836	6.00728	14.52564			
1	4	10.2868	5.00164	15.28844			
1	5	12.065	0	12.065	5	12.065	4
1	6	13.85584	0	13.85584			

2	0	0	8.0368	8.0368	0	8.0368	0
2	1	5.75164	7.01836	12.77			
2	2	7.50728	6.00728	13.5146			
2	3	9.26836	5.00164	14.27			
2	4	11.0368	0	11.0368			
2	5	12.815	0	12.815			
3	0	0.75	7.01836	7.76836	0	7.76836	1
3	1	6.50164	6.00728	12.5089			
3	2	8.25728	5.00164	13.2589			
3	3	10.01836	0	10.0184			
3	4	11.7868	0	11.7868			
3	5	13.565					
4	0	1.5	6.00728	7.50728	0	7.50728	2
4	1	7.25164	5.00164	12.2533			
4	2	9.00728	0	9.00728			
4	3	10.76836	0	10.7684			
4	4	12.5368					
4	5	14.315					
5	0	2.25	5.00164	7.25164	0	7.25164	3
5	1	8.00164	0	8.00164			
5	2	9.75728	0	9.75728			
5	3	11.51836					
5	4	13.2868					
5	5	15.065					

Demand	**3**						
	Stage	4	**Comp.**				
i	**x**(i)	***h* (i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	**x^***	**f** (i)	
0	3	7.01836	13.10584	20.1242			
0	4	8.7868	12.065	20.8518			
0	5	10.565	8.0368	18.6018	5	18.6018	2
0							
0							
1	2	6.00728	13.10584	19.1131			
1	3	7.76836	12.065	19.8334			
1	4	9.5368	8.0368	17.5736	4	17.5736	2
1	5	11.315	7.76836	19.0834			
1							
2	1	5.00164	13.10584	18.1075			
2	2	6.75728	12.065	18.8223			
2	3	8.51836	8.0368	16.5552	3	16.5552	2
2	4	10.2868	7.76836	18.0552			
2	5	12.065	7.50728	19.5723			
3	0	0	13.10584	13.1058	0	13.1058	0
3	1	5.75164	12.065	17.8166			
3	2	7.50728	8.0368	15.5441			
3	3	9.26836	7.76836	17.0367			
3	4	11.0368	7.50728	18.5441			

Demand	**3**						
	Stage	4	**Comp.**				
i	**x**(i)	***h* (i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	**x** *	**f** (i)	
0	3	7.01836	13.10584	20.1242			
0	4	8.7868	12.065	20.8518			
0	5	10.565	8.0368	18.6018	5	18.6018	2
0							
0							
1	2	6.00728	13.10584	19.1131			
1	3	7.76836	12.065	19.8334			
1	4	9.5368	8.0368	17.5736	4	17.5736	2
1	5	11.315	7.76836	19.0834			
1							
2	1	5.00164	13.10584	18.1075			
2	2	6.75728	12.065	18.8223			
2	3	8.51836	8.0368	16.5552	3	16.5552	2
2	4	10.2868	7.76836	18.0552			
2	5	12.065	7.50728	19.5723			
3	0	0	13.10584	13.1058	0	13.1058	0
3	1	5.75164	12.065	17.8166			
3	2	7.50728	8.0368	15.5441			
3	3	9.26836	7.76836	17.0367			
3	4	11.0368	7.50728	18.5441			

4	0	0.75	12.065	12.815	0	12.815	1
4	1	6.50164	8.0368	14.5384			
4	2	8.25728	7.76836	16.0256			
4	3	10.01836	7.50728	17.5256			
4	4	11.7868	7.25164	19.0384			
5	0	1.5	8.0368	9.5368	0	9.5368	2
5	1	7.25164	7.76836	15.02			
5	2	9.00728	7.50728	16.5146			
5	3	10.76836	7.25164	18.02			
5	4	12.5368					
Demand	**1**						
	Stage	3	**Comp.**				
i	**x**(i)	***h* (i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	**x^***	**f** (i)	
0	1	5.00164	18.6018	23.6034			
0	2	6.75728	17.5736	24.3309			
0	3	8.51836	16.55516	25.0735			
0	4	10.2868	13.10584	23.3926	4	23.3926	3
1	0	0	18.6018	18.6018	0	18.6018	0
1	1	5.75164	17.5736	23.3252			
1	2	7.50728	16.55516	24.0624			
1	3	9.26836	13.10584	22.3742			

2	0	0.75	17.5736	18.3236	0	18.3236	1
2	1	6.50164	16.55516	23.0568			
2	2	8.25728	13.10584	21.3631			
2	3	10.01836	12.815	22.8334			
3	0	1.5	16.55516	18.0552	0	18.0552	2
3	1	7.25164	13.10584	20.3575			
3	2	9.00728	12.815	21.8223			
3	3	10.76836	9.5368	20.3052			
4	0	2.25	13.10584	15.3558	0	15.3558	3
4	1	8.00164	12.815	20.8166			
4	2	9.75728	9.5368	19.2941			
4	3	11.51836					
5	0	3	12.815	15.815	0	15.815	4
5	1	8.75164	9.5368	18.2884			
5	2	10.50728					
5	3	12.26836					

Demand	**5**						
	Stage	2	**Comp.**				
i	**x**(i)	***h* (i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	**x^***	**f** (i)	
0	5	9.065	23.39264	32.4576			
0	6	10.85584	18.6018	29.4576	6	29.4576	1
0							
0							
0							
0							
1	4	8.0368	23.39264	31.4294			
1	5	9.815	18.6018	28.4168	5	28.4168	1
1	6	11.60584	18.3236	29.9294			
1							
1							
1							
2	4	8.7868	18.6018	27.3886	4	27.3886	1
2	5	10.565	18.3236	28.8886			
2	6	12.35584					
2							
2							
2							
3	3	7.76836	18.6018	26.3702	3	26.3702	1
3	4	9.5368	18.3236	27.8604			
3	5	11.315					
3	6	13.10584					
3							
3							

4	2	6.75728	18.6018	25.3591	2	25.3591	1
4	3	8.51836	18.3236	26.842			
4	4	10.2868					
4	5	12.065					
4	6	13.85584	15.35584	29.2117			
4							
5	1	5.75164	18.6018	24.3534	1	24.3534	1
5	2	7.50728	18.3236	25.8309			
5	3	9.26836					
5	4	11.0368					
5	5	12.815	15.35584	28.1708			
5	6	14.60584					
Demand	**5**						
	Stage	1	**Comp.**				
i	**x**(i)	***h* (i+x-d)+c(x)**	**f** (i+x-d)	**C+D**	**x***	**f** (i)	
0	5	9.065	29.45764	38.5226	5	38.5226	0
0	6	10.85584	28.4168	39.2726			
0	7	12.66268	27.3886	40.0513			
0	8	14.48936	26.37016	40.8595			
0	9	16.3402	25.35908	41.6993			
0	10	18.22	24.35344	42.5734			
0							
0							
0							
0							
1	4	8.0368	29.45764	37.4944	4	37.4944	0
1	5	9.815	28.4168	38.2318			
1	6	11.60584	27.3886	38.9944			
1	7	13.41268	26.37016	39.7828			
1	8	15.23936	25.35908	40.5984			
1	9	17.0902	24.35344	41.4436			
1	10	18.97					
1							
1							
1							

2	4	8.7868	28.4168	37.2036	4	37.2036	1
2	5	10.565	27.3886	37.9536			
2	6	12.35584	26.37016	38.726			
2	7	14.16268	25.35908	39.5218			
2	8	15.98936	24.35344	40.3428			
2	9	17.8402					
2	10	19.72					
2							
2							
2							
3	3	7.76836	28.4168	36.1852	3	36.1852	1
3	4	9.5368	27.3886	36.9254			
3	5	11.315	26.37016	37.6852			
3	6	13.10584	25.35908	38.4649			
3	7	14.91268	24.35344	39.2661			
3	8	16.73936					
3	9	18.5902					
3	10	20.47					
3							
3							
4	2	6.75728	28.4168	35.1741	2	35.1741	1
4	3	8.51836	27.3886	35.907			
4	4	10.2868	26.37016	36.657			
4	5	12.065	25.35908	37.4241			
4	6	13.85584	24.35344	38.2093			
4	7	15.66268					
4	8	17.48936					
4	9	19.3402					
4	10	21.22					
4							
5	1	5.75164	28.4168	34.1684	1	34.1684	1
5	2	7.50728	27.3886	34.8959			
5	3	9.26836	26.37016	35.6385			
5	4	11.0368	25.35908	36.3959			
5	5	12.815	24.35344	37.1684			
5	6	14.60584					
5	7	16.41268					
5	8	18.23936					
5	9	20.0902					
5	10	21.97					

Using Ockham's razor, the optimal solutions are encapsulated in the following optimalitytable

Tables 1, 2 and 3: Optimal Inventory policies for specified pertinent data

Table 1

Optimal Inventory Policy Table for $\boldsymbol{T = 6, h}$**= 0.75,** $\boldsymbol{k_0}$= 4, $\boldsymbol{k_1}$= 1, $\boldsymbol{k_2}$= 0.0015, $\boldsymbol{k_3}$= 0.00012, $\boldsymbol{k_4}$=0.00002					
Stage $\boldsymbol{T}$	**Demand**	**Vector of entering inventory levels** $\{\boldsymbol{i_t}\}$	**Corresponding vector of optimal prod. Levels** $\{\boldsymbol{x_t(i_t)}\}$	**Corresponding vector of minimum cost from stage** ***t*****to stage 6**	**Vector of ending inventory levels** $\{\boldsymbol{i_{t+1}}\}$
1	5	(0, 1, 2, 3, 4, 5)	(5, 4, 4, 3, 2, 1)	(38.52, 37.49, 37.20, 36.19, 35.17, 34.17)	(0,0,1,1,1,1)
2	5	(0, 1)	(6, 5)	(29.46, 28.42)	(1, 1)
3	1	1	0	18.60	0
4	3	0	5	18.60	2
5	2	2	0	8.04	0
6	4	0	4	8.04	0

Interpretation: If the enteringin period 1 is 0, then the optimal production policy is to produce 5, 6, 0,5, 0, and 4 units in periods 1, 2, 3, 4,5 and 6 respectively with total minimum cost of 38.52 monetary units. A similar interpretation applies to the entering inventories 1 to 5.

Table 2

Optimal Inventory Policy Table for *T = 6, h* = 0.6, $\boldsymbol{k_0}$= 4,$\boldsymbol{k_1}$= 1, $\boldsymbol{k_2}$= 0.001, $\boldsymbol{k_3}$= 0.0001, $\boldsymbol{k_4}$= 0.00001

Stage *t*	**Demand**	**Vector of entering inventory levels** $\{\boldsymbol{i_t}\}$	**Corresponding vector of optimal prod. levels** $\{\boldsymbol{x_t}(\boldsymbol{i_t})\}$	**Corresponding vector of minimum cost from stage 1 to stage 6**	**Vector of ending inventory levels** $\{\boldsymbol{i_{t+1}}\}$
1	5	(0, 1, 2, 3, 4, 5)	(5, 4, 4, 3, 4, 1)	(37.983,36.964,36.537,35.525,34.517,33.5136)	(0,0,1,1,1, 1)
2	5	(0, 1)	(6, 5)	(28.9393, 27.9125)	(1, 1)
3	1	1	0	18.2687	0
4	3	0	5	18.2687	2
5	2	2	0	8.02496	0
6	4	0	4	8.02496	0

Table 3

Optimal Inventory Policy Table for *T = 4, h* = 0.5, $\boldsymbol{k_0}$= 3, $\boldsymbol{k_1}$= 1, $\boldsymbol{k_2}$= 0, $\boldsymbol{k_3}$= 0, $\boldsymbol{k_4}$= 0

Stage *t*	**Demand**	**Vector of entering inventory levels** $\{\boldsymbol{i_t}\}$	**Corresponding vector of optimal prod. levels** $\{\boldsymbol{x_t}(\boldsymbol{i_t})\}$	**Corresponding vector of minimum cost from stage 1 to stage 6**	**Vector of ending inventory levels** $\{\boldsymbol{i_{t+1}}\}$
1	1	(0, 1, 2, 3, 4, 5)	(4, 0, 0, 0, 0, 0)	(19.5, 16, 15.5, 15, 12.5, 12.5)	(3, 0, 1, 2, 3, 4)
2	3	(0, 1, 2, 3, 4)	(5, 4, 3, 0, 0)	(16, 15, 14, 11, 10.5)	(2, 2, 2, 0, 1)
3	2	(0, 1, 2)	(6, 5, 0)	(11, 10, 7)	(4, 4, 0)
4	4	(0, 4)	(4, 0)	(7, 0)	(0, 0)

The solution templates act as a supervisor program for problems in the same class. In particular they reveal,

as observed from the last table, the sub-optimality of Winston's solution, Winston ( ), with respect to the entering inventory 0 in period 1,with $f_1(0) = 20$.

4. SUMMARY

So far, this study has shown that manual computational process for optimal production scheduling problems is quite tedious and prone to errors. Also, it is virtually impossible to solve problems with increasing horizon length in manual computations. Electronic implementation is the only way forward for solving practical problems of reasonable sizes and the undertaking of sensitivity analyses. This is imperative for contract bidding and subsequent execution where the utility and power of the solution template can be easily demonstrated in just a matter of minutes, subject to correct data inputs and any modifications or revisions thereof.

5. CONCLUSION AND RECOMMENDATIONS

This article designed and automated prototypical solution templates for optimal policy prescriptions for a certain class of inventory problems, with an algorithmic exposition on the interface and solution process.

Consequently, relevant practical problems of any conceivable size can now be solved instantly as soon as the pertinent data have been organized and stored at the appropriate Excel cell locations, resulting in tremendous savings in time, cost and energy. Furthermore, any desired levels of sensitivity analyses can be easily undertaken and accomplished with great rapidity, needless to say that long horizon lengths can be assigned henceforth in inventory problems that hitherto could hardly be contemplated due to the 'curse of dimensionality' .

Finally, this study deployed the templates to obtain optimal production schedules for three problems with entering inventory in the set $\{0,1,2,\cdots,5\}$, in just a matter of minutes.

Optimal production inventory problems should never be manually solved because it is quite tedious, cumbersome, and prone to errors, especially for large problem instances. Furthermore it does not lend itself to sensitivity analyses. Therefore, for practical purposes it is recommended that electronic solution templates be deployed in the search for, and the implementations of optimal production policy prescriptions for the stated class of problems; thanks in particular to the breakthrough research results in Ukwu[1-5] and Ukwu et al. [6, 7].Furthermore, it is recommended that further research should be carried out on the feasibility of devising solution templates for optimal production schedules with respect to other relevant classes of inventory problems.

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