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FIXED POINT IN MENGER CONE METRIC SPACES

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ABSTRACT

We define Menger Cone metric space and find some fixed point results for weak contraction condition we also illustrate an example in support of our result.

Key Words: Menger Cone Metric Space, Cauchy Sequence, Fixed Point,

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1. INTRODUCTION

There have been a number of generalizations of metric space. In 1942 Menger [2] used distribution functions instead of nonnegative real numbers as values of the metric, the notion of probabilistic metric space correspond to situations when we do not know exactly the distance between the two points but we know probabilities of possible values of this distance. A probabilistic generalization of metric spaces appears to be interest in the investigation of physical quantities and physiological threshold. It is also a fundamental importance in probabilistic functional analysis. Schweizer and Sklar [4] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [5]. The development of fixed point theory in PM-spaces was due to Schweizer and Sklar [3], [4].

The concept of metric spaces is generalized by Huang and Zhang, [1] replacing the set of real numbers with an ordered Banach space, resulted the definition of the cone metric spaces. They also described the convergence of sequences and introduced the notion of completeness in cone metric spaces. They have proved some fixed point theorems of contractive mappings on complete cone metric space with the assumption of normality of a cone. Subsequently, various authors have generalized the results of Huang and Zhang and have studied fixed point theorems for normal and non-normal cones. There exist a lot of work involving fixed points used the Banach contraction principle. This principle has been extended kind of contraction mappings by various authors.

By exploiting recent work [6] and [7], we have proved some fixed point results for weak contraction condition supported by an example.

2. PRELIMINARY

Definition 2.1: Let (E, τ) be a topological vector space and P a subset of E, P is called a cone if

1. P is non-empty and closed, $P \neq \{0\}$,

2. For $x, y \in P$ and $a, b \in R \Longrightarrow ax + by \in P$ where $a, b \geq 0$

3. If $x \in P$ and $-x \in P \Longrightarrow x = 0$

For a given cone$P \subseteq E$, a partial ordering $\geq$ with respect to P is defined by $x \geq y$ if and only if $x - y \in P, x > y \, if \, x \geq y$ and $x \neq y$, while $x \gg y$ will stand for $x - y \in int\, P, int\, P$ denotes the interior of P.

Definition 2.2 A probabilistic metric space (PM space) is an ordered pair (X, F) consisting of a nonempty set X and a mapping F from $X \times X$ into the collections of all distribution functions .For $x, y \in X$ we denote the distribution function $F(x, y)$ by $F_{x,y}$ and $F_{x,y}(u)$ is the value of $F_{x,y}$ at u in R. The functions $F_{x,y}$ assumed to satisfy the following conditions:

2.2.1 $F_{x,y}\ (u) = 1\ \forall\, u > 0$ iff $x = y$,

2.2.2 $F_{x,y}(0) = 0\ \forall\, x, y$ in X,

2.2.3 $F_{x,y} = F_{y,x}\quad \forall\, x, y$ in X,

2.2.4 If $F_{x,y}(u) = 1$ and $F_{y,z}(v) = 1$

then $F_{x,z}(u + v) = 1\ \forall\, x, y, z$ in X and $u, v > 0$

Definition 2.3 A commutative, associative and non-decreasing mapping t: $[0,1] \times [0,1] \to [0,1]$ is a t-norm if and only if $t(a, 1) = a$ for all $a \in [0,1]$, $t(0,0) = 0$ and $t(c, d) \geq t(a, b)$ for $c \geq a,\ d \geq b$

Definition 2.4 A Menger space is a triplet (X, F, t), where (X, F) is a PM-space, is a t-norm and the generalized triangle inequality for all x, y, z in X $u, v > 0$

$$F_{x,y}(u + v) \geq t\ (F_{x,y}(u), F_{y,z}(v))$$

The concept of neighborhoods in Menger space is introduced as

Definition 2.5 Let (X, F, t) be a Menger space. If $x \in X, \varepsilon > 0$ and $\lambda \in (0,1)$, then (ε, λ) -neighborhood of x, called $U_x(\varepsilon, \lambda)$, is defined by

$$U_x(\varepsilon,\lambda) = \{y \in X: F_{(x,y)}(\varepsilon) > (1-\lambda)\}$$

An (ε,λ)-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \lambda \in (0,1)\}$ of neighborhood.

Remark: If t is continuous, then Menger space (X,F,t) is a Housdroff space in (ε,λ)-topology.

Let (X,F,t) be a complete Menger space and $A \subset X$. Then A is called a bounded set

if for $u > 0$

$$\lim_{u\to\infty} inf \,{}^{F_{(x,y)}}_{x,y\in A}(u) = 1$$

Definition 2.6 A sequence $\{x_n\}$ in (X,F,t) is said to be convergent to a point x in X if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ for all $n \geq N$ or equivalently $F\,(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$.

Definition 2.7 A sequence $\{x_n\}$ in (X,F,t) is said to be Cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon,\lambda)$ such that $F(x_n, x_m;\ \varepsilon) > 1-\lambda\ \forall\, n,m \geq N$.

Definition 2.8A Menger space (X,F,t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X.

Lemma 1 Let $\{x_n\}$ be a sequence in a Menger space (X,F,t), where t is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$, if there exists a constant $k(0,1)$ such that $\forall\, p > 0$ and $n \in N$

$$t\big(F(x_n, x_{n+1}; kp)\big) \geq t\left(F\big(x_{n-1,}x_n; p\big)\right),$$

then $\{x_n\}$ is Cauchy sequence.

Lemma 2 If (X,d) is a metric space, then the metric d induces, a mapping $F: X \times X \to L$ defined by $F(p,q) = H(x - d(p,q)), p,q \in R$. Further if t: $[0,1] \times [0,1] \to [0,1]$ is defined by $t(a,b) = min\{a,b\}$, then (X,F,t) is a Menger space. It is complete if (X,d) is complete.

Definition 2.9: Let M be a nonempty set and the mapping $d: M \to X$ and $P \subset X$ be a cone, satisfies the following conditions:

2.9.1 $F_{x,y}(u) > 1 \ \forall \ x, y \in X \Leftrightarrow x = y$

2.9.2 $F_{x,y}(u) = F_{y,x}(u)$

2.9.3 $F_{x,y}(u+v) \geq t\left(F_{x,z}(u), F_{z,y}(v)\right) \ \forall \ x, y \in X.$

2.9.4 for any $x, y \in X, (x, y)$ is non-increasing and left continuous.

3. Main Results

Theorem 3.1: Let (X, d) be a complete Menger Cone Metric space and P a normal cone with normal constant K. Suppose M be a nonempty separable closed subset of Menger cone metric space X and let T and S be commuting mapping defined on M satisfying the contraction

$\|F_{Tx,Ty}(u)\| \leq \lambda \|F_{Sx,Sy}(u)\|$ for all $x, y \in X$............3.1.1

And range of S contains range of T and if S is continuous, then T have unique common fixed point in X.

Proof: For each $x_0 \in X$ and $x_1 \in X$ considered such that

$y_0 = Tx_0 = Sx_1$. Therefore in general, $y_n = Tx_n = Sx_{n+1}$

$$\|Fy_{n,}y_{n-1}(u)\| = \|T_{Tx_{n},Tx_{n-1}}(u)\| \leq \lambda \|F_{Sx_{n},Sx_{n-1}}(u)\| = \lambda \|Fy_{n-1,}y_{n-2}(u)\|$$

$$\Rightarrow \|Fy_{n,}y_{n-1}(u)\| \leq \lambda \|Fy_{n-1,}y_{n-2}(u)\|$$

$$\leq \lambda^2 \|Fy_{n-2,}y_{n-3}(u)\| \leq \lambda^3 \|Fy_{n-3,}y_{n-4}(u)\|$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

$$\leq \lambda^{n-1} \|Fy_{1,}y_0(u)\|$$

Now for $n > m$

$$Fy_{n,}y_m(u) \leq Fy_{n,}y_{n-1}(u_1) + Fy_{n-1,}y_{n-2}(u_2) + Fy_{n-2,}y_{n-3}(u_3) + \cdots \ldots \ldots ..$$

$+\cdots \ldots\ldots \left\|Fy_{m+1},y_m(u_{n-m})\right\|$

Since P is normal cone

$$\left\|Fy_n,y_m(u)\right\| \le K\left[\left\|\begin{matrix} Fy_n,y_{n-1}(u_1)+Fy_{n-1},y_{n-2}(u_2)+Fy_{n-2},y_{n-3}(u_3)+\cdots \\ +\cdots Fy_{m+1},y_m(u_{n-m}) \end{matrix}\right\|\right]$$

$$\left\|Fy_n,y_m(u)\right\| \le K\left[\begin{matrix} \left\|Fy_n,y_{n-1}(u_1)\right\|+\left\|Fy_{n-1},y_{n-2}(u_2)\right\|+\left\|Fy_{n-2},y_{n-3}(u_3)+\right\|+\cdots \\ +\cdots \left\|Fy_{m+1},y_m(u_{n-m})\right\| \end{matrix}\right]$$

$$\left\|Fy_n,y_m(u)\right\| \le K[\lambda^{n-1}+\lambda^{n-1}+\lambda^{n-1}+\cdots \ldots+\lambda^m]\left\|Fy_1,y_0(u)\right\|$$

$$\left\|Fy_n,y_m(u)\right\| \le \frac{K\lambda^{n-m}}{1-\lambda}$$

$\Longrightarrow \left\|Fy_n,y_m(u)\right\| \to 0$ as $m \to \infty$

Therefore sequences $\{x_n\}=\{Tx_n\}=\{Sx_{n+1}\}$ is Cauchy sequence and X in complete therefore there exist p in X such that

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_{n+1} = p$$

Now S is continuous and T and S are commuting mappings, we get

$$Sp = S\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} S^2\, x_n$$

$$Sp = S\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} ST\, x_n = \lim_{n\to\infty} TS\, x_n$$

Now from (1) we have

$$\left\|FF_{TSx_n,Sp}(u)\right\| \le \lambda\left\|F_{S^2x_n,Sp}(u)\right\|$$

On taking $n\to\infty, we\ get$

$$\left\|F_{Sp,Tp}(u)\right\| \le \lambda\left\|F_{Sp,Sp}(u)\right\|$$

Since $0 < \lambda < 1$, $\left\|F_{Sp,Tp}(u)\right\| = 0 \implies Sp = Tp$

Again from (1) we have

$$\|TF_{x_n,Tp}(u)\| \leq \lambda \|F_{Sx_n,Sp}(u)\|$$

$\|F_{p,Tp}(u)\| \leq \lambda \|F_{p,Sp}(u)\| = \lambda \|F_{p,Tp}(u)\|$

$\Rightarrow Tp = p.$

$\Rightarrow Sp = Tp = p(\omega).$

For uniqueness let there exists another fixed point q in X such that from (3.1.1)

$$\|F_{p,q}(u)\| = \|F_{Tp,Tq}(u)\| \leq \lambda \|F_{Sp,Sq}(u)\|$$

Hence for all $0 < \lambda < 1$ we have $p = q$.

Theorem 3.2: Let (X, d) be a complete Menger cone metric space and P a normal cone with normal constant K. Suppose M be a nonempty separable closed subset of cone metric space X and let T and S be commuting operators defined on M satisfying contraction:

$$\|F_{Tx,Ty}(u)\| \leq \lambda \|F_{Sx,Sy}(u)\|$$

for all $x, y \in X$ and $0 < \lambda < {}^1/_2$ … … … … … (3.2.1)

$\|F_{Tx,Ty}(u)\| \leq \lambda \|F_{Tx,Sx}(u)\| + \|F_{Ty,Sy}(u)\|$

for all $x, y \in X$ and $0 < \lambda < {}^1/_2$ … … … … … (3.2.2)

$\|F_{Tx,Ty}(u)\| \leq \lambda \|F_{Tx,Sy}(u)\| + \|F_{Ty,Sx}(u)\|$

for all $x, y \in X$ and $0 < \lambda < {}^1/_2$ … … … … … (3.2.3)

and range of S contains range of T and if SX is continuous, then T and S have unique point of coincidence . If T and S weakly compatible, S and T have unique common fixed point in X.

Proof:- For each $x_0 \in X$ and $x_1 \in X$ considered such that $y_0 = Tx_0 = Sx_1$.

Therefore in general, $y_n = Tx_n = Sx_{n+1}$

As per theorem 3.1 and for all the cases (3.1),(3.2),(3.3) we have

$\Longrightarrow \|Fy_n,y_{n-1}(u)\| \leq \lambda \|Fy_{n-1},y_{n-2}(u)\|$ … … (3.2.4)

Indeed by (3.2.1) it follows that

$$\|Fy_n,y_{n-1}(u)\| = \|F_{Sx_{n+1},Sx_n}(u)\| = \|F_{Tx_n,Tx_{n-1}}(u)\|$$

$$\leq \lambda \|F_{Sx_n,Sx_{n-1}}(u)\| = \lambda \|Fy_{n-1},y_{n-2}(u)\|.$$

Indeed by (3.2.2) it follows that

$$\|Fy_n,y_{n-1}(u)\| = \|F_{Sx_{n+1},Sx_n}(u)\| = \|F_{Tx_n,Tx_{n-1}}(u)\|$$

$$\leq \lambda[\|F_{Tx_n,Sx_n}(u_1)\| + \|F_{Tx_{n-1},Sy_{n-1}}(u_2)\|]$$

$$\leq \lambda[\|Fy_n,y_{n-1}(u_1)\| + \|Fy_{n-1},y_{n-2}(u_2)\|]$$

$\|Fy_n,y_{n-1}(u)\| \leq h\left(Fy_{n-1},y_{n-2}(u)\right)$ where $h = \frac{\lambda}{\lambda-1} \in (0,1)$.

Indeed by (3.2.3) it follows that

$$\|Fy_n,y_{n-1}(u)\| = \|F_{Tx_n,Tx_{n-1}}(u)\|$$

$$\leq \lambda[\|F_{Tx_n,Sx_{n-1}}(u_1)\| + \|F_{Tx_{n-1},Sy_n}(u_2)\|]$$

$$\leq \lambda[\|Fy_n,y_{n-2}(u_1)\| + \|Fy_{n-1},y_{n-1}(u_2)\|]$$

$$\leq \lambda[\|Fy_n,y_{n-1}(u_1)\| + \|Fy_{n-1},y_{n-2}(u_2)\|]$$

$\|Fy_n,y_{n-1}(u)\| \leq h\left(Fy_{n-1},y_{n-2}(u_2)\right)$ where $h = \frac{\lambda}{\lambda-1} \in (0,1)$.

Now, by (3.4) for all cases we get

$$\left\|Fy_{n},y_{n-1}(u)\right\| = \lambda\left\|Fy_{n-1},y_{n-2}(u)\right\|$$

$$\leq \lambda^2\left\|Fy_{n-2},y_{n-3}(u)\right\| \leq \lambda^3\left\|Fy_{n-3},y_{n-4}(u)\right\|$$

… …

$$\leq \lambda^{n-1}\left\|Fy_{1},y_0(u)\right\|$$

Now for $n > m$

$$Fy_{n},y_m(u) \leq Fy_{n},y_{n-1}(u_1) + Fy_{n-1},y_{n-2}(u_2) + Fy_{n-2},y_{n-3}(u_3) + \cdots \ldots$$

$$+ \cdots \ldots \left\|Fy_{m+1},y_m(u_{n-m})\right\|$$

Since P is normal cone

$$\left\|Fy_{n},y_m(u)\right\| \leq K\left[\left\|\begin{matrix} Fy_{n},y_{n-1}(u_1) + Fy_{n-1},y_{n-2}(u_2) + Fy_{n-2},y_{n-3}(u_3) + \cdots \\ + \cdots Fy_{m+1},y_m(u_{n-m}) \end{matrix}\right\|\right]$$

$$\left\|Fy_{n},y_m(u)\right\| \leq \left[\begin{matrix} \left\|Fy_{n},y_{n-1}(u_1)\right\| + \left\|Fy_{n-1},y_{n-2}(u_2)\right\| + \left\|Fy_{n-2},y_{n-3}(u_3) +\right\| + \cdots \\ + \cdots \left\|Fy_{m+1},y_m(u_{n-m})\right\| \end{matrix}\right]$$

$$\left\|Fy_{n},y_m(u)\right\| \leq K[\lambda^{n-1} + \lambda^{n-1} + \lambda^{n-1} + \cdots \ldots + \lambda^m]\left\|Fy_{1},y_0(u)\right\|$$

$$\left\|Fy_{n},y_m(u)\right\| \leq \frac{K\lambda^{n-m}}{1-\lambda}\left\|Fy_{1},y_0(u)\right\|$$

$\Longrightarrow \left\|Fy_{n},y_m(u)\right\| \to 0$ as $m \to \infty$

Therefore sequences $\{x_n\} = \{Tx_n\} = \{Sx_{n+1}\}$ is Cauchy sequence and $S(X)$ is complete therefore there exist p in X such that $Sp = q$. Now we will show that for all cases $T(p) = q$.

From 3.2.1

$$\left\|F_{Sx_n,Tp}(u)\right\| = \left\|F_{Tx_{n-1},Tp}(u)\right\| \leq \lambda\left\|F_{Sx_{n-1},Sp}(u)\right\|$$

By taking $n \to \infty$, we get

$$\Longrightarrow \|F_{SpTp}(u)\| \leq \lambda \|F_{Sp,Sp}(u)\| = 0.$$

$$\Longrightarrow \|F_{SpTp}(u)\| = 0. \text{ Hence } Sp = Tp.$$

Now for unique coincidence let us consider another point of coincidence p_1 in X such that $T_{p_1} = S_{p_1} = q_1$ Now,

$$\left\|F_{S_{p\,1,}Sp}(u)\right\| = \left\|F_{T_{p\,1,}Tp}(u)\right\| \leq \lambda \left\|F_{S_{p\,1,}Sp}(u)\right\|$$

$$\Longrightarrow \left\|F_{S_{p\,1,}Sp}(u)\right\| = 0. \text{ Hence } Sp = Tp.$$

$$\Rightarrow \left\|F_{S_{p\,1,}Sp}(u)\right\| = 0. \text{ Hence } S_{p_{1,}} = Sp = Tp = Tp_{1,}.$$

Now, from 3.2.2 it follows

$$\|F_{Sx_n,Tp}(u)\| = \|F_{Tx_{n-1},Tp}(u)\|$$

$$\leq \lambda[\|F_{Tx_{n-1},Sx_{n-1}}(u_1)\| + \|F_{Sp,Tp}(u_2)\|].$$

$$\Longrightarrow \|F_{Sp,Tp}(u)\| \leq \lambda \|F_{Sp,Sp}(u)\| + \|F_{Tp,Sp}(u)\| = \|F_{Tp,Sp}(u)\|.$$

$$\Longrightarrow Tp = Sp.$$

Again for uniqueness let us consider another point of coincidence p_1 in X such that $Tp_{1,} = Sp_{1,} = q_{1,}$. Now

$$\left\|F_{S_{p\,1,}Sp}(u)\right\| = \left\|F_{T_{p\,1,}Tp}(u)\right\|$$

$$\leq \lambda \left[\left\|F_{T_{p\,1,}Sp_1}(u_1)\right\| + \left\|F_{T_{p,}Sp}(u_2)\right\|\right].$$

$$\Rightarrow \left\|F_{S_{p\,1,}Sp}(u)\right\| = 0. \text{ Hence } S_{p_{1,}} = Sp = Tp = Tp_{1,}.$$

Again from (3.2.3)

$$\left\|F_{Sx_{n},Tp}(u)\right\| = \left\|FT_{x_{n-1,}}(\omega), Tp(\omega)\right\|$$

$$\leq \lambda\left[\left\|F_{Tx_{n-1},Sp}(u_1)\right\| + \left\|F_{Tp,Sx_{n-1,}}(u_2)\right\|\right].$$

By taking $n \to \infty$, we get

$$\left\|F_{Sp,Tp}(u)\right\| \leq \lambda\left[\left\|F_{Tp,Sp}(u_1)\right\| + \left\|F_{Tp,Sp}(u_2)\right\|\right].$$

$$\left\|F_{Sp,Tp}(u)\right\| \leq \lambda.2.\left\|F_{Tp,Sp}(u)\right\|.$$

Since $0 < \lambda < 1/2$ therefore $\left\|F_{Sp,Tp}(u)\right\| = 0$. Hence $Sp = Tp$.

For uniqueness let us consider another point of coincidence p_1 in X such that $Tp_{1,} = Sp_{1,} = q_{1,}$. Now

$$\left\|F_{S_{p_{1,}},Sp}(u)\right\| = \left\|F_{T_{p_{1,}},Tp}(u)\right\|$$

$$\leq \lambda\left[\left\|F_{T_{p_{1,}},Sp_1}(u_1)\right\| + \left\|F_{T_p,Sp}(u_2)\right\|\right]$$

$$= \lambda\left[\left\|F_{S_{p_{1,}},Sp_1}(u)\right\| + \left\|F_{S_p,Sp_{1,}}(u)\right\|\right]$$

$$\Rightarrow \left\|F_{S_{p_{1,}},Sp}(u)\right\| \leq \lambda.2.\left\|F_{S_{p_{1,}},Sp}(u)\right\|$$

Since $0 < \lambda < 1/2$ therefore $\left\|F_{S_{p_{1,}},Sp}(u)\right\| = 0$. Hence $Sp_1 = Sp = Tp = Tp_1$.

By the use of preposition 1.4 of [1] in all above cases we can find that p is unique common fixed point of T and S.

4. Example: Let $M = R$ and $P = \{x \in M: x \geq 0\}$. Let $X = [0, \infty)$ and define mapping as $d: X \times X \to M$ by $F_{x,y}(u) = |x - y|$ Then (X, F, t) is a Menger cone metric space. Define operator T from X to X as $T(x) = x/2$. Also sequence of mapping $x_n : X \to X$ is defined by $x_n = \{1 + 1/n\}$ for $every\, n \in N. T$ Satisfies all condition of the theorem 3.1 and hence 1 is fixed point of the space.

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