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# Five Dimensional Stiff Fluid Cosmological Model in Lyra Geometry

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## Abstract:

In recent years many efforts have been made to construct alternative theories of gravitation. Lyra proposed modification of Riemannian geometry; in this theory both scalar and tensor fields have intrinsic geometrical significance. In this paper, considering a five dimensional spherically symmetric Kaluza-Klein metric with stiff fluid distribution in Lyra geometry, we have obtained the exact cosmological models for two cases, namely, constant displacement vector and time dependent displacement vector. Moreover, some physical and kinematical properties of the models are discussed.

***Keywords:*** *Five dimensional cosmological model, stiff fluid, Lyra geometry.*

## 1. Introduction

The higher dimensional cosmological models play an important role in describing the universe in its early stages of evolution. The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interactions of the universe. Therefore in recent years many efforts have been made to construct alternative theories of gravitation. Einstein's idea of geometrizing gravitation in general theory of relativity motivated others to geometrize other physical things. Weyl (1918) proposed a modification of Riemannian manifold in order to unify gravitation and electromagnetism, but due to the non integrability of length transfer, this theory was never considered seriously. Lyra (1951) proposed a modification of Riemannian geometry in which he introduced a gauge function to remove the non integrability of length of a vector under parallel transport. In this theory both the scalar and tensor fields have intrinsic geometrical significance. Various five dimensional cosmological models in Lyra manifold are constructed by Rahaman et al. (2002-03) and Singh (2004), Reddy (2005-07), Khadekar et al. (2005), Mohanty, et.al (2008-11), KalpanaPawar et al. (2014).

In this paper, considering a five dimensional spherically symmetric Kaluza-Klein metric with stiff fluid distribution in Lyra geometry, we have obtained the exact cosmological models for two cases, namely, constant displacement vector and time dependent displacement vector. We have also discussed some physical and kinematical properties of the models.

## 2. Metric and Field Equations

We consider the five dimensional spherically symmetric Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)\left[\frac{1}{1-k\,r^2}\,dr^2 + r^2\,d\theta^2 + r^2 sin^2\theta\,d\phi^2\right] - B^2(t)\,d\mu^2 \qquad (1)$$

where $A$ and $B$ are functions of cosmic time t only and the fifth coordinate is taken to be space-like.

The field equations in normal gauge for Lyra's manifold as proposed by Sen (1957), Sen and Dunn (1971) are given by

$$R_j^i - \frac{1}{2}\,R\,\delta_j^i + \frac{3}{2}\Phi^i\Phi_j - \frac{3}{4}\delta_j^i\Phi_k\Phi^k = -\chi\,T_j^i\,, \qquad (2)$$

where $\Phi_k$ is the displacement vector given by

$$\Phi_k = (\,\beta, 0, 0, 0, 0\,) \qquad (3)$$

and $T_j^i$, the energy momentum tensor given by

$$T_j^i = (\,p + \rho\,)u^i u_j - p\,\delta_j^i \qquad (4)$$

together with the co-moving coordinates

$$u^i u_i = 1 \qquad (5)$$

Here $p,\ \rho$ and $u^i$ are respectively the isotropic pressure, energy density and five velocity vector of the cosmic fluid distribution.

The field equations (2) together with (3), (4) and (5) for the space-time metric (1) yield the following equations:

$$3\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}}{A\,B}+\frac{k}{A^2}\right)-\frac{3}{4}\beta^2=\chi\,\rho \qquad (6)$$

$$\left(\frac{\dot{A}^2}{A^2}+2\,\frac{\dot{A}\dot{B}}{A\,B}+\frac{k}{A^2}\right)+2\,\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{3}{4}\beta^2=-\chi\,p \qquad (7)$$

$$3\left(\frac{\dot{A}^2}{A^2}+\frac{k}{A^2}\right)+3\,\frac{\ddot{A}}{A}+\frac{3}{4}\beta^2=-\chi\,p \qquad (8)$$

where overhead dot denotes differentiation with respect to $t$.

## 3. Solutions of Field equations and Cosmological Models

In this section we have derived exact solutions of the field equations (6) to (8) for stiff fluid ( $p=\rho$ ) . Here there are four unknowns $A, B, \beta$ and $p$ involved in three field equations (6) to (8). In order to derive explicit solutions, we consider the following cases:

**Case (I):** $\beta=$ constant

In this case, on solving (6) to (8) for $p=\rho$, we obtain

$$3\,\frac{\dot{A}}{A}=-\,\frac{\ddot{B}}{\dot{B}}=a\,, \qquad (9)$$

where$a$ is an arbitrary constant.

Integration of (9) yield

$$B=-\,\frac{e^{-a\,t\,+\,b}}{a} \qquad (10)$$

and

$$A=e^{\frac{a}{3}t\,+c} \qquad (11)$$

where$b$ and $c$ are constants of integration.

Using (10) and (11) in (6) and (8), we get

$$p=-\,\frac{\gamma}{\chi} \qquad (12)$$

where$\gamma=\frac{2}{3}a^2+\frac{3}{4}\beta^2$ .

Hence, metric (1) assumes the form

$$ds^2=dt^2-e^{2\frac{a}{3}t\,+2\,c}\left[\frac{1}{1-k\,r^2}\,dr^2+r^2\,d\theta^2+r^2 sin^2\theta\,d\phi^2\right]-\frac{e^{-2\,a\,t\,+2\,b}}{a^2}\,d\mu^2\,. \qquad (13)$$

For this metric, we get a scalar expansion, $=u^i{}_{;\,i}=0$ .

**Case (II):** $\beta = \beta(t)$

In this case, we consider $B \propto A$, i.e.,

$$B = \sigma A \tag{14}$$

where$\sigma$ is an arbitrary constant. Equation (14) give

$$\dot{B} = \sigma \dot{A}\ , \quad \frac{\ddot{B}}{B} = \frac{\ddot{A}}{A} \tag{15}$$

On solving (6) to (8) for $p = \rho$ with (14) and (15), we obtain

$$\frac{\ddot{A}}{A} = -3\ \frac{\dot{A}}{A}$$

Integrating, we get

$$A = [\,4\,(a_1 t + a_2)]^{1/4} \tag{16}$$

where$a_1$ and $a_2$ are constants.

Therefore from (14), we get

$$B = \sigma\,[\,4\,(a_1 t + a_2)]^{1/4} \tag{17}$$

Using (16) and (17) in (6) and (8), we get

$$\chi\,\rho = \frac{6\,{a_1}^2}{[\,4\,(a_1 t + a_2)]^2} - \frac{3}{4}\beta^2 \tag{18}$$

It is impossible to obtain the values of $\rho$ and $\beta$ separately.

If we assume,

$$\beta = \frac{1}{a_3\ t + a_4} \tag{19}$$

Then from (18), we get

$$\chi\,\rho = \frac{6\,{a_1}^2}{[\,4\,(a_1 t + a_2)]^2} - \frac{3}{4}\frac{1}{[\,a_3\ t + a_4\,]^2} \tag{20}$$

In this case , the line element (1) assumes the form

$$ds^2 = dt^2 - [\,4\,(a_1 t + a_2)]^{1/2}\left[\frac{1}{1-k\,r^2}\ dr^2 + r^2\ d\theta^2 + r^2 sin^2\theta\ d\phi^2\right] - \sigma^2[\,4\,(a_1 t +$$

*a2]1/2 dμ2* (21)

For this metric, we get a scalar expansion, $= {u^i}_{;\,i} = \frac{a_1}{a_1 t + a_2}$ .

## 4. Discussion and Conclusion

In case of $\beta =$ constant, we have obtained exact solution of the field equations as in (13), which shows that at an initial epoch $t = 0$ a metric becomes flat and as time increases the three space coordinates expands while the fifth coordinate, i.e., the extra dimension contracts, at infinite time the extra dimension becomes unobservable. Moreover, in this case the scalar expansion $u^{i}{}_{;i} = 0$. This behaviour is similar to that of cosmic string model obtained by Reddy.

In case of $\beta = \beta(t)$, equation (21) indicates that the line element is flat at an initial epoch $t = 0$, while the space coordinates expand with increase in time. In this case the scalar expansion $u^{i}{}_{;i} = \frac{a_1}{a_1 t + a_2}$. This model does not approach isotropy for large values of $t$.

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