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ANEVALUATIONOF QUASI-TWO DIMENSIONAL COLLECTIVE EXCITATION SPECTRUM OF CONDENSATE USING VARIATIONAL MODEL BASED ON GAUSSIAN TRAIL WAVE FUNCTION

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Abstract

Using hybrid and Gaussian model, m= o low lyig mode frequency (ω_-) and high lying mode frequency (ω_+) were calculated Our theoretically evaluated results are in good agreement with the experimental data and with the other workers.

Introduction

Bose-Einstein condensation of dilute atomic gases has been achieved in a variety of magnetic and optical dipole force traps with different geometries. There is considerable interest in studying the properties of these ultracold gases under conditions where the confinement gives a system with dimensionality less than 3.

Recent experiments in optical lattices have observed the properties of a one-dimensional Tonks gas in which bosons show fermionic properties. [1,2] Many other experiment: phase coherence between of lattice wells was observed;[3,4] collective excitations of a one-dimensional gas were studied and three-body recombination rates in a correlated ID degenerate Bose gas measured.[5] All these experiments were carried out with many individual condensates in a lattice of tightly confining potential tubes formed at the inter-section of two optical standing waves. Tunnelling between individual wells was controlled through the beam intersities. A single optical potential well was used to confine a mixture of BEC and Frem gas where the BEC was found to have a one-dimensional character.[6]

Other experiments used a one- of BECs formed by a single standing wave. Each individual condensate was confined to an extreme pancake shaped potential well and had quasi-two dimensional properties; the tunnelling between the well could be controlled by adjusting the intensity of the standing wave an oscillating atomic current in an array of Josephson junctions was studied,[7] number-squeezed states were created[8] and interference between independent condensates was observed[9]

Two-dimensional Bose-condensates in a single potential were studied[10-12] However, the new physics in this regime remains to be explored: a two-dimensional Bose gas in a homogenous potential does not undergo Bose-Einstein condensation(BEC);instead there is a Berenzinskill-Thouless transition(a topological phase transition mediated by the spontaneous formation of vortex pairs), a system that is superfluid even though it does not possess long-range order. This is counter to the usual picture os superfluidity in three dimensions explained in terms of a macroscopic wave function describing the whole system. A recent theoretical paper[13] discusses of the condensate coherence length on temperature. It shows that even for very low temperature . at a fraction of the critical temperature, the coherence length is much smaller than the condensate size due to strong phase fluctuations. Early experiments on the KT transition were carried out with films of superfluid^{4}He[14-15] and more recent once include the observation of quasi condensates in thin layers of spin-polarized hydrogen.[16]

In recent experiments, BECs created in conventional tree-dimensional magnetic traps have been put into the quasi-two dimensional(Q2D) regime through the addition of an optical potential. In this limit the interaction energy. proportional to the chemical potential μ,is on the order of or smaller than the harmonic oscillator level spacing. Along the tightly confined axial direction the characteristics of the condensate of the condensate are those of an ideal gas and the condensate width equals to harmonic oscillator length. Only when compressing the trap much further to a point where the condensate width becomes comparable to the scattering length one finds that the coupling constant g is changing and becomes dependent on the density, [17,18] However, such a tight compression has not been achieved in recent experiments. The crossover to the Q2D regime was first observed in [10] and in [11] continuously removing atoms form a highly anisotropic trap to decrease the interaction energy. In the experiment described in[12] the Q2D crossover is observed by gradually increasing the trap anisotropy form moderate to very large values whilst keeping the number of atoms fixed.

Mathematical Formula used in the Calculation

Condensates are usually trapped in harmonic potentials given by

$$V_{ext} = \frac{m}{2}\omega_O^2 \sum_i \lambda_i^2 X_i^2 \qquad \text{------------- (1)}$$

where the $\lambda_i(t)$ denote the trap anisotropies which can in general depend on time. A quasi-two dimensional trap has $\lambda_Z >> \lambda_{XY}$ For large anisotropies the condensate shape along the z direction is very similar to the Gaussian profile of an ideal gas. However, along the weakly confined x and y axes the condensate has parabolic shape characteristic the hydrodynamic regime. To determine the dynamics of the quasi-two dimensional condensate we use a variation method [17] and define the trial wave function

$$\psi = A_n \sqrt{1 - \frac{x^2}{1_x^2} - \frac{y^2}{1_x^2} e - z^2 / 21_z^2 e^i (\beta_x X^2 + \beta_y y^2 + \beta_z Z^2)} \quad \text{------------- (2)}$$

where the normalization constant A_n is given by

$$A_n^2 = \frac{2}{1_x 1_y 1_z \pi^{3/2}} \quad \text{------------- (3)}$$

The condensate with (t) and phase $\beta_i(t)$ parameters are functions of time and their time evolution completely describes that of the condensate. The condensate density profile is at all time restricted to a parabolic shape in the radial plane and a Gaussian shape along the highly compressed axial direction. The Lagrangian density for the nonlinear Schrodinger equation is given by

$$\mathcal{L} = \frac{1}{2} i\hbar \left(\frac{\partial \psi^*}{\partial t} \psi - \psi^* \frac{\partial \psi}{\partial t} \right) + \frac{\hbar^2}{2m} |\nabla \psi|^2 + V_{ext}(r,t) |\psi|^2 + \frac{1}{2} gN |\psi|^4 \quad \text{---------- (4)}$$

with the nonlinearity parameter $g = 4\pi\hbar^2 a / m$, where a is the scattering length, N is the number of atoms in the condensate and m is the atomic mass. After inserting the trial wave function(2) and into Eq.(4) the corresponding lagrangian is found through integration L= $\int \mathcal{L} d^3 x;$ the four terms of Eq.(4) lead to

$$L = L_1 + L_2 + L_3 + L_4$$

$$= \frac{\hbar^2}{m} \left(\frac{\beta_x 1^2{}_x}{3} + \frac{\beta_y 1^2{}_y}{3} + \beta_z 1_z^2 \right) + \frac{\hbar}{m} \left(\frac{\beta^2{}_x 1^2{}_x}{3} + \frac{\beta^2{}_y 1^2{}_y}{3} + \beta_z^2 1_z^2 + \frac{1}{4 I_z^2} \right) +$$
$$\frac{m}{4} \left(\frac{\omega_z^2 1_x^2}{3} + \frac{\omega_y^2 1_y^2}{3} \omega_z^2 1_z^2 \right) + \frac{\sqrt{2} gN}{3 1_x 1_y 1_z \pi^{3/2}} \quad \text{---------- (5)}$$

where we obtained the quantum pressure term[18] for the x and y directions(where this term is divergent due to the sharp boundaries of the condensate wave function in the hydrodynamic regime) but retained it for the z direction where the condensate assumes the Gaussian shape of an ideal non-interacting gas (as the term proportional to $1/1_z^2$. The quantum presuure term is crucial in describing the dynamics. The total energy per particle E_{tot} and the chemical potential μ are given by

$$E\ tot = E_{kin}+E_{pot}+E_{int},\ \mu = E_{kin}+E_{pot}+E_{int} \qquad \text{---------- (6)}$$

where E_{kin}, E_{pot} and E_{int} are the kinetic, potential and interaction energy, given by the last three terms of the Lagrangian(5), respectively, the Euler lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{I}_i} = \frac{\partial L}{\partial I_i}, \frac{d}{dt}\frac{\partial L}{\partial \dot{\beta}_i} = \frac{\partial L}{\partial \beta_i} \qquad \text{---------- (7)}$$

Yield the dynamic equagtion for the condensate whith 1_i and phase β_i we find for the widths

$$\dot{I}_i = \frac{2\hbar}{m}\beta_i 1_i \qquad \text{---------- (8)}$$

After differentiating Eqs.(8) once more with respect to time one can express the resulting second equation in terms of the 1_i alone

$$\ddot{I}_i = \omega_i^2(t)i_i + \left(\frac{2}{\pi}\right)^{3/2}\frac{gN}{m}\frac{1}{1_i 1_x 1_y 1_z}\left(1-2/3\delta_{iz}\right)+\frac{\hbar^2}{m^2}\frac{1}{1_i^3}\delta_{iz} \qquad \text{---------- (9)}$$

where δ_{iz} =1 for i=z and 0 otherwise. It is convenient to express the above equation in diemensionless quantities. so we introduces the dimensionless time τ and widths d_i defined by

$$d_i = \frac{1_i}{a_0}, \qquad \tau = t\omega_0, \qquad \text{---------- (10)}$$

where a_0 = $\sqrt{\hbar/(m\omega_o)}$ is the harmonic oscillator length . in terms of quantities Eq(9) can be rewritten as

$$\ddot{d}_i = \lambda_i^2(t)d_i + \left(\frac{C_p}{d_i d_x d_y d_z}\right)\left(1-\frac{2}{3}\delta_{iz}\right)+\frac{1}{d_z^3}\delta_{iz} \qquad \text{---------- (11)}$$

wherer the constant Cp = $8\sqrt{(2/\pi)}(a/a_o)N$. To find ground state of Eqs.(11) one has to set the left side equal to zero and solve the remaining coupled nonlinear equations:

$$d_{i0}^2\lambda_{i0}^2 = \left(\frac{C_p}{d_x o d_y o d_z o}\right)\left(1-\frac{2}{3}\lambda_{iz}\right)+\frac{1}{d_{z0}^2}\delta_{iz} \qquad \text{---------- (12)}$$

This cannot be done analytically but it is straight toforward to find a numerical solution.The $\lambda_{io} = \lambda_i$ (o), i= x,y,z are defined as the trap anisotropies at time t=0 when the condensate is in the ground state. The $d_{io}=d_i(o)$ are the ground state solution of Eq. (11) i.e. the solutions for the condensate widths d_i when the time derivative is set to zero.

After some algebra and using various symmetries the three coupled equations can be reduced to one polynomial equation introducing new dimensionless units D_i defined as the ground state condensate widths 1_i0 normalised by the axial harmonic oscillator length a_z, i.e., $D_i=1_i0/a_Z$ the polynomial equation can be written as

$$\gamma^8 = \frac{1}{3}\left(\frac{Cp\lambda_{xO}\lambda_{yO}}{\lambda_z^{3/2}}\right)^{1/2}\gamma^3 + 1 \qquad \text{---------- (13)}$$

where $D_Z = \gamma^2$ There is only one real and positive solution to this equation For the x and y widths we find

$$D_x = \left(\frac{C_p}{D_z}\frac{C_p\lambda yo\lambda_{zO}^{5/2}}{\lambda_{xO}^3}\right)^{1/4} \quad , \quad D_y = D_x\frac{\lambda_x o}{\lambda_y o} \qquad \text{---------- (14)}$$

One now examines the case where the anisotropy becomes very large. A solution to Eq.(13) is then given by neglecting the first term on the right- hand side (RHS) and solving the remaining equation We find that $\gamma^2 = D_Z = 1$ and thus the approximate solution is given by the axial harmonic oscillator length

$$I_{zo} = \sqrt{\frac{\hbar}{m\omega_z}} \qquad \text{---------- (15)}$$

It is the minimum width the condensate shape can attain and it is also the solution for the width of a noninteracting gas.For this reason the gas along the z direction is said to have the characteristics of an ideal noninteracting gas.

Now, one calculates the chemical potential form Eq.(6) and the terms of the Lagrangian(5) and obtain after some algebra

$$\bar{\mu} = \frac{1}{2}m\omega_x^2 1_{xO}^2, \qquad \tilde{\mu} = \mu - \frac{\hbar\omega_z}{2} \qquad \text{---------- (16)}$$

Where one used $\omega_x^2 1_{x0}^2 = \omega_y^2 1_{yo,}^2$ the expression for I_{zo}[Eq.15] and other symmetries of Eqs.(12) one finds that the relation $I_{io} = \sqrt{2\tilde{\mu}/m\omega_i^2}$, $i = x, y$ is similar to that of a hydrodynamic gas[19] only that for the quasi two-dimensional gas one uses the chemical potential shifted by an amount $\hbar\omega_z/2$ to calculate the radial width. inserting solution(15) for the axiatl width into Eq.(14) one obtains explicit expressions for the radial width

$$I_{x0}^2 = a_0^2 \left(8\sqrt{\frac{2}{\pi}} N \frac{a}{a_0} \frac{\lambda_{yo} \lambda_{zo}^{1/2}}{\lambda_{xo}^3} \right)^{1/2} \qquad \text{---------- (17)}$$

Another,as yet unexplored, method to observe the transition of Q2D is probe the collective excitation spectrum. An ansatz for the trial wave function of the type(2) allows for the description of the three qadrupolar modes[19] In an axially symmetric trap they are given by the m=2 mode and them=0 low and high-lying modes, where m denotes the angular momentum quantum number. In order to calculate mode frequencies one linearise the dynamic equations of the hybrid mode(11) around the ground state. Making the ansatz.

$$d_i = d_{io} + \in_i \qquad \text{---------- (18)}$$

inserting it into Eqs.(11) expanding up to first order $\in$, and using Eqs(12) to simplify and combine certain terms, one obtains after some algebra

$$\begin{matrix} \ddot{\in}_x \\ \ddot{\in}_y \\ \ddot{\in}_z \end{matrix} = (-) \begin{pmatrix} \frac{3C_p}{d_{x0}^3 d_{y0} d_z o} & \frac{C_p}{d_{x0}^2 d^2{}_{y0} d_z o} & \frac{C_p}{d_{x0}^2 d^2{}_{y0} d_z o} \\ \frac{C_p}{d_{x0}^2 d^2{}_{y0} d_z o} & \frac{3C_p}{d_{x0} d^3{}_{y0} d_z o} & \frac{C_p}{d_{x0} d^2{}_{y0} d^2{}_z o} \\ \frac{C_P/3}{d_{x0}^2 d_{y0} d^2{}_z o} & \frac{C_p/3}{d_{x0} d^2{}_{y0} d^2{}_z o} & \frac{C_p/3}{d_{x0} d_{y0} d^3{}_z o} + \frac{4}{d_{z0}^4} \end{pmatrix} \times \begin{pmatrix} \in_x \\ \in_y \\ \in_z \end{pmatrix} \qquad \text{---------- (19)}$$

Calculating the eigenvalue and eighenfrequencies of the above matrix one finds the collective excitation frequencies and modes. This can be easily done numerically. For simplicity that lends it self to an easy analyticaltreatment one assumes cylindrical symmetry $\lambda_{xo} = \lambda_{yo} \equiv \lambda_{rO}$, $d_{xO} = d_y o \equiv d_{zO}$ and reduce the set of three equations to a set of two

$$\begin{pmatrix} \ddot{\in}_r \\ \ddot{\in}_z \end{pmatrix} = -\begin{pmatrix} 4 & Dd_{zO} \\ \frac{2}{3} Dd_{zO} & 4\lambda_{zO}^3 - \frac{1}{3} Dd_{ro} \end{pmatrix} \begin{pmatrix} \in_r \\ \in_z \end{pmatrix} \qquad \text{---------- (20)}$$

where D= Cp/$\left(d_{ro}^3 d_{zo}^3\right)$ one can diagonalize the above matrix and find the eigenvalue which yields for the eigenfrequencies ω^2 :

$$\frac{\omega^2}{\omega_0^2} = 2\lambda_{z0}^2 - \frac{1}{6} Dd_{ro} + 2 \pm \sqrt{\left(-2\lambda_{zO}^2 - \frac{1}{6} Dd_{ro} + 2 \pm \right)^2 + \frac{2}{3} D^2 d_{zO}^2} \qquad \text{-------- (21)}$$

Inserting expression (15) and (17) for the ground state width into the equation above one obtains and analytic expression for the collective excitation frequencies in the Q2D regime:

$$\frac{\omega 2}{\omega_0^2} = 2\lambda_{z0}^2 - \frac{1}{6}\sqrt{\frac{C_p\lambda_{z0}^2}{\lambda^{5/2}_{z0}}} + 2 \pm \sqrt{-\left(2\lambda_{z0}^2 + \frac{1}{6}\sqrt{\frac{C_p\lambda_{z0}^2}{\lambda_{zO}^{5/2}}} + 2\right)^2 + \frac{2}{3}\sqrt{C_p\lambda_{r0}^6\,\lambda_{zO}^{5/2}}}$$

-------- (22)

The two frequencies given by Eq.(21) describe the high and low lying m=0 modes of the collective excitation spectrum. The high lying m=0 mode is an in-phase compressional model mode along all directions (breathing mode) The low lying m=o corresponds to a radial oscillation of the width which is out of phase with an oscillation along the trap axis. The third mode(not described in Eq.(21) in an axially symeetric trap is the m=2 mode. It corresponds to a quadrupole type excitation in the radial plane and its frequency is given $\omega = \sqrt{2\omega_r}$, irrespective of the axial frequency.

Discussion of Results

In this paper we have evaluated the m=o high lying mode frequency ω_+ and low lying mode freqneucy (ω-) as a function of radial frequency($(\omega_r / 2\pi)$ by taking two modes into account namely the hybrid variatinal model and Gaussian variational model. evaluated results are shown in table T_1 and T_2 respectively. Our results for both (ω_r / ω_z) and (ω_r / ω_z) decreases with radial frequency $(\omega_r / 2\pi)$.the results are alos very much identical for both these models. One observes the change of the collective excitation frequencies of the two m=0 modes for increasing ω_r.For very small ω_r in the Q2D regime, the mode frequencies approach the ideal or non interacting gas limit.

Table T^1

Evaluated result of m=o high lying mode frequency ω_+ as function of $(\omega_r / 2\pi)$

Radial trap frquency $(\omega_r / 2\pi)$	m=0 high lying mode $(\omega_r / 2\pi)$	
	Hybrid model	Gaussian Model
0	2.054	2.022
10	2.006	2.003
15	1.958	1.962
20	1.934	1.944
25	1.907	1.918
50	1.866	1.873

100	1.835	1.840
125	1.808	1.815
150	1.786	1.796
200	1.762	1.777
225	1.745	1.752
250	1.732	1.744
300	1.706	1.716

Table T_2

Evaluated result of m=O low lying mode frequency ω_- as a function of radial trap frequency $(\omega_r / 2\pi)$

Radial trap frequency $\omega_r / 2\pi$	m=0 low lying mode $(\omega_r / 2\pi)$	
	Hybrid model	Gaussian Model
0	1.986	1.992
10	1.964	1.987
15	1.952	1.965
20	1.932	1.952
25	1.908	1.922
50	1.854	1.884
100	1.814	1.842
125	1.786	1.805
200	1.762	1.774
225	1.754	1.762
250	1.750	1.754
300	1.726	1.743

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