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The Concept OfQuasi Regularity In A Near Ring:

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ABSTRACT

Quasi-regularity captures what it mean for an element of a ring to be “bad”; that is have desirable properties. Although a “bad element” is necessarily quasi regular, quasi regular element need not be bad, in a rather vague sense.

The concept of invertibilty of an element (right or left or two-sided) requires the ring to possess a unity element. However the concept can be generalized for rings without unity element. Suppose a non-zero element of a ring R has inverse b, so that ab=1.

If we put a=1+x and b=1+y then (1+x). (1+y)=1, which reflects to the condition x+y+xy=0. Hence the right inevitability of an element a with inverse b simplifies in a condition x+y+xy=0 which explicitly does not contain unity element 1. This is a generalization of the concept of right inveritibility for rings without unity element. The concept of quasi-regularity was introduced by Baer and Levtzki independently.

INTRODUCTION :-

QUASI-REGULARITY IN NEAR-RING

The quasi-regularity of an element in a general ring can be treated as a generalization of the inevitability concept a ring without unity element.

An element x of a ring R is right quasi regular (left quasi-regular) if there exist y∈R such that x+y+xy=0 (y+x+yx=0) and under this condition y is aid to be right qusi-inverse of x (left quasi-inverse of x) in R.

Alternatively, an element x of R is right quasi-regular (rqr) if and only if x∈ $\{x+xr\}_{r\in R} = R$.

A right ideal A is said to be qusi-regular (qr) if each element of A is rqr. In a arbitrary ring R, the join of all the qr right ideals of R is a rqr two-sided ideal and radical of a ring R is the join of all the qr, left ideals of R, both left quasi-regularity and left quasi-inverse are defined analogously.

An R-subgroup B of a near-ring R is said to be a two-sided R-subgroup if RB ⊆ B. If B is a subset of R, then (B:R) denotes the set of all elements of R such that Rr⊆ B. If B is a an R subgroup of R then (B:R) is the largest two-sided R sub-group contained in B.

An R-subgroup B of a near-ring R is called nilpotent if there exists a positive integer n such that $[b_1,\ldots\ldots\ldots,b_n]=0$ all sequences $[b_1,\ldots\ldots\ldots,b_n]$ of element of B.

A right ideal B of a near-ring R is called modular if B is maximal as a R-subgroup.

If I be the collection of all modular ring ideals of R then the radical of R be defined by $J(R) = \cap B$ $(B\in I)$. Clearly I is empty, then R is its own radical, then in this case R is called a radical near-ring.

An non-empty set A f a near-ring R is an ideal of R if and only if A is a right ideal and $RA \subseteq A$. The radical of a near-ring R is an ideal.

In a near-ring the concepts of radical and quasi-regularity don't have the same formulation in the wake of truncated distributive law and absence of commutative law of addition.

The following definition of quasi-regularity of a near-ring was introduced by Beidleman as follows.

1.1 **DEFINITION :** An element r of a near-ring R is called quasi-regular if and only if there exist an element $r'\in R$ such that $(e-r)\ r' =e$, where e is the identify element of R.

Similarity if R has a unity element then an element r of a near-ring R is quasi-regular if there exists $r'\in R$ such that $(1-r)r'=1$

We find that this definition is different from the usual definition of right quasi-regularity.

If we put $x=1-r'$ then the condition $(1-r)r'=1$ does not reduce to

$$r+x-rx=0.$$

This definition is not valid for near-rings without unity element. So we hall generalize the definition of quasi-regularity of near-ring without unity element.

1.2 **DEFINITION :** An element r of a near-ring R is said to be a right quasi-regular if a is in the right ideal generated by the set $\{x-rx\}_{x\in R.}$

Since the right ideal generated by the set $\{x-rx\}_{x\in R}$contains r then this right ideal coincides with R. Since r is rqr is a near-ring R if and only if the right ideal generated by the set $\{x-rx\}_{x\in R}=R$.

We see that the definition [1.2] reduces to definition [1.1] if R has a unity element.

For near-ring with unity Beidlleman's [21] quasi-regularity given by definition [1.1] is stronger than right quasi-regularity given by definition [1.2]. An R subgroup X of R is quasi-regular if and only if each of its element of X is quasi-regular.

THEOREM: If X is a quasi-regular R-subgroup of R, then X is contained in the radical J (R) of R.

PROOF: Let X is not contained in the radical J (R) of R then there exist a modular right ideal X', such that $X \nsubseteq X$.'

We know by well-known result that if X is a R-subgroup and X' a sub module of R-module M, then the additive subgroup $X+X'=R$ of M.

If $e=x'+x$ where e is the identity element and $x'\in X'$ and $x\in X$, then

$$e-x=x'+x-x= x'\in X'.$$

Now X is quasi-regular then there exist an element y of R such that

$e=(e-x)y=x'y\in X'$, which leads to a contradiction.

Hence X is contained in the radical J (R) of R.

If R be a near-ring and X a proper R-subgroup of R. Since R contains an identity e then by Zorn's lemma it follows that X is contained in a maximal R-subgroup. Particularly R contains a maximal R-subgroup then the collection L of all maximal R-subgroup of R is non-empty.

1.4 DEFINITION: The R-subgroup A=∩ X where X∈L of all maximal R-subgroup of R, is called radical subgroup of R.

THEOREM: The radical subgroup A of a near-ring R is quasi-regular R-subgroup which contains all quasi-regular right ideals of R.

PROOF : Let X be an element of A. Putting, (e-x) R=R.

For, if (e-x) is proper R-subgroup then (e-x) R is contained in maximal R-subgroup X.

Hence, e=(e-x) + a∈X, which is a contradiction.

Since (e-x) R=R there exist an element y ∈ R such that (e-x)y=e and hence A is a quasi-regular R-subgroup.

Next, suppose A' is a non-zero quasi-regular right ideal of R.

If A' ⊈ A, then there exist a maximal R-subgroup X such that A is not contained in X.

Also by a well-known result, R=A'+X.

Since X is a maximal R-subgroup.

Let e = x+a, where x∈X and a'∈A'

Thus, since A is quasi-regular, then there exists an element y∈R such that e =(e-a') y = xy∈X, which is also a contradiction.

Hence, A' is contained in A.

Now we shall generalize a result of Beidleman that the quasi-regularity of distributively generated near-ring will satisfy the DCC (descending chain condition.)

This topic are concerning theorem :-

THEOREM: Let R be a distributively generated near-ring satisfying the DCC (Descending Chain Condition) on R-subgroup. If B is quasi-regular R-subgroup, then B is nilpotent.

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