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Concept of Strong Regularity of a Ring

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ABSTRACT

The concept of strongly regular rings. Give the main characterizations of strongly regular rings. A class of rings which coincides with the class of strongly regular rings in absence of nilpotent elements is called a strongly regular rings.

INTRODUCTION :-

STRONGLY REGULAR RINGS:-

Speicaltypes of von-neumann regular rings include unit regular and strongly von-neumann regular rings. A ving is called unit regular if for every element a in R there is a unit U in R, such that a = aua.

A ring R, is called strongly von-neumann regular if for every a in R, there is some X in R with the property a = aax.

The condition is left – right symmetric. Strongly von-Neumann regular rings are Unit regular. Every strongly von-neumann regular ring is a sub direct product of division rings in some sense, this more closely mimics the properties of commutative von-Neumann regular rings. Von-Neumann regular and strongly non-Neumann are equivalent is general. The following are equivalent for a ring R:

(i) R is Von-Neumann regular and every idempotent in R is central.

(ii) Every principal left ideal of R is generated by a central idempotent.

(iii) R is Von-Neumann regular and reduced.

(iv) R is strongly Von-Neumann regular.

The concept of strongly regular ring was introduced by Arens&Kaplanski by generalizing a property of a regular ring.

DEFINITION:

A ring is called strongly regular if every element of R there exist at least one element x is R such that, a=a^2x.

It can be seen that every strongly regular ring is regular.

Since R is strongly regular if and only if one of the following conditions is satisfied.

[A] For every element a∈R there exist a∈aR and there exist a central idempotent a such that aR=eR.

[B] B is regular ring without non-zero nilpotent elements. It is easy to see that there exist two-sided ring which is neither commutative nor a division ring.

1.2 **DEFINITION:** A ring is called a right (left) duo ring if every right (left) ideal of R is an ideal.

1.3 **DEFINITION:** A ring R is called right (left) V-ring if $R^2=R$ and every right (left) ideal of R is an intersection of maximal right (left) ideals of R.

It has been every two-sided ring can be represented as a sub direct sum of sub directly irreducible two-sided ring.

Following result of LaJos&Szasz are characterizations of two-sided strongly regular ring

THEOREM: An associative ring A is strongly regular if and only if relation (a) L∩R = L A R holds for every left ideal L and right ideal R of A.

PROOF: For any associative ring the following three condtions are equivalent with each other.

(i) A is strongly regular ring.

(ii) L ∩ R = LR for any left ideal L and right ideal R of A.

(iii) A is two-sided regular ring.

Using these result in our proof.

NECESSARY CONDITION: Since A be a strongly regular ring then A satisfies the condition (ii) and (iii). Suppose L and R be the left and right ideal of A. If a be any element of L∩ R, then there exist an element x of A, such that a=axa.

Also a∈LAR i.e. L ∩ R C LAR.

Then, L ∩ R C LAR C LR= L ∩ R whence L ∩ R = LAR.

SUFFICIENT CONDITION: Let A be an associative ring satisfying the condition (a) for any left ideal L and right ideal R of A.

If R=A holds then (a) implies L ∩ R = LA ^{2}, whence every left ideal L of A is also right ideal of A. In a similar way, for L=A,

(a) implies A ∩ R = A^2 R, whence every right ideal R of A is a left ideal of A. therefore, a is a two-sided ring which implies the condition. (iii)

Therefore, L ∩ R = LAR C LR C L ∩ R,

So, L ∩ R = LR

Hence (a) implies the condition (ii) which means that A is a strongly regular ring.

Following theorem given by Chiba &Tominaga is a generalized form of the above theorems.

THEOREM: The following conditions are equivalent:

(i) R is strongly regular.

(ii) R is a regular ring and is a sub direct sum of division rings.

(iii) I ∩ x = Ix for very left ideal I and every right ideal x of R.

(iv) R is a right duo, right V-ring

(v) x ∩ x’ = xx for each right ideal x,x’ of R.

(vi) R is a right duo ring such that every ideal is idempotent.

(vii) R contains no non-zero nilpotent elements and R/p is regular for every prime ideal $p \subseteq R$.

(viii) R is regular, right duo ring.

(ix) R contains non non-zero nilpotent elements and every completely prime ideal, subset of R is a maximal right ideal.

Before proving the theorem, we shall use several familiar results and summarized a Lemma.

LEMMA : Suppose R be a ring without non-zero nilpotent elements and let a, b be elements of R.

(a) If ab=0 and ba=0 and so the right annihilator r (a) coincides with l(a).

(b) If r is prime ring then R contain no non-zero divisors.

(c) If a is non-zero then R/r(a) contains no non-zero nilpotent elements and the residue class of a modr(a) is a non-zero divisor.

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