International Research Journal of Mathematics, Engineering and IT

ISSN: (2348-9766)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 5.489 Volume 6, Issue 5, May 2019
Website- www.aarf.asia, Email : editor@aarf.asia , editoraarf@gmail.com

AN APPROACH FOR THE RATE OF CONVERGENCE FOR STANCU- MODIFIED BETA OPERATOR

Vikash Kumar
Research Scholar
Department of Mathematics
Sri Satya Sai University of Technology & Medical Sciences, M.P.

ABSTRACT

In this paper, we shall study about simultaneous approximation for the linear combinations of Stancu Type Generalization for Modified Beta Operators. We obtain a direct result in terms of higher order modulus of continuity. To prove the main result, we use the technique of the linear approximation method i.e. SteklovMean.

Keywords: Stancu Type Generalization of Modified Beta Operators; Linear Combinations; Modulus of Continuity.

AMS Subject Classification: 41A25, 41A35

1. INTRODUCTION

Let f beafunctiondefinedon[0,∞).TheModifiedBetaOperatorsareintroducedbyGupta and Ahmad [3]as

$$P_n(f,x) = \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p_{n,v}(t)f(t)\,dt \qquad x \in [0,\infty) \qquad \ldots (1.1)$$

where $b_{n,v}(x) = \frac{1}{B(v+1,n)} x^v(1+x)^{-(n+v+1)}$,,,,,,,

$$p_{n,v}(t) = \binom{n+v-1}{v} t^v(1+t)^{-(n+v)}$$

and $B(v+1,n) = \frac{v!(n-1)!}{(n+v)!}$, also $B(v,n) = \int_0^{\infty} \frac{s^{v-1}}{(1+s)^{n+v}}\,dx$,

These operators are introduced by Gupta and Ahmad [3] to approximate Lebesgue function on the [0, ∞)as-

© Association of Academic Researchers and Faculties (AARF)
A Monthly Double-Blind Peer Reviewed Refereed Open Access International e-Journal - Included in the International Serial Directories.

LetCy[0,∞)={ƒ∈[0,∞):|ƒ(t)|≤Mt^y ƒorsoɴeyΣ0andsoɴeconstantMΣ0},wedefinethenor m

$\|.\|_y$ on $C_y[0,\infty)$ by $\|f\|_y = \sup_{0 \in t \in \infty} |f(t)| \pm t^y$

Here we shall apply Stancu [7] type generalization of Bernstein [1] polynomials as-

where $$p^{\alpha}_{n,\alpha}(x) = \binom{n}{k} \frac{\prod_{s=0}^{k-1}(x+\alpha s)\prod_{s=0}^{n-k-1}(1-x+\alpha s)}{\prod_{s=0}^{n-1}(1+\alpha s)} \quad \text{…. (1.3)}$$

We get the Bernstein polynomials by putting □ □ 0 , starting with two parametersα & þ satisfying 0 ≤ α ≤ þ in1983.

The other generalization of Stancu Operators was given in [8] and studied the linear positive operators as follows-

$$S_n^{\alpha,\beta}(f,x) = \sum_{k=0}^{n} p_{n,k} f\left(\frac{k+\alpha}{n+\beta}\right), \qquad 0 \le x \le 1 \quad \text{…. (1.4)}$$

where $$p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad \text{…. (1.5)}$$

It is the Bernstein basis function.

Recently Ibrahim [5] introduced Stancu Chlodowsky polynomials and investigated convergence and approximation properties of theseoperators.

Now Stancu type generalization for Operators (1.1) as follows-

$$P_n^{\alpha,\beta}(f,x) = \frac{n-1}{n} \sum_{v=0}^{\infty} b_{n,v}(x) \int_0^{\infty} p_{n,v}(t) f\left(\frac{nt+\alpha}{n+\beta}\right) dt, \qquad x \in [0,\infty] \quad \text{…. (1.6)}$$

where $b_{n,v}$ (x) and $p_{n,v}$ (t) are defined as earlier. The operators $P_n^{(\alpha,þ)}$ are called Modified Beta Stancu Operators. For α = þ = 0, the operators (1.2) reduce to operators (1.1).

ItiseasilyverifiedthattheoperatorsP$_n$arelinearpositiveoperators.AlsoP $_n$ (1, x) = 1,it turns out the order of approximation for the operators (1.2) are at best O (1/n), howsoever smooth the function may be. Thus to improve the order of approximation, we consider the linear combination of operators (1.1) as describedfurther.

For d_O, d_1, d_2, … , d_v arbitrary but fixed distinct positive integers, the linear combination $P_n(f, v, x)$ of $P_{d_j n}(f,x)$, j = 0, 1,2, … , n are defined by

$$P_n(f, v, x) = \sum_{j=0}^{v} C(j, v) P_{d_j n}(f, x) \qquad \dots (1.7)$$

$$\text{where} \quad C(j, v) = \prod_{\substack{i=0 \\ i \neq j}}^{v} \frac{d_j}{d_j - d_i}, \qquad v \neq 0 \text{ and } C(0,0) =$$

Alternately the above linear combination may be defined as-

$$P_n(f, v, x) = \begin{vmatrix} 1 & d_0^{-1} & d_0^{-2} & \dots & d_0^{-v} \\ 1 & d_1^{-1} & d_1^{-2} & \dots & d_1^{-v} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & d_v^{-1} & d_v^{-2} & \dots & d_v^{-v} \end{vmatrix}^{-1} \begin{vmatrix} P_{d_0 n} & d_0^{-1} & d_0^{-2} & \dots & d_0^{-v} \\ P_{d_1 n} & d_1^{-1} & d_1^{-2} & \dots & d_1^{-v} \\ \dots & \dots & \dots & \dots & \dots \\ P_{d_v n} & d_v^{-1} & d_v^{-2} & \dots & d_v^{-v} \end{vmatrix}, \qquad \dots (1.8)$$

2. BASICRESULTS

In this section, we study some definitions and certain lemmas by using Stancu operators to prove our main theorems. We shall extend the results of Maheshwari and Gupta [6] by applying Stancu type ofgeneralization.

Here we mention two definitions named as Steklov mean and k^{th} order modulus of continuity, which will be beneficial in finding our results.

Steklov Mean-Let us assume that $0 < a < a_1 < b_1 < \infty$, for sufficiently small ð > 0, the

© Association of Academic Researchers and Faculties (AARF)
A Monthly Double-Blind Peer Reviewed Refereed Open Access International e-Journal - Included in the International Serial Directories.

$(2k + 2)^{th}$ order.

Steklov mean $g_{2k+2,i\delta}$ corresponding to $g \in C_y[0, \infty)$ is defined by

$$g_{2k+2,i\delta}(t) = \delta^{-(2k+2)} \int_{i\delta/2}^{\delta/2} \int_{i\delta/2}^{\delta/2} \dots \dots \int_{i\delta/2}^{\delta/2} [g(t) - \Delta_y^{2k+2} g(t)] \prod_{i=1}^{2k+2} dt,$$

$$\text{where} \qquad 5 = \frac{1}{2k+2} \sum_{i=1}^{2k+2} i\,, \qquad \text{and } i \in [a, b]$$

It is easily checked [2, 3, 5] that

i. $g_{2k+2,\delta}$ has continuous derivatives up to order $(2k + 2)$ on $[a,b]$.

ii. $\|g^{(r)}_{2k+2,\delta}\|_{C[a_1,b_1]} \leq K\delta r_{..} (g, \delta, a, b), \qquad r = 1,2, \dots, (2k + 2),$

iii. $\|g - g_{2k+2,\delta}\|_{C[a_1,b_1]} \leq KW_{2k+2}(g, \delta, a, b),$

iv. $\|g_{2k+2,\delta}\|_{C[a_1,b_1]} \leq K\|g\|_y$

where 'K' is an arbitrary constant and in this paper it will have different values at different places.

k^{th} **Order Modulus of Continuity**- The k^{th} order moment of continuity $m_k(f,\delta)$ for a function continuous on an interval [a, b] is defined by

$$m_k(f,\delta)=\text{Sup}\{|\Delta_h^k f(x)|:|h| \leq \delta, x, x+kh \in I\}$$

For $k=1$, $m_k(f,\delta)$ is written simply as $m_i(\delta)$ or $m(f,\delta)$.

Lemma-2.1 – For $m \in N \cup \{0\}$ if

$$U_{n,m}(x) = \frac{1}{-}\sum_{v=0}^{\infty} b_{n,v}(x)\left(\frac{v}{\quad} - x\right)^m$$

then $\qquad (n+1)U_{n,m+1}(x) = x(1+x)\left\{U'_{n,m}(x) + mU_{n,m+1}(x)\right\}$

Consequently

(i) $U_{n,m}(x)$ is a polynomial in x of degree $\leq m$.

(ii) $U_{n,m}(x) = O\left(n^{-[m+1/2]}\right)$, Where [£] denotes the integral part of £.

Lemma-2.2-LettheN th ordermomentbedefinedby

$$T_{n,N}(x) = \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+b} - x\right)^N dt \qquad \text{... (1.5)}$$

then $T_{n,0}(x) = 1$, $T_{n,1}(x) = \frac{\alpha}{n+b} - x$ and

$$(n - N - 2)T_{n,N+1} = x(1+x)[T'_{n,N}(x) + 2NT_{n,N-1}(x)] + [(1+2x)(N+1)+x]T_{n,N}(x),$$
$$n > N + 2$$

Further, for all $x \in [0, \infty)$, $T_{n,m}(x) = O\left(n^{-[(m+1)/2]}\right)$

ROOF: TheproofofLemma-2.1caneasilybeobtainedbyusingthedefinitionof$T_{n,N}(x)$ fromLemma-2.2.so,first,fortheproofofLemma-2.2weproceedasfollows.

Differentiating (1.5) with respect to x and multiplying by x(1 + x) on both sides-

$$\ldots_{n,N} \quad \frac{n-1}{n}\sum_{v=0}^{\infty} \ldots_{n,v}(x)\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+b}\ldots\right)^N \ldots dt - Nx(1+$$

$xTn,N-1x$

Using relations

1) $x(1+x)b'_{n,v}(x) = [v-(n+1)x]b_{n,v}(x)$,

$$\frac{nt+\alpha}{n+b} \quad \frac{nt+\alpha}{n+b} \quad {}_{n,v}\, t = [v - n\,(\frac{nt+\alpha}{n+b} \quad {}_{n,v},$$

3) $$\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p'_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^N dt$$

$$= -\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^{N-1} dt$$

$$= -NT_{n,N-1}(x)$$

we obtain-

$$x(1+x)[T'_{n,N}(x) + NT_{n,N-1}(x)]$$

$$= \frac{n-1}{n}\sum_{v=0}^{\infty}[v-(n+1)x]\, b_{n,v}(x)\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^N dt$$

$$= \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty}[(v-n\,\frac{nt+\alpha}{n+b}) + n\,(\frac{nt+\alpha}{n+b}-x) - x]\, p_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^m dt$$

$$= \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty}\frac{nt+\alpha}{n+b}(1+\frac{nt+\alpha}{n+b})\, p'_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^N dt + nT_{n,N+1}(x)$$

$$- xT_{n,N}(x)$$

$$= \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty}[(1+2x)\left(\frac{nt+\alpha}{n+\beta}-x\right)+\left(\frac{nt+\alpha}{n+b}-x\right)^2$$

$$+x(1+x)]\,p_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^m dt + nT_{n,N+1}(x) - xT_{n,N}(x)$$

$$=(1+2x)\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p'_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^{m+1} dt$$

$$+\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p'_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^{m+2} dt$$

$$+x(1+x)\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\int_0^{\infty} p'_{n,v}(t)\left(\frac{nt+\alpha}{n+b}-x\right)^{m} dt + nT_{n,N+1}(x)$$

$$-xT_{n,N}(x)$$

$$= -(N+1)(1+2x)T_{n,N}(x) - (N+2)T_{n,N+1}(x) - Nx(1+x)T_{n,N-1}(x)$$
$$+ nT_{n,N+1}(x) - xT_{n,N}(x)$$

This leads to Lemma-2.2. Obviously $T_{n,m}(x) = O\left(n^{-[m+1]/2}\right)$

Lemma-2.3- There exists the polynomial $q_{i,j,r}(x)$ independent of n & v, such that

$$x^r(1+x)^r \frac{d^r}{dx^r}\left(x^v(1+x)^{-n-v}\right) = \sum_{\substack{2i+j\le r\\ i,j\ge 0}} n^i (v-nx)^j q_{i,j,r}(x)\, x^v (1+x)^{-n-v}$$

Lemma-2.4- Let f be r times differentiable on $[0,\infty)$ such $f^{(r-1)} = O(t^q)$ for some $\alpha>0$ as $t\to\infty$ then

for r = 1,2,3 and n > q + r, we have

$$\ldots \frac{\ldots}{n!\,(n-2)!}\sum_{v=0}^{\infty} b_{n+r,v}(x)\int_0^{\infty} p_{n-r,v+r}(t)\, f^{(r)}\left(\frac{nt+\alpha}{n+b}\right)\ldots$$

PROOF- We have

$$P_n^r(f,x) = \frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}^r(x)\int_0^{\infty} p_{n,v}(t)\, f\left(\frac{nt+\alpha}{n+b}\right)dt$$

© **Association of Academic Researchers and Faculties (AARF)**
A Monthly Double-Blind Peer Reviewed Refereed Open Access International e-Journal - Included in the International Serial Directories.

By using Leibnitz theorem-

$$P_n^r(f,x)=\frac{n-1}{n}\sum_{i=0}^{r}\sum_{v=i}^{\infty}\binom{r}{i}\frac{(n+v+r-i)!}{(n-1)!\,(v-i)!}(-1)^{r-i}x^{v-i}(1+x)^{-n-v-1-r-i}\int_0^{\infty}p_{n,v}(t)\,f\left(\frac{nt+\alpha}{n+\beta}\right)dt$$

$$=\frac{n-1}{n}\sum_{v=0}^{\infty}\frac{(n+v+r)!}{(n-1)!\,v!}\cdot\frac{x^v}{(1+x)^{n+v+r+1}}\int_0^{\infty}\sum_{i=0}^{r}(-1)^{r-i}\binom{r}{i}p_{n,v+i}(t)f\left(\frac{nt+\alpha}{n+\beta}\right)dt$$

$$=\frac{(n-1)(n+r-1)!}{(n)!}\sum_{v=0}^{\infty}b_{n+r,v}(x)\int_0^{\infty}\sum_{i=0}^{r}(-1)^{r-i}\binom{r}{i}p_{n,v+i}(t)f\left(\frac{nt+\alpha}{n+\beta}\right)dt$$

Again, by using Leibnitz theorem, we get

$$p^r_{n-r,v+r}(t)=\frac{(n-1)!}{(n-r-1)!}\sum_{i=0}^{r}(-1)^i\binom{r}{i}p_{n,v+i}(t)$$

Hence,

$$P_n^r(f,x)=\frac{(n-r-1)!\,(n+r-1)!}{(n-2)!\,n!}\sum_{v=0}^{\infty}b_{n+r,v}(x)\int_0^{\infty}(-1)^r p^r_{n-r,v+r}(t)\,f\left(\frac{nt+\alpha}{n+\beta}\right)dt$$

Integrating r times, we get the required result.

Lemma-2.5 Let $f\in C_\gamma[0,\infty)$, if $f^{(2k+r+2)}$ exists at a point $x\in[0,\infty)$, then

$$\lim_{n\to\infty}n^{v+1}\left[P_n^r(f,(d_0,d_1,\dots d_v),x)-f^r(x)\right]=\sum_{i=r}^{2v+r+2}Q(i,v,r,x)f^i(x)$$

where Q(i, v, r, x) are certain polynomials in x

3. MAINRESULTS

In this section we shall prove the following main results.

Theorem 3.1- Let $f^r \in C_y[0, \infty)$ and $0 < a < a_1 < b_1 < b < \infty$ then for ‘nsufficiently large

$$\|P_n^r(f, v, x) - f^r\|_{C[a_1,b_1]} \leq \mathrm{Max}\{C_1\, m_{2v+2}(f^r, n^{-1/2}a, b), C_2\, n^{-(v+1)}\|f\|_y\}$$

where $C_1 \equiv C_1(v, r)$ and $C_2 \equiv C_2(v, r, f)$

Proof: First, we have by linearity property of the operators, we have

$$\|P_n^r(f, v) - f^n\|_{C[a_1,b_1]} \leq \|P_n^r(f - f_{2v+2,\eth}, v)\|_{C[a_1,b_1]} + \|P_n^r(f_{2v+2,\eth}(d_0, d_1, \ldots, d_v), \ldots) - f^n_{2v+2,\eth}\|_{C[a_1,b_1]} + \|f^r - f^r_{2v+2,\eth}\|_{C[a_1,b_1]}$$

$$= B_1 + B_2 + B_3, \text{ (Say)}$$

By property (iii) of Steklov mean, we have

$$B_3 \leq K m_{2v+2}(f^r, \eth, a, b)$$

Next, by Lemma-2.5, we have

$$B_2 \leq K n^{-(v+1)} \sum_{j=r}^{2v+r+2} \|f^j_{2v+2,\eth}\|_{C[a,b]}$$

ByinterpolationpropertyduetoGoldbergandMeir[2]foreachj=r,r+1,…,2v+r+ 2, wehave-

$$\|f^j_{2v+2,\eth}\|_{C[a,b]} \leq K\{\|f_{2v+2,\eth}\|_{C[a,b]} + \|f^{2v+r+2}_{2v+2,\eth}\|_{C[a,b]}\}$$

© **Association of Academic Researchers and Faculties (AARF)**
A Monthly Double-Blind Peer Reviewed Refereed Open Access International e-Journal - Included in the International Serial Directories.

Therefore by properties (ii) and (iv) of Steklov mean, we have-

$$B_2 \leq Kn^{-(v+1)}\{\|f\|_y + ð^{-(2v+2)} m_{2v+2}(f^r, ð)\}$$

Finally, we shall estimate B, choosing a*, b* satisfying the conditions,

$$0 < a < a^* < a_1 < b_1 < b^* < \infty$$

Alsolet f beacharacteristicfunctionoftheinterval[a*,b*],thenTypeequationhere.

$$B_1 \leq \|P_n^r[\psi(\frac{nt+\alpha}{n+b})\{f(\frac{nt+\alpha}{n+b}) - f_{2v+2,ð}(\frac{nt+\alpha}{n+b})\}]\|_{C[a_1,b_1]}$$

$$+ \|P_n[\{1 - f(\frac{nt+\alpha}{n+b})\}\{f(\frac{nt+\alpha}{n+b}) - f_{2v+2,ð}(\frac{nt+\alpha}{n+b})\}, v, .]\|_{C[a_1,b_1]}$$

$$= B_4 + B_5, \quad \text{(Say)}$$

We may note here that to estimate $B_4$and B_5, it is enough to consider their expressions without the linearcombinations.

By Lemma-2.4, we have

$$P_n[f(\frac{nt+\alpha}{n+b})\{f(\frac{nt+\alpha}{n+b}) - f_{2v+2,ð}(\frac{nt+\alpha}{n+b})\}, x] = \frac{(n-r-1)!(n+r-1)!}{(n-2)!n!}\sum_{v=0}^{\infty} b_{n+r,v}(x)\int_0^{\infty} p_{n-r,v+r}(t).f(\frac{nt+\alpha}{n+b})\{f^r - f_{2v+2,ð}(\frac{nt+\alpha}{n+b})\}\, dt$$

Hence.

$$\|P_n[f(\frac{nt+\alpha}{n+b}).f(\frac{nt+\alpha}{n+b}) - f_{2v+2,ð}(\frac{nt+\alpha}{n+b}), .]\|_{C[a,b]} \geq K\|f^r - f_{2v+2,ð}\|_{C[a^*,b^*]}$$

Now. for $x \in [a_1, b_1]$ & $t \in [0, \infty)\setminus[a^*, b^*]$, we choose a $ð_1$ ...... $|(\frac{nt+\alpha}{n+b}) - x| \geq ð_1$

Therefore by Lemma-2.3 and Schwarz inequality, we have-

$$I = P_n^r|\{1 - f(\frac{nt+\alpha}{n+þ})\}\{f(\frac{nt+\alpha}{n+þ}) - f_{2v+2,\delta}(\frac{nt+\alpha}{n+þ})\}, x|$$

$$\leq \frac{n-1}{n} \sum_{\substack{2i+j\check{S}r \\ i,j\check{S}O}} n^i \frac{|q_{i,j,r}(x)|}{x^r(1+x)^r} \sum_0^\infty b_{n,v}(x).|v - nx|^j \int_0^\infty p_{n,v}(t)\{1 - f(\frac{nt+\alpha}{n+þ})\}|f(\frac{nt+\alpha}{n+þ}) - f_{2v+2,\delta}(\frac{nt+\alpha}{n+þ})| dt$$

$$I = P_n^r|\{1 - f(\frac{nt+\alpha}{n+þ})\}\{f(\frac{nt+\alpha}{n+þ}) - f_{2v+2,\delta}(\frac{nt+\alpha}{n+þ})\}, x|$$

$$\leq \frac{n-1}{n} \sum_{\substack{2i+j\check{S}r \\ i,j\check{S}O}} n^i \frac{|q_{i,j,r}(x)|}{x^r(1+x)^r} \sum_0^\infty b_{n,v}(x).|v - nx|^j \int_0^\infty p_{n,v}(t)\{1 - f(\frac{nt+\alpha}{n+þ})\}|f(\frac{nt+\alpha}{n+þ}) - f_{2v+2,\delta}(\frac{nt+\alpha}{n+þ})| dt$$

$$\leq K\|f\|_y (\frac{n-1}{n}) \sum_{\substack{2i+j\check{S}r \\ i,j\check{S}O}} n\, b^j_{n,v}(x)|v - nx|^j \int_{|\frac{nt+\alpha}{n+þ} - x| \geq \delta_1} p_{n,v}(t)dt,$$

$$\leq K\delta_1^{-2c}\|f\|_y \frac{n-1}{n} \sum_{\substack{2i+j\check{S}r \\ i,j\check{S}O}} n^i \sum_{v=0}^{\infty} b_{n,v}(x)|v \quad \frac{1}{2} \quad \infty$$

© **Association of Academic Researchers and Faculties (AARF)**
A Monthly Double-Blind Peer Reviewed Refereed Open Access International e-Journal - Included in the International Serial Directories.

$$\frac{nt+\alpha}{} \quad - nx|^{i} \left(\int_0^{\infty} p_{n,v}(t)dt\right)^{\frac{1}{4c}} \left(\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+\beta} - x\right)^{2} dt\right)$$

$$\leq K\delta_1^{-2c}\|f\|_y \sum_{\substack{2i+j\leq r\\ i,j\geq 0}} n^i \left(\sum_{v=0}^{\infty} b_{n,v}(x)(v - nx)^{j}\right)^{1/2} \cdot \left\{\frac{n-1}{n}\sum_{v=0}^{\infty} b_{n,v}(x)\left(\int_0^{\infty} p_{n,v}(t)\left(\frac{nt+\alpha}{n+\beta} - x\right)^{4c} dt\right)\right\}^{1/2}$$

Hence, by Lemma-2.1 & 2.2, we have-

$$I \leq K\|f\|_y O(n)^{i+\frac{1}{2}+c} \leq Kn^{-y}\|f\|_y$$

Where $q = (s - n/2)$. Now choose $\delta > 0$ such $q \geq (v+1)$, then

$$I \leq Kn^{-(v+1)}\|f\|_y$$

Therefore by property (iii) of Steklov mean, we get-

$$B_1 \leq K\|f^r - f^r_{2v+2,\delta}\|_{C[a^*,b^*]} + Kn^{-(v+1)}\|f\|_y$$

$$\leq Km_{2v+2}(f^r, \delta, a, b) + Kn^{-(v+1)}\|f\|_y$$

Hence with $\delta = n^{-1/2}$, the theorem follows.

REFERENCES

(1) Agrawal PN, Gupta V. Bull. Greek Math. Soc 1989; 30:21-29.

(2) Goldberg S, Meir V. Proc. London Math. Soc. 1971; 23:1-15.

(3) Gupta V, Ahmad A. Simultaneous approximation by Modified Beta Operators. Istanbul Univ. Fen. Fak. Mat. Derg. 1995; 54:11-22.

(4) HewittE,StrombergK.RealandAbstractAnalysis,McGrawHill,NewYork1956.

(5) Ibrahim B. Approximation by Stancu- Chlodowsky polynomials. Comput. Math. Appl. 2010; 59:274-282.

(6) Maheshwari P, Gupta V. Estimation on the Rate of Convergence for Modified Operators. Indian J.Pure aApl. Math. 2003; 34 (6):927-934.

(7) Stancu DD. Approximation of function by means of a new generalized Bernstein Operator. Calcolo 1983; 20:211-229.

(8) Stancu DD. Approximation of function by a new class of linear polynomials Operators. Rev. Romaine. Math. Pures Appl. 1968; 13:1173-1194.