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FUZZY FIELDS AND FUZZY LINEAR SPACES

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Abstract: This paper deals with the Fuzzy Fields and Fuzzy Linear Spaces.

Keywords: Fuzzy set, Linear Space, Fuzzy Fields, Fuzzy Set, Function

Introduction:The concept of fuzzy sets and fuzzy set operations was first introduced by Zadehand subsequently several authors including Zadeh have discussed various aspects of the theory and application of fuzzy sets such as a fuzzy topological spaces, similarity relations and fuzzy orderings, algebraic properties of fuzzy sets, fuzzy measures, probability measures of fuzzy events, fuzzy mathematical programming, fuzzy dynamic programming and decision making on a fuzzy environment. Fuzzy groups were introduced by Rosenfeld and subsequently discussed by Anthony and Sherwood, Osman and Wu. Fuzzy rings and fuzzy ideals have been studied by Liu. Here we study the concepts of fuzzy fields and fuzzy linear spaces.

Preliminaries :

Let X be a set and let L be a lattice. In particular L could be the closed interval [0,1]. A fuzzy set A in X is characterised by a membership function $\mu_A : X \rightarrow L$, which associates with eachpoint $x \in X$ its 'grade or degree of membership' $\mu_A(x) \in L$. We first quote some definitions which will be needed in the sequel.

Definition Let A and B be fuzzy set in X. Then

A = B if $\mu_A(x) = \mu_B(x)$ for all $x \in X$,

$A \subset B$ if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$,

$C = A \cup B$ if $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,

$D = A \cap B$ if $\mu_D(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,

More generally, if L is a complete lattice, then for a family of

$A = \{A_i : j \in J\}$, the union, $C = \cup_{j \in J} A_i$, and the intersection, $D = \cap_{j \in J} A_i$ defined by

$$\mu_C(x) = \sup_{i \varepsilon I} \mu_A(x),\ \mu_D(x) = \inf_{i \varepsilon I} \mu_A(x), x \in X.$$

We denote by K_c the fuzzy set in X with membership function $\mu K_c(X) = c$ for $x \in X$. The fuzzy sets K_1 and K_o respectively correspond to the set X and empty set θ .

Definition**:** Let f be a mapping from a set X into a set Y. Let B be a fuzzy in Y, with membership functionμB. Then the inverse image of B, $f^{-1}[B]$, is fuzzy set in X with membership function defined by

$\mu f^{-1}[B](x) = \mu_B(f(x))$ for all $x \in X$

Let A be a fuzzy set in X with membership function μ_A.Then the image A, f[A], is the fuzzy set in Y with membership function defined by

$$\mu_{f[A]}(y) = \begin{cases} \sup_{z \in f} \mu a(z) \; if \; f(y) \neq \emptyset \\ 0 \end{cases}$$

Otherwise,

for all $y \in Y$, Where $f^{-1}(y) = \{x : f(x) = y\}$.

Definition:

Let X be a group and G a fuzzy set in X with membership function μ_G.

The G is a fuzzy group in X if the following conditions are satisfied:

(i) $\mu_G(xy) \geq \min\{\mu_G(x)(y)\}$ for all $x, y \in X$.

(ii) $\mu_R(x^{-1}) \geq \mu_G(x)$ for all $x \in X$.

Definition:

Let X be a ring and R a fuzzy set in X with membership function μ_R .

Then R is a fuzzy ring in X iff the following conditions are satisfied :

(i) $\mu_R(x+y) \geq \min\{\mu_R(x), \mu_R(y)\}$ for all $x, y \in X$.

(ii) $\mu_R(-x) \geq \mu_R(x)$ for all $x \in X$,

(iii) $\mu_R(xy) \geq \min\{\mu_R(x), \mu_R(y)\}$ for all $x, y \in X$.

Fuzzy fields and fuzzy linear spaces:

We now introduce the concepts of fuzzy fields and fuzzy linear spaces.

Definition:

Let X be a field and F a fuzzy set in X with membership function μ_F.

Then F is a fuzzy field in X if the following conditions are satisfied:

(i) $\mu_F(x+y) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all $x, y \in X$.

(ii) $\mu_F(-x) \geq \mu_F(x)$ for all $x \in X$,

(iii) $\mu_F(xy) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all $x, y \in X$.

(iv) $\mu_F(x^{-1}) \geq \mu_F(x)$ for all $0 \neq x \in X$,

(v) $\mu_F(0) = 1$, $\mu_F(1) = 1$.

Definition:

Let X be a field and F a fuzzy field in X with membership function μ_F.

Let Y be a linear space over F and V a fuzzy subset of Y with membership function μ_V. Then V is a fuzzy linear space in Y if the following properties hold:

(i) $\mu_V(x+y) \geq \min\{\mu_V(x), \mu_V(y)\}$ for all $x, y \in X$.

(ii) $\mu_V(\lambda x) \geq \min\{\mu_F(\lambda), \mu_V(y)\}$ for all $\lambda \in F$ and all $x \in Y$.

(iii) $\mu_V(0) = 1$.

If F is an ordinary field (in particular if F=X), then the condition (ii) above will be replaced by the following axiom:

(iv) $\mu_V(\lambda x) \geq \mu_V(x)$ for all λ in F or X and all $x \in Y$.

Proposition: F is a fuzzy field in X if

(i) $\mu_F(x-y) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all $x \in Y$,

(ii) $\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all $x \in X$, $0 \neq y \in Y$.

Proposition:

Let X and Y be fields and f a homomorphism from X into Y. Let F be a fuzzy field in Y. Then the inverse image f^{-1} (F) of F is a fuzzy field in X.

Proof. For all x, y $\in$ X,

$$\mu f^{-1}[F](x-y)=\mu_F(f(x-y))=\mu_F(f(x)-f(y))$$

$$\geq \quad \text{Min } \{\mu_F(f(x)), \mu_F(f(y))\}.$$

$$= \quad \text{Min } \{ \mu f^{-1}[F](x), \mu f^{-1}[F](y)\}$$

Similarly it can be shown that for all x$\in$X, 0 $\neq$y $\in$ Y,

$$\mu f^{-1}[F](xy^{-1}) \geq \text{Min } \{ \mu f^{-1}[F](x), \mu f^{-1}[F](y)\}.$$

This completes the proof.

For images, we need the following property [8]. A fuzzy set A in X is said to have the sup property, if for any subset T $\subset$ X, there exists $t_0 \in T$ such that $\mu_A(t_0) = \sup_{t\in T}\mu_A(t)$.

Let X and Y be fields and homomorphism of X into Y. Let F be a fuzzy field in X that has the sup property. Then the image f[F] of F is a fuzzy field in Y.

Proof:

Let u,v$\in$Y. If either $f^{-1}(u)$ or $f^{-1}(\upsilon)$ is empty, then the inequality in proposition above is trivially satisfied. Suppose neither $f^{-1}(u)$ nor $f^{-1}(\upsilon)$ is empty. Let $r_{0} \in f^{-1}(u)$, $s_{0} \in f^{-1}(\upsilon)$ be such that

$$\mu_F(r_0) = \sup_{t \in f^{-1}(u)} \mu_F(t), \text{ and}$$

$\mu_F\ (s_0) = \begin{matrix} sup \\ t \in f \end{matrix} - 1(v)\mu_F(t)$

Then

$\mu_{f[F]}\ (u\text{-}v) = \begin{matrix} sup \\ w \in f \end{matrix} - 1\ (u\text{-}v)\ \ \mu_F(w)$

$\geq \quad \text{Min}\ \{\mu_F(r_0),\ \mu_F(s_0)$

$= \quad \text{Min}\ \{\mu_{f[F]}(u),\ \mu_{F[F]}(\upsilon)\}$

Similarly it can be shown that for $x \in Y$ and $0 \neq \upsilon \in Y$,

$\mu_{f[F]}(u\upsilon^{-1}) \geq \text{Min}\ \{\mu_{f[F]}(u),\ \mu_{F[F]}(\upsilon)\}.$

This completes the proof.

If F is a fuzzy field in a field X, then $\mu_F(-x) = \mu_F\ (x)$ for all $x \in X$

Also $\mu_F(x^{-1}) = \mu_F\ (x)$ for all $0 \neq x \in F$.

V is a fuzzy linear space in Y (over a fuzzy field F in X) if.

$\mu_V(\lambda x + \mu y) \geq \min\ \{\mu_V(x),\ \mu_V(y)\}$ for all x, y F.

If L is a complete lattice, then the intersection of a family of fuzzy linear spaces is a fuzzy linear space.

Proof.

Let $\{V_i{:}j\ \ J\}$ be a family of fuzzy linear spaces and

let $V = \cap_{jJ}\ V_i$. Then

$\mu_v(x\text{-}y) = \begin{matrix} inf \\ i\varepsilon I \end{matrix} \mu_v(x+y)$

$\geq \quad \underset{i\varepsilon J}{inf}$ [min {μ_V, (x),μ_{Vi}(y)}]

$= \quad \min [\underset{j\varepsilon J}{inf}\ \mu_{vi}(x), \underset{j\varepsilon J}{inf}\ \mu_{vi}(y)]$

$= \quad \min [\mu v(x), \mu v\ (y)]$

And,

$\mu_V(\lambda x) = \underset{j\varepsilon J}{inf}\ \mu_{vi}(\lambda x),$

$= \underset{j\varepsilon J}{inf} [\min\{\mu_F(\lambda), \mu_{vi}(x)\}]$

$= \quad \min[_F(\lambda), \underset{j\varepsilon J}{inf}\ \mu_{vi}(x)]$

$= \quad \min [\mu_F(\lambda), \mu_{vi}(x)]$

This completes the proof.

Let Y and Z be linear spaces over a fuzzy field F in a field X and f a linear transformation of Y into Z. Let W be a fuzzy linear space in Z. Then the inverse image f^{-1}[W] of W is a fuzzy linear space in Y.

Proof. For all x,yϵ Y,

$\mu f^{-1}[W](\lambda x + \mu y) = \mu_W(f(\lambda x + \mu y))$

$= \quad \mu w[\lambda f(x) - \mu f(y))]$

$= \quad \min \{\min\{\mu_F(\lambda), \mu_W\ (f(x))\}, \min \{\mu_F(\mu), \mu_W(f))\}\}$

$\geq \quad \min \{\min\{\ \mu_F(\lambda),\ (x)\}, \min \{\mu[w]\ (y)\}\}.$

Let Y and Z be linear spaces over a fuzzy field F in a field X and f a linear transformation of Y into Z. Let V be a fuzzy linear space in Y that has the sup property.
Then the image f[V] of V is a fuzzy linear space in Z.

Proof :

Let u, υ∈ W. If either f^{-1} (u) of f^{-1}(υ) is empty then the inequality of Proposition is satisfied. Suppose neither f^{-1}(u) not f^{-1}(υ) is empty. Then

$$\mu f[v]\,(\lambda u + \mu v) = \sup_{w \varepsilon f(\lambda u + \mu v)} \mu v(w)$$

$\geq$ min {min{$\mu_F(\lambda)$, $\mu_{[v]}$ (u)}, min {$\mu_F(\mu)$, $\mu_{f[v]}$ (υ)}}.

This completes the proof.

If V is a fuzzy linear space in a linear space Y over an ordinary field F in X, then $\mu_V(\lambda x) = \mu_V$ (x) for all x Y and all 0 λ X .

Proof :

For all x ∈ Y we have $\mu_V(\lambda x) \geq \mu_V$ (x). For 0 ≠λ ∈ F and all $=\mu_V$ (x) for 0 ≠λ= X and all x ∈ Y we have

μ_V (x) = $\mu_V(\lambda^{-1} \lambda x) \geq \mu_V$ (λx.)

Hence μ_V (x) = μ_V (x) for all 0 ≠λ∈ X and all x∈Y. this completes the proof.

Conclusions :The concept of topological group has been applied to the theory of fuzzy sets by Foster by introducing the notion of fuzzy topological groups. In this paper the concept of fuzzy linear spaces has been introduced and it is expected that several results from Linear Algebra and Functional Analysis can be extended to the concept of Fuzzy sets. Particularly, it is hoped that, in a natural way, the concept of fuzzy linear spaces will give rise to the notions like fuzzy linear topological spaces and fuzzy normed linear spaces.The concepts of fuzzy vector spaces and fuzzy topological vector spaces have been discussed in Katsaras and Liu.But it may be noted that our definitions are different from those introduced ahead.

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