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COMMON FIXED THEOREM FOR TWO MAPPINGS ON FUZZY METRIC SPACE

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Abstract

The purpose of this paper is to prove a common fixed point theorem for two coincidentally commuting mapping on Fuzzy matric space.

The notion of Fuzzy sets was introduced by Zadeh's [ ]. This laid foundation of Fuzzy mathematics, the theory of Fuzzy metric space has been extensively studied and developed by Sessa [] Junk and Rhodes [ ] Dhae [ ].

Chauhan and Singh [ ] on 1997 proved a fixed point theorem for a continuous self mapping T of a complete S-Fuzzy metric space (X, S, *) satisfying the condition.

$S(Tx, Ty, Tz, Kt) \geq S(x, y, z, t)$ for all x, y ∈ X, 0 <K < 1, t>0.

$\lim_{t\to\infty} S(x, y, z, t) = 1$, then T has a unique fixed point in X.

We have extended this theorem to a pair of self mappings T_1 and T_2 on X and showed that T_1 and T_2 have a unique common fixed point.

PRELIMINARIES :

How we set some definitions to be used an our result.

DEFINITION (i): The three tuple (X, S, *) is said to a S-Fuzzy metric space if X is an arbitrary sets, * is continuous t-norm and S is a Fuzzy set on X^3 x (0,∞) satisfying the following conditions.:

$S(x, y, z, t) > 0$ (non-negative) ---------------------- (i)

$S(x, y, z, t) = 1$ iff x=y=z (coincidence)----------------------------(ii)
$S(x, y, z, t) = S(y, z, x, t)$ (symmetry)------------------------------(iii)
$S(x, y, z, r+s+ k) \geq S(x, y, w, r) * S(x, w, z, s)$
$*S(w, y, z, t)\ \forall\, x, y, z, w \in X$
and r, s, k>0 (Tetrahedral inequality.)-------------------(iv)

DEFINITION (ii) : A sequency (yn) is a S-Fuzzy metric space (x, s, *) is a cauchy sequence iff for Each ∈ > 0,t> 0, there exists n_0 ∈ N such that

$S(y_n, y_m, y_p, t) > 1 - \in$ for all n, m, p $\geq n_0$.

DEFINITION (iii) : A S-Fuzzy metric space in which every cauchy sequence is convergent is called a complete S-Fuzzy metric space.

DEFINITION (iv): Let X be a arbitrary set. Two maps T_1 and T_2 : X → x are said to be coincidentally commuting if they commute at coincidence point.

Main Result:

Let (X, S, *) be a S-Fuzzy metric space and let T_1, T_2 : X → x be map that the following conditions:

$T_1(x) \subseteq T_2(x)$ (1.1)

and any one of $T_1(x)$ and $T_2(x)$ is complete. (1.2)

$S(T_1x, T_1y, T_1z\ \alpha\ t) \geq S(T_2x, T_2y, T_2z, t)$

for all x, y, z ∈ x and 0<1, t> 0 and for

t> 0, $\lim_{t\to\infty} S(x, y, z, t) = 1$ (1.3)

Then T_1 and T_2 have a unique common fixed point, provided T_1 and T_2 are coincidentally commuting mapping on X.

Proof: Suppose x ∈ X be an arbitrary point in X.

By (1.1) we get $T_1x_0 = T_2x_1 = y_1$.

By induction:

$y_{n+1} = T_{1xn} = T_{2xn+1}$, n = 0, 1, 2, …………….. .

Since $T_1(x) \subseteq T_2(x)$.

If $y_t = y_{t+1}$ for some r ∈ N then

$y_r = T_1x_{r-1} = T_{1xr} = T_2x_r = T_2x_{r+1} = y_{r+1}$ = u for some u ∈ X .

We show that u is a common fixed point of T_1 and T_2 .

Since $T_1x_r = T_2x_r$ and T_1 and T_2 are coincidentally commuting Mapping.

We have,

$T_1u = T_1T_2x_r = T_2u$.

From (1.3), we have

$$
\begin{aligned}
S(T_{1u}, T_1u, u, \alpha\ t) &= S(T_{1u}, T_1u, T_1x, \alpha\ t) \\
&\geq S(T_2u, T_{2u}, T_2x, t) = (T_1u, T_{1u}, T_1x_r, t) \\
&\geq S(T_2u, T_{2u}, T_2z, t/\ \alpha) \\
&\geq S(T_2u, T_{2u}, T_2x, t/\ \alpha\ n)
\end{aligned}
$$

Letting n → ∞ we have

$T_1u = u = T_2u$.

This shows that u is a fixed point of T_1 and T_2 .

If yn ≠ y_{n+1} for each n = 0, I, 2, …………………….. .

For each P ∈ N, t > 0 we have by. (1.3)

$$
\begin{aligned}
S(y_1, y_2, y_{p+1}, \alpha\ t) &= S(x_0, T_1x_1, T_1x_p, \alpha\ t) \\
&\geq S(T_2x_0, T_2x_1, T_2x_p, \alpha\ t) \\
&= S(y_0, y_1, y_p, t)
\end{aligned}
$$

And

$$
\begin{aligned}
S(y_2, y_2, y_{p+2}, \alpha\ t) &\geq S(y_0, y_2, y_{p+1}, t) \\
&\geq S(y_0, y_2, y_p, t/\ \alpha)
\end{aligned}
$$

Or,

$$S(y_2, y_3, y_{p+2}, \alpha\ t) \geq S(y_0, y_2, y_p, t/\ \alpha^2)$$

Proceeding in this way for p, q ∈ N and t> 0 we have

$S(y_n, y_{n+p}, y_{n+p+q}, 3t) \geq S(y_n, y_{n+1}, y_{n+p+q}, t)*S(y_{n+1}, y_{n+p}, y_{n+p+q}, t)$ $*S(y_n, y_{n+p}, y_{n+1}, t)$

$\geq S(y_0, y_1, y_p, t/\ \alpha^n)*S(y_0, y, y_{p+q}, t/\ \alpha^n)*S(y_{n+1}, y_{n+p}, y_{n+p+q}, t)$

$\geq S(y_0, y_1, y_p, t/\ \alpha^n)*S(y_0, y, y_{p+q}, t/\ \alpha^n)*S(y_{n+1}, y_{n+2}, y_{n+p+q}, t) *S(y_{n+1}, y_{n+p}, y_{n+2}, t) *S(y_{n+2}, y_{n+p}, y_{n+p+q}, t)$

$\geq S(y_0, y_1, y_p, t/\ \alpha^n)*S(y_0, y_1, y_{p+q}, t/\ \alpha^n)*S(y_0, y_1, y_{p+q-1}, t/\ \alpha^{n+1}) *S(y_0, y_1, y_{p-1}, t/\ \alpha^n) *S(y_{n+2}, y_{n+p}, y_{n+p+q}, t)$

$\geq S(y_0, y_1, y_p, t/\ \alpha^n)*S(y_0, y_1, y_{p+q}, t/\ \alpha^n)*S(y_0, y_1, y_{p-1}, t/\ \alpha^{n+1}) *S(y_0, y_1, y_{n+p-1}, t/\ \alpha^{n+1})$……………………… $*S(y_0, y_1, y_{p+1}, t/\ \alpha^{n+p-1})$

Taking limit as n → ∞ , we have

$S(y_n, y_{n+p}, y_{n+p+q}, 3t) \geq 1*1*1*1*1*$……………………*1

Which implies that

$S(y_n, y_{n+p}, y_{n+p+q}, 3t) \to 1$, then $\{y_n\}$ is Cauchy sequence in X.

Since $T_2(x)$ is complete so there exists a point $u \in T_2(x)$ such that

$\lim_{n\to\infty} y_n = \lim_{n\to\infty} T_2 x_n = u$

Now we show that u is a common fixed point of T_1 and T_2 .

Since $u \in T_2(x)$ so there exists point $P \in X$ such that $T_2 p = u$

By (1.3)

$$S(T_1P, T_2P, T_2P, k) = \lim_{n\to\infty} S(T_1P, T_1x_n, T_1x, k) \text{ for } t>0$$
$$\geq \lim_{n\to\infty} S(T_2P, T_2x, T_1x_n, t)$$
$$= S(T_2P, u, u, t)$$
$$= S(u, u, u, t) \text{ which mapping that } T_1P = T_2P.$$

Now we show that $u = T_1P = T_2P$ is a common fixed point of T_1 and T_2 .

Since $T_1P = T_2P$ and T_1 ,T_2 are coincidentally commuting mapping.

We have $T_1u = T_1T_2P = T_2T_1P = T_2u$

We claim, $T_1u = T_2u = u$.

From (1.3) we have

$$\begin{aligned} S(T_1u, T_2u, u, \alpha t) &= S(T_1u , T_2T_1P, u , \alpha t) \\ &= S(T_1u , T_1T_2P , T_1T_2, \alpha t) \\ &= S(T_1u , T_1u , T_1P, \alpha t) \\ &\geq S(T_2u , T_2u , T_2P, t) \\ &= S(T_1u , T_1T_2P , T_1P, t) \\ &= S(T_1u , T_1u , T_1P, t) \\ &\geq S(T_1u , T_2u , u, t/ \alpha) = S(T_1u , T_2u , u, t/ \alpha) \\ &= \ldots\ldots\ldots\ldots S(T_1u , T_2u , u, t/ \alpha^n) \end{aligned}$$

$0< \alpha < 1$ as $n\to \infty$.

This we have by Chauhan and Singh [ ] $T_1u = T_2u = u$.

For uniqueness if u and v are two points common to T_1 and T_2, we have

$$\begin{aligned} S(u, u, v, \alpha t) &= S(T_1u , T_1u, T_1v, \alpha t) \\ &\geq S(T_2u , T_2u, T_2v, t) \\ &= S(u, u, v, t) \\ &\geq S(u, u, v, t/ \alpha^n) \text{ as } n \to \infty . \end{aligned}$$

have u = v

This completes the proof.

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