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# ELASTIC CABLE-CONNECTED SATELLITES SYSTEM UNDER SEVERAL INFLUENCES OF GENERAL NATURE: JACOBEAN INTEGRAL OF MOTION IN CIRCULAR ORBIT

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***Abstract***

*Jacobean integral of motion of a system of two artificial cable connected satellites in circular orbit under several influences of general nature is obtained. Cable connecting the two satellites is light, flexible, non-conducting and elastic in nature. Several influences of general nature are earth's magnetic field, solar radiation pressure, shadow of the earth and earth's oblateness. These influences are the perturbative forces acting simultaneously on the system concerned. We do not consider nutation and wobbling of the circular orbit of centre of mass of the system.*

***Keywords:*** *Two cable connected satellites, Circular orbit, Elastic cable, Jacobean integral.*

## 1. Introduction:

The work is a physical and mathematical idealization of real space system. We establish Jacobean integral of motion of the system under the influence of shadow of the earth, solar radiation pressure, oblateness of the earth and earth's magnetic field in circular orbit. The influence of the above mentioned perturbations on the system has been studied singly and by a combination of any two or three of them by various workers, but never conjointly all at a time. Therefore, these could not give a real picture of motion of the system. This fact has initiated the present research work. Central attractive force of the earth will be the main force and all other forces, being small enough are considered here as perturbing forces. Since masses of the satellites are small and distances between the satellites and other celestial bodies are very large, the gravitational forces of attraction between the satellites and other celestial bodies including the sun have been neglected. The satellites are considered as charged material particles.

Beletsky and Novikova (1969) are the pioneer workers in the area of researches related to a cable connected satellites system. They studied about motion of a system of two cable-connected satellites in the central gravitational field of force relative to its centre of mass. This study assumed that the two satellites are moving in the plane of the centre of mass. This problem was further investigated in two and three dimensional cases by Singh and Demin (1972) and Singh (1973). Das et al. (1976) studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit. Kumar and Bhattacharya (1995) studied the stability of equilibrium positions of two cable connected satellites under the influence of solar radiation pressure, earth's oblateness and earth's magnetic field. Sinha and Singh (1987) investigated the effect of solar radiation pressure on the motion and stability of the system of two inter-connected satellites when their centre of mass moves in circular orbit. Singh et al. (2001) studied non-linear effects on the motion and stability of an inter connected satellites system orbiting around an oblate earth. Kumar and Srivastava (2006) studied evolutional and non-evolutional motion of a system of two cable-connected artificial satellites under some perturbative forces. Kumar and Prasad (2015) studied about nonlinear planer oscillation of a cable-connected satellites system and non-resonance. Kumar and Kumar (2016) studied equilibrium positions of a cable-connected satellites system under several influences.

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2. **Treatment of the problem and Jacobean integral:**

We write the equations of motion of one of the two satellites when the centre of mass moves along Keplerian elliptical orbit in Nechvile's (1926) co ordinate system as

$$X''-2Y'-3X\rho=-\frac{A}{\rho}\cos i-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos\left(v-\alpha\right)+\frac{12\mu k_2}{R^5}\rho X$$

$$-\lambda_\alpha\left[\rho-l_0\left(X^2+Y^2\right)^{-1/2}\right]X$$

and

$$Y''+2X'=-\frac{A\rho'}{\rho^2}\cos i+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\left(v-\alpha\right)-\frac{3\mu k_2}{R^5}\rho Y$$

$$-\lambda_\alpha\left[\rho-l_0\left(X^2+Y^2\right)^{-1/2}\right]Y \tag{1}$$

With the condition of constraint

$$X^2+Y^2\le\frac{l_0^{\,2}}{\rho^2} \tag{2}$$

Also,

$$\rho=\frac{1}{1+e\cos v},\qquad \lambda_\alpha=\left[\frac{m_1+m_2}{m_1m_2}\right]\frac{\lambda}{l_0},\ k_2=\frac{\bar{\varepsilon}\,\mathrm{R_e}^{\,2}}{3},\ \bar{\varepsilon}=\alpha_R-\frac{m}{2},\ m=\frac{\Omega^2\,\mathrm{R_e}}{g_e}$$

$$A=\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu p}} \tag{3}$$

Where $m_1$ and $m_2$ are masses of the two satellites. $\mu$ is the product of mass of the earth and gravitational constant. $\lambda$ is the modulus of elasticity for connecting cable. $Q_1$, $Q_2$ are charges of the two satellites. $\gamma$ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If $\gamma$ is equal to zero, then the system is affected by the shadow of the earth. If $\gamma$ is equal to one, then the system is not within the said shadow. $B_1$, $B_2$ are the absolute values of the forces due to the direct solar pressure on $m_1$ and $m_2$ respectively. $\ell_0$ is original length of the cable, $\alpha_R$ is the earth's oblateness, $\Omega$ is angular velocity of the earth's rotation, $\mathrm{R_e}$ is equatorial radius of the earth and $g_e$ is the force of gravity. $e$ is the eccentricity of the orbit of centre of mass of the system, $v$ is the true anomaly as the independent variable in place of time *t*. $i$ is inclination of the orbit with the equatorial plane, $\in$ is inclination of the oscillatory plane of the masses $m_1$ and $m_2$ with the orbital plane of the centre of mass of the system and $\alpha$ is inclination of the ray. $\mu_E$ is the value of magnetic moment of the earth's dipole. $p$ is the focal parameter. Prime represent differentiation with respect to v.

If motion of one of the satellites we determined with the help of equation (1), motion of the other satellite of mass $m_2$ can be determined by

$$m_1\bar{\rho}_1 + m_2\bar{\rho}_2 = 0 \tag{4}$$

$\bar{\rho}_j\,(j=1,2)$ is the radius vector of the particle $m_j\,(j=1,2)$ with respect to the centre of mass of the system.

In the case of circular orbit of centre of mass of the system, we have $e=0$, $\rho=1$ and $\rho'=0$. Therefore the equations (1) become

$$X''-2Y'-3X = -A\cos i-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos\left(v-\alpha\right)+\frac{12\mu k_2}{R^5}X-\lambda_\alpha\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]X$$

and

$$Y''+2X' = \gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\left(v-\alpha\right)-\frac{3\mu k_2}{R^5}Y-\lambda_\alpha\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]Y \tag{5}$$

With the condition of constraint

$$X^2+Y^2 \le l_0^2 \tag{6}$$

The system of two satellites is allowed to pass through the shadow beam during its motion. Let us assume that $\theta_2$ is the angle between the axis of the cylindrical shadow beam and the line joining the centre of the earth and the end point of the orbit of the centre of mass within the earth's shadow, considering the positive direction towards the motion of the system. The system starts to be influenced by the solar pressure when it makes an angle $\theta_2$ with the axis of the shadow beam and remains under the influence of solar pressure till it makes an angle $\left(2\pi-\theta_2\right)$ with the axis of the cylindrical shadow beam. Thereafter, the system will enter the shadow beam and the effect of solar pressure will come to an end.

Next, the small secular and long periodic effects of solar pressure together with the effects of earth's shadow on the system may be analyzed by averaging the periodic terms in (5) with respect to $\upsilon$ from $\theta_2$ to $\left(2\pi-\theta_2\right)$ for a period when the system is under the influence of the sun rays directly i.e. $\gamma=1$ and from $-\theta_2$ to $+\theta_2$ for a period when the system passes through the shadow beam i.e. $\gamma=0$.

Thus after averaging the periodic terms, we write the equations (5) as

$$X''-2Y'-3X=-A\cos i+\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}+\frac{12\mu k_2}{R^5}X-\lambda_\alpha\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]X$$

and

$$Y''+2X'=\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\sin\alpha\sin\theta_2}{\pi}-\frac{3\mu k_2}{R^5}Y-\lambda_\alpha\left[1-l_0\left(X^2+Y^2\right)^{-1/2}\right]Y \tag{7}$$

Equations (7) do not contain the time explicity. Therefore Jacobean integral of motion of the system exists. In order to obtain the Jacobean integral of motion of the system, we multiply the first and second equations of (7) by 2 X' and 2Y' respectively, add them and then integrate the final equation. In this way, we obtain

$$X'^2+Y'^2-3X^2=-2AX\cos i+\frac{2}{\pi}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\theta_2\left(X\cos\alpha+Y\sin\alpha\right)$$
$$+\frac{3\mu K_2}{R^5}\left(4X^2-Y^2\right)-\lambda_\alpha\left[\left(X^2+Y^2\right)-2l_0\left(X^2+Y^2\right)^{1/2}\right]+h \tag{8}$$

Where, h is a constant of integration, generally known as Jacobean constant.

Next, we examine the surface of zero velocity of motion of the system. For this purpose, we take help from the Jacobean integral (8). It is as follows

$$3X^2-2AX\cos i+\frac{2}{\pi}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\theta_2\left(X\cos\alpha+Y\sin\alpha\right)$$
$$+\frac{3\mu K_2}{R^5}\left(4X^2-Y^2\right)-\lambda_\alpha\left[\left(X^2+Y^2\right)-2l_0\left(X^2+Y^2\right)^{1/2}\right]+h=0 \tag{9}$$

We, therefore, conclude that satellite $m_1$ moves inside the boundary of different curves of zero velocity, represented by (9) for different Values of Jacobean constant h.

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3. **Result and Discussion:**

Aim of the present paper is to obtain Jacobean integral of motion of a system of two cable-connected satellites under the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth and earth's magnetic field in circular orbit. The cable connecting the two satellites is light, flexible, non- conducting and elastic in nature. The satellites are considered as charged material particles. As the body of the satellites is made up of metal, the satellites cut magnetic lines of force of the earth during the motion. According to Lorentz force charges get developed on the two satellites. But magnitude of the charges is very small. Thus, electrostatic interaction between the satellites is not taken into account. Motion of the system is studied relative to the centre of mass.

4. **Conclusion:**

Equation (8) is the required Jacobean integral of motion of the system. It is also known as the first integral of motion of the system. It has wide applications in the further studies of the problems related to two elastic cable connected artificial satellites.

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