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Wavelet Analytical Approach to Human Face Expressions and Basic Emotions

Bijendra Singh and Anil Kumar*
Department of Physics, Hindu College, Moradabad, Affiliated to M.J.P. Rohilkhand University, Bareilly, U.P., INDIA
*Email: akumarmbd@gmail.com

Abstract: Wavelet transform is an important tool and being frequently used in the various field of image processing due to its ability of simultaneously providing the spatial and frequency representation of an image. The emotions develop behavioural tendencies in the poser as well as perceiver. An image signal is passed through an analysis filter bank consisting of a low pass and high pass filter at each decomposition stage and followed by a decimation operation. In each level of wavelet decomposition, the image is decomposed into four sub-bands corresponding to approximation $LL1$ and detail $HL1$, $LH1$ and $HH1$. Face expressions related to six basic emotions are taken as sample images. The wavelet and statistical analysis of sample images are performed and interpreted. The wavelet analytical results of the face expressions with different emotions are consistent with the actual emotions of an individual.

Keywords: Approximation, detail, emotion, face expression, image, wavelet

1. Introduction

Fourier transforms has been an important analytical tool for long times in science and engineering. But it well analyses stationary signal and it is not suitable to analyse non-stationary and transient signals. Window Fourier transforms (WFT) or short time Fourier transforms (STFT) analyses non-stationary signal taking its small part as stationary. In STFT, a signal is multiplied by a time-frequency window function and thereafter its Fourier transform is performed [1]. The resolution in STFT is restricted by Heisenberg uncertainty principle. To overcome the discrepancies of FT and STFT, the concept of wavelet transforms is proposed in 1980s. The formalization and emergence of wavelet theory is the result of a multidisciplinary effort that brought together Mathematicians, Physicists and Engineers, who recognized that they were independently developing similar ideas. For signal processing, this connection has created a flow of ideas that goes well beyond the construction of new bases or transforms. The world of transients is considerably larger and more complex than the garden of stationary signals. Wavelet theory involves representing general functions in terms of simpler and fixed building blocks at different scales and positions. This has been found to be a useful approach in several different areas of sub-band filtering techniques, quadrature mirror filters, pyramid schemes, etc. in signal processing, while in mathematical physics similar ideas are studied as part of the theory of coherent states [2-3]. Wavelet theory represents a useful synthesis of these different approaches. A wavelet is a wave-like oscillation with amplitude that begins from zero, increases, and then decreases back to zero. It can typically be visualized as a brief oscillation like one recorded by a seismograph or heart monitor. Generally, wavelets are intentionally crafted to have specific properties that make them useful for signal processing. Wavelets can be combined using a reverse, shift, multiply and integrate technique called convolution, with portions of a known signal to extract information from the unknown signal. As a mathematical tool, wavelets can be used to extract information from many different kinds of data, including but certainly not limited to audio signals and images. Sets of wavelets are generally needed to analyse data fully. Thus, set of wavelets is useful in wavelet-based analysis and synthesis algorithms where it is desirable to recover the original information with minimal loss. The two dimensional (2D) or image wavelet transforms has great importance in multimedia information retrieval, biometrics, pattern classification, machine learning, computer vision and medical data processing. It is frequently used for feature extraction due to its ability to simultaneously provide spatial and frequency representation of image [4].

Happiness, sadness, disgust, fear, surprise and anger are six basic emotions. Facial expressions are salient social cues in everyday interaction. On basis of the study of human faces the prediction of the emotions of an individual has been an extreme curiosity for long times. The emotions develop behavioural tendencies in the poser and perceiver, namely approach and avoidance. Happiness and anger are directly associated with approach and avoidance respectively, however, behavioural tendencies in response to sadness and disgust are more complicated [5]. In wavelet analysis of face expressions, the facial signal is processed using wavelet transforms and the emotions of the person at that time is recognized. The response or behaviour of perceiver is also highly important in reaction of findings related to these kinds of social stimuli. By testing their impact on behaviour of any person, the different emotional expressions are divided into positive and negative cues based on theoretical speculations. In their review of historical conceptualizations of approach and avoidance motivation, Elliot and Covington [6] concluded that in humans, however, this link might at least be influenced by other, more conscious, motivational mechanisms, such as relationship and mood state. Based on data of a large sample, the expression of happiness was perceived as an invitation to reciprocate smiles and to cooperate. However, comparing more automatic to more controlled processes, we expect to clarify differential processes in reactions to these diverging facial emotional expressions. With help of 2D wavelet transforms, we can analyse the human face expressions and basic emotions of an individual very well.

2. Two-dimensional (2D) wavelet transforms

Wavelet is defined as a small wave which can be dilated and translated. In wavelet transforms the convolution of a signal and wavelets takes place.A multi resolution analysis (MRA) is a radically new recursive method for performing discrete wavelet analysis [7-9].By MRA, the orthogonal decomposition of signal space V_0 is as follows: -

$$V_0 = V_{j_0} \oplus \sum_{j=1}^{j_0} W_j$$

where $j \in \mathbb{Z}$ and j_0 represents the order or level of decomposition. We can approximate a discrete signal in space of square summable sequences $\ell^2(\mathbb{Z})$ as follows:-

$$f(x) = \sum_k a_{j_0,k} \phi_{j_0,k}(x) + \sum_{j=1}^{j_0} \sum_k d_{j,k} \psi_{j,k}(x)$$

Where,

$$a_{j_0,k} = \langle f, \phi_{j_0,k} \rangle = \int f(x) \phi_{j_0,k}(x) dx$$

$$d_{j,k} = \langle f, \psi_{j,k} \rangle = \int f(x) \psi_{j,k}(x) dx$$

Are called approximation and detail coefficients respectively. By the first decomposition step, the signal is decomposed in two sets of approximation and detail coefficients.

A digital image $s_0[m, n]$ described in a 2D discrete space is derived from an analogue image $s_0(x, y)$ in a 2D continuous space through a sampling process that is frequently referred to as digitization. The 2D continuous image $s_0(x, y)$ is divided into M rows and N columns [10]. The intersection of a row and a column is termed a pixel. The value assigned to the integer coordinates $[m, n]$ with $\{m = 0, 1, 2, \ldots, M-1\}$ and $\{n = 0, 1, 2, \ldots, N-1\}$ is $s_0[m, n]$. In 2D wavelet transform, the scaling and wavelet function are two variable functions $\phi(x, y)$ and $\psi(x, y)$. The scaling function is the low frequency component of the previous scaling function in two dimensions. However, the wavelet function is related to the order to apply the filters. The functions can be easily rewritten as follows: -

$\phi(x, y) = \phi(x)\phi(y)$

$\psi^1(x, y) = \psi(x)\phi(y)$

$\psi^2\ (x, y) = \phi(x)\psi(y)$

$\psi^3\ (x, y) = \psi(x)\psi(y)$

If we define the functions as separable functions, it is easier to analyse the 2D function and we can focus on the design of 1D wavelet and scaling functions. The analysis and synthesis equations for approximation and details are modified as follows: -

$a[j, m, n] = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{x=0}^{N-1} f(x, y)\ \phi_{j,m,n}(x, y)$

$$d^i[j, m, n] = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{x=0}^{N-1} f(x, y)\ \psi^i{}_{j,m,n}(x, y)$$

Where $i = 1,2,3$. Any discrete image can be expressed in terms of approximation and detail coefficients as follows: -

$$f(x, y) = \frac{1}{\sqrt{MN}} \sum_{m}^{M-1} \sum_{n}^{N-1} a[j_0, m, n]\phi_{j_0,m,n}(x, y) + \frac{1}{\sqrt{MN}} \sum_{i=1,2,3} \sum_{j=1}^{j_0} \sum_{m}^{M-1} \sum_{n}^{N-1} d^i[j, m, n]\psi^i{}_{j,m,n}(x, y)$$

3. Methodology

In 2D-wavelet transforms of an image, the scaling and wavelet functions can be described as,

$$\phi_{j,m,n}(x, y) = 2^{j/2}\phi\left(2^j x - m, 2^j y - n\right)$$
$$\psi^i{}_{j,m,n}(x, y) = 2^{j/2}\psi^i\left(2^j x - m, 2^j y - n\right)$$

for $1 \le i \le 3$. The wavelet functions $\left\{\psi^1{}_{j,m,n}, \psi^2{}_{j,m,n}\psi^3{}_{j,m,n}\right\}$ form an orthonormal basis of the subspace of details as follows: -

$W_j{}^2 = \left(V_j \otimes W_j\right)\left(W_j \otimes V_j\right)\oplus(\left(W_j \otimes W_j\right)$

at scale j. The Lebesgue space $L^2(\mathbb{R})$ can be expressed as follows:-

$L^2(\mathbb{R}^2) = \sum_j W_j{}^2$

In 1D wavelet transform, the analysis filter bank consists a low pass and high pass filter at each decomposition stage. When a signal is passed through these filters, it is divided into two sub-bands. The low pass filter corresponding to an averaging (approximation) operation, extracts the coarse information of the signal, while the high pass filter corresponding to a differencing (detail) operation extracts the fine information of the signal [11]. The 2D wavelet transform is considered as superposition of two 1D wavelet transforms, one is along X-dimension and other is along Y-dimension. An image signal is analysed by passing it through these analysis filter banks followed by a decimation operation. Because of decimation, the total size of the transformed image is same as the original size.In this way, the image is divided into four sub-bands denoted by LL1, $HL1$, $LH1$, and $HH1$ after one level decomposition (Fig. 1).

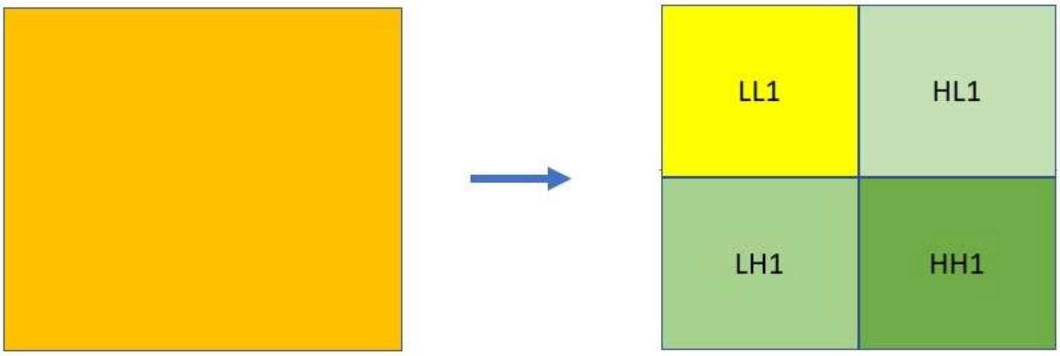

Figure 1: Decomposition of an image (level-1)

The $LL1$, $HL1$, $LH1$ and $HH1$ are each $M/2 \times N/2$ submatrices. The trend $LL1$ consists of scaling coefficients, whereas the fluctuations $HL1$, $LH1$ and $HH1$ consist of wavelet coefficients for each of the three kinds of wavelet basis functions. The trend $LL1$ contains scaling coefficients for the scaling basis $\{\phi_{j,M-1}(x)\phi_{k,M-1}(y)\}$ and occupies the upper left quadrant of the transform.The wavelet transforms with a cascade of filtering is followed by sub-sampling by a factor of 2. The $LL1$, $HL1$, $LH1$ and $HH1$ are each $M/2 \times N/2$ submatrices. The trend $LL1$ consists of scaling coefficients, whereas the fluctuations $HL1$, $LH1$ and $HH1$ consist of wavelet coefficients for each of the three kinds of wavelet basis functions. The trend $LL1$ contains scaling coefficients for the scaling basis $\{\phi_{j,M-1}(x)\phi_{k,M-1}(y)\}$ and occupies the upper left quadrant of the transform [13].The trend LL1 has great importance here due to containing average characteristics of the image. The wavelet and statistical analysis of sub-images corresponding to trend LL1 of face expressions related to basic emotions are performed and interpreted.

4. Results and Discussion

Six types of face expressions corresponding to basic emotions happiness, sadness, disgust, fear, surprise and anger are considered as the test images. The 2D wavelet analysis of trend of all these images are performed throughHaar wavelet (level-1) with help of software MATLAB in figures 2-7 [12].The trend $LL1$ consists of scaling coefficients is clearly a low pass version of the original image and represents the average behaviour of the image.

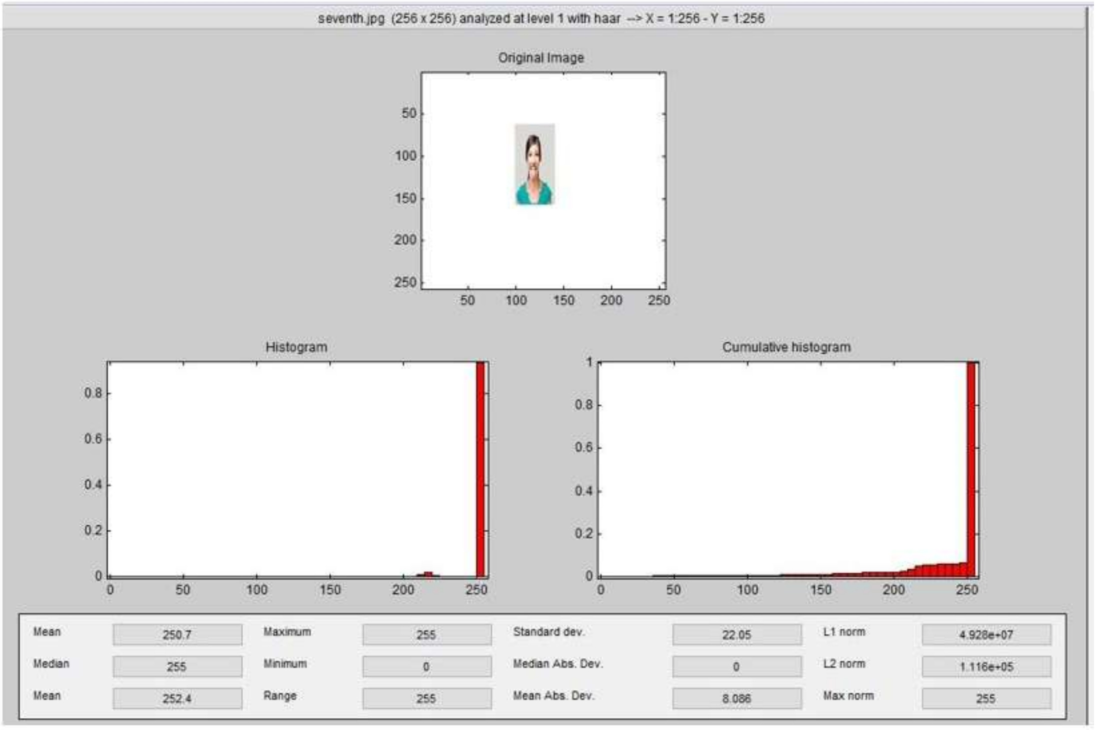

Figure 2: Waveletanalysis of Happiness

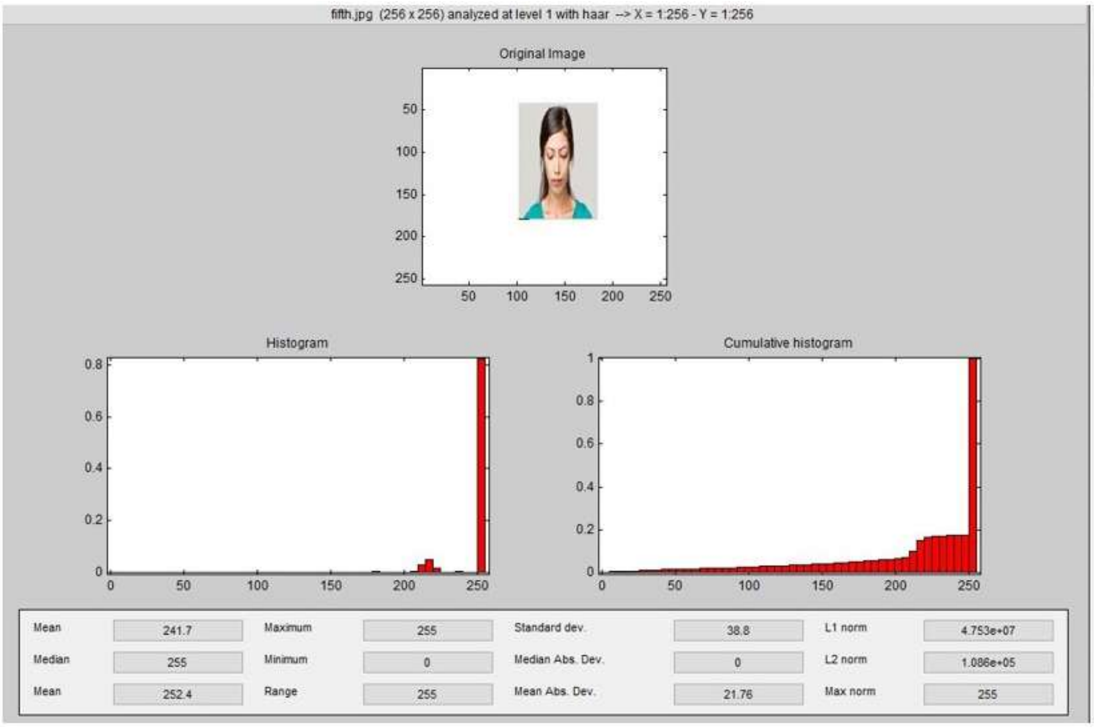

Figure 3: Wavelet analysis of Sadness

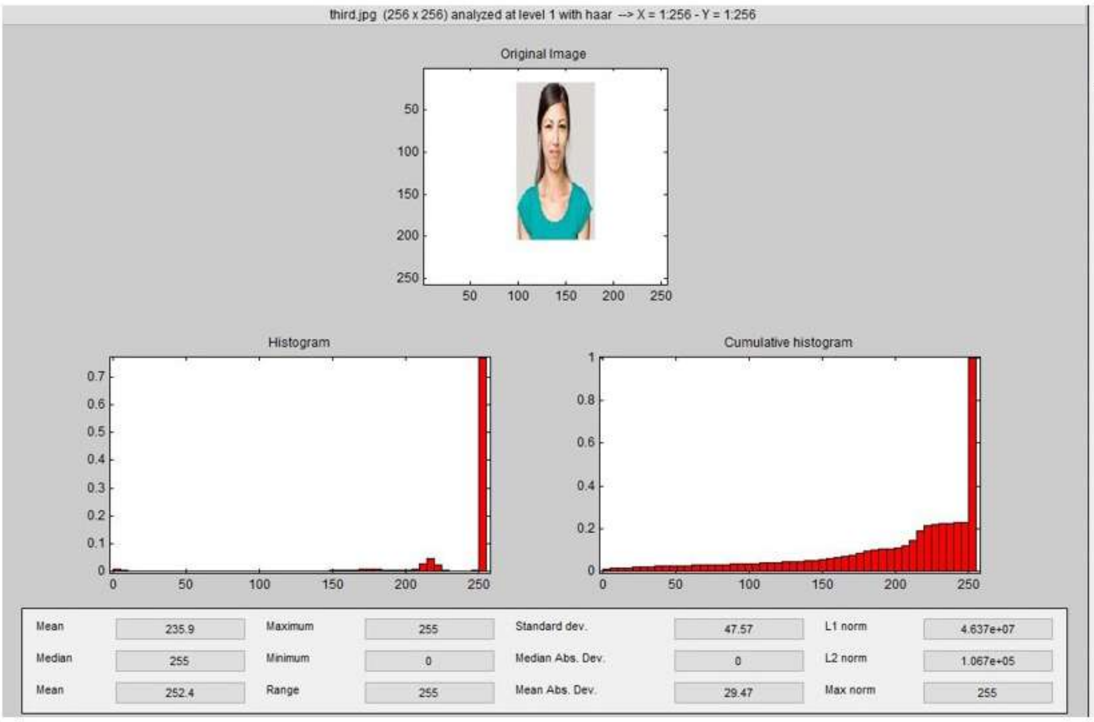

Figure 4: Wavelet analysis of Disgust

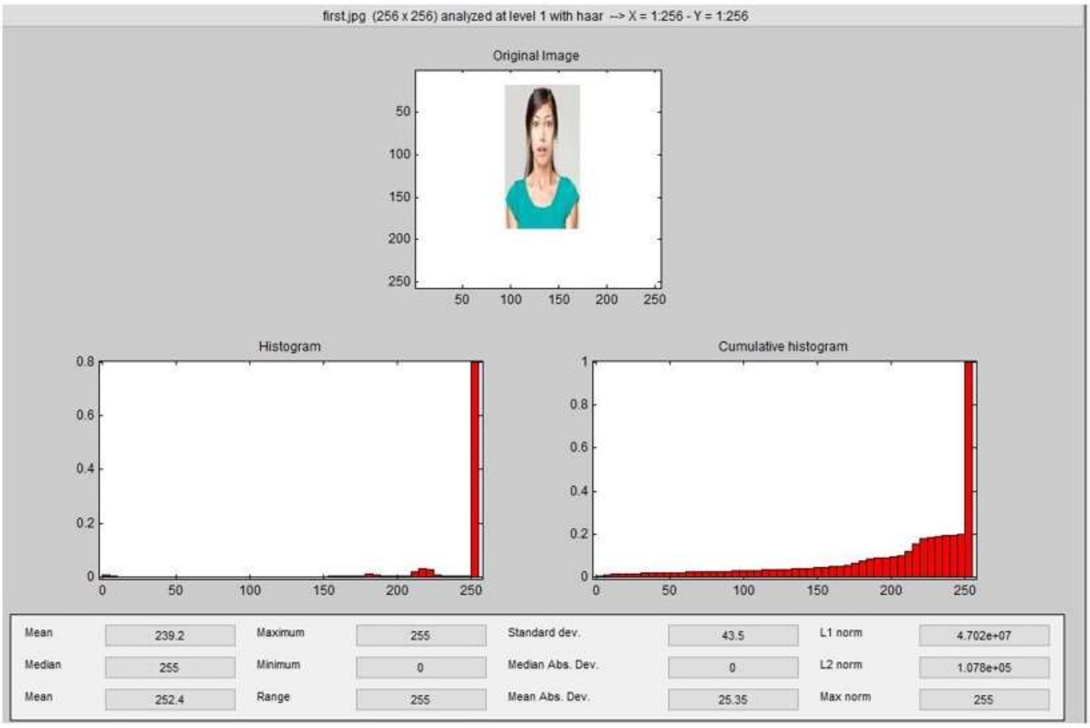

Figure 5: Wavelet analysis of Fear

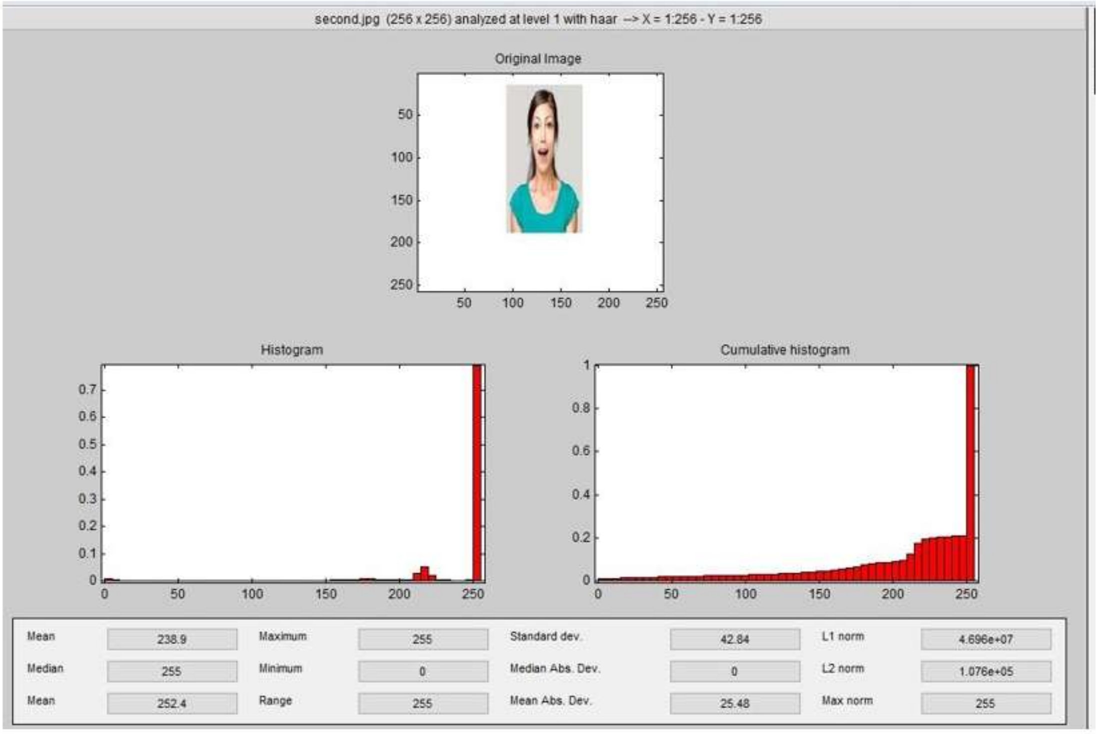

Figure 6: Wavelet analysis of Surprise

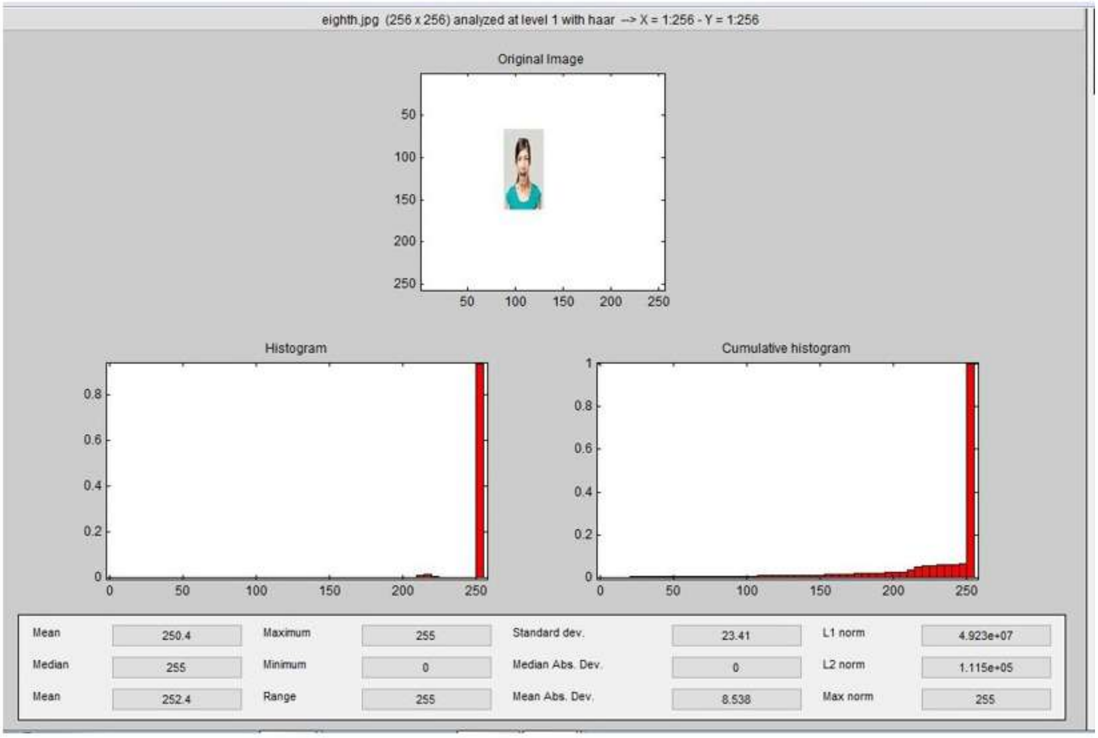

Figure 7: Wavelet analysis of Anger

The histogram represents density of the given data. It is an estimation of the probability distribution of variables. Cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin. Few statistical parameters of the wavelet approximated image corresponding to the six basic emotions are determined and enlisted in the table 1.

Table 1: Few statistical parameters of wavelet approximatedbasic emotions

S.No.	Emotion	Mean	Standard Deviation	L1 Norm	L2 Norm
1	Happiness	250.7	22.05	4.928x10^7	1.16x10^5
2	Sadness	241.7	38.8	4.753x10^7	1.086x10^5
3	Disgust	235.9	47.57	4.636x10^7	1.067x10^5
4	Fear	239.2	43.5	4.702x10^7	1.078x10^5
5	Surprise	238.9	42.84	4.696x10^7	1.076x10^5
6	Anger	250.4	23.41	4.923x10^7	1.15x10^5

The $L1$ -norm of a signal is a measure of sparsity or compressibility of the signal and represents the sparse sensing of any signal. In signal analysis, $L2$ -norm is a measureof vector difference of a signal and interpreted as a measure of energy associated with the signal. The wavelet analysis of the face expression corresponding to emotion; happiness out of all basic emotions provides the maximum value of the mean, $L1$ -norm and $L2$ -norm, while minimum value of standard deviation. Obviously, the maximum sparsity and energy are associated with the face expression corresponding to happiness. It is also clear that happiness represents to minimum deviation of wavelet coefficients of approximation from the mean value.

5. Conclusion

Imagesare processed via 2D wavelet transformsusing low pass and high pass filters. The low pass filtered image occupies the upper left quadrant and represents the low pass version or average of the image. The average versions of the imagesare obtained by using Haar wavelet, level-1 consists of face expression corresponding to all basic emotions. The wavelet coefficients belonging to these average versions providemaximum sparsity and energy associated in the face expression of happiness out of all emotions. Taking into account these results, it is concluded that the wavelet analytical approach to the face expressions provides a simple and accurate framework for extracting the face expressions and expecting the emotions of an individual.

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