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PARAMETRIC OPTIMIZATION OF *FINNED* COUNTER-FLOW DEHUMIDIFIER COUPLED WITH THERMO-ELECTRIC COOLER (TEC)

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Abstract

A counter-flow dehumidifier with uniform rectangular fins was connected to a generic thermoelectric cooler to optimize for optimum water extraction and dehumidification of air. The temperature of the cold face of the thermo couple was kept lower than that of ambient temperature of air so that temperature gradient is formed on the cold side. In this way the incoming air saturates and condenses on the cold surface of the thermoelectric cooler. The heat (Q_L) at temperature T_SC, accumulated on the cold side is pumped to the hot side at temperature T_SH. The heat, Q_H, on the hot side must be continuously expelled by the electric power provide by the thermoelectric couple through Peltier effect. To accomplish this, a counter flow cool air is fed to the hot face of the thermoelectric cell. The numerical analysis indicates that augmenting the dehumidifier with a number of fins just for a small amount of electric power input produces a better amount of water and dehumidifies the incoming air at a faster rate as compared to the dehumidifier without fins.

Key words: fin effectiveness, fin efficiency, fins, thermoelectric cooler, saturation humidity ratio.

Introduction

A vast range of thermoelectric materials have been investigated for a wide range of applications [Refs.1-5]. As a result of their versatility and simplicity, many researchers have delved into optimizing their efficiencies [refs.1, 2, 7, 8 and 9]. A common Thermo-Electric heat pump [Figs.1 & 2] which is commercially available is considered to dehumidify moist air in conjunction with production of potable water. The dehumidification and collection of potable water is enhanced by incorporating an optimum number of fins. This technique of dehumidification and collection of drinking water is most valuable in places where hot and muggy/humid weather is common. The counter flow heat exchanger with fins [Fig. 3,] is represented by two counter flow channels whereby warm air pumped in on one side and cooled air on the other side. In the middle of the channel sits the Thermo-Electric Couple [Fig.1 & 2] whose cold face with temperature, T_{SC} , is exposed to the ambient and humidity ratio of T_i and ω_i, respectively. The cold surface temperature, T_{SC} is kept lower than the incoming ambient temperature, T_i, so that condensation takes place to produce water (refs. 10 and 11). An electric power input is provided to the thermo-electric cell [Fig. 2] so that the cold side heat, $\dot{Q}_L$, which has temperature, T_{SC}, is pumped to the hot side of the thermo-Electric element with heat, $\dot{Q}_H$, and of temperature, T_{SH}.

In order to study the viability of Thermo-Electric heat pump for this purpose, optimization and analysis is carried out using multiple fins with different temperature scales. For the counter flow heat exchanger [Fig. 3], the first and second laws of Thermodynamics analysis are used. Thereafter, an analytical closed form expression of the temperature distributions and humidity or vapor equations are derived. This process is simplified by using the analogy between heat and mass transfer on the wet/cold side of the Thermo-electric element. Since the cold side temperature of the element is lower than the incoming dew point temperature of the air, condensation of water vapor takes place on the cold side of the element. Also, the saturation humidity ratio of the cold side, ω_{SC}, is kept lower than the incoming air humidity, ω_i. As a consequence of this humidity difference, mass transfer occurs from the incoming air to the cold side of the thermo-Electric element [Refs. 5 and 6].

Geometric Description of Thermo-Electric Cooler with uniform rectangular fins:

As presented on Figs. 4, 5 and 6, and Table 1, the various area definitions are calculated as follows:

$$\begin{aligned}
&A_{c=cross_\sec_area} = A_{b=base_area} = w*t \\
&A_{no_fin=TEC_surf} = l_{TEC} * w \\
&A_{un_fin_convection} = l_{TEC} * w - n_{fin} w * t; \quad \{n_{fin} = number_of_fins\} \\
&A_{fin_surf} = 2*l_{fin} * w + w * t; \quad \{the\ 2\ indicates\ top_and_bottom_surfaces\} \\
&A_{tot_convection} = A_{un_fin_convection} + n_{fin} * A_{fin_surf}
\end{aligned} \tag{1}$$

Heat transfer analysis of rectangular finned with uniform cross-sectional area:

The task of a fin is to extract heat from the body it is attached to and expel the heat from its own exposed surface to the surrounding medium via a convection mechanism. Using energy balance, the quantity of heat conducted and convected by the fin is derived from a differential element of the fin [Fig. 6] as:

$$\begin{aligned}
&\{Rate\ of\ heat\ conduction\ through\ the\ cross-sectional\ area, A_c\ at\ x\} = \\
&\{Rate\ of\ heat\ conduction\ through\ the\ cross_sectional\ area, A_c\ at\ x+\Delta x\} \\
&+\{Rate\ of\ heat\ convection\ from\ differential\ circumferential\ area, dA_s\} + TEC_rate_of_heat
\end{aligned}$$

Mathematically, this can be expressed as:

$$\dot{Q}_x = \dot{Q}_{x+\Delta x} + dQ_{conv} + \dot{Q}_{TEC} \qquad (2)$$

where $\dot{Q}_{TEC}$ *is the heat genereted by the TEC* and is obtained from the manufacture's data.

Furthermore, by using Fourier's law for conduction and Newton's law for the convection:

$$\dot{Q}_x = -kA_c \frac{d[T(x)-T_\infty]}{dx} \qquad (3)$$

$$\dot{Q}_{x+\Delta x} = -kA_c \frac{d[T(x)-T_\infty]}{dx} - k\frac{d}{dx}\left[A_c \frac{d[T(x)-T_\infty]}{dx}\right] \qquad (4)$$

$$dQ_{conv} = h[T(x)-T_\infty]dA_s \qquad (5)$$

Note that the differential circumferential surface area is:

$$dA_s = pdx \qquad (6)$$

Where p is the perimeter of the fin.

Inserting Eqs. (3) through (6) into Eqn. (2) and considering constant cross-sectional area of the rectangular fin, we obtain the following heat equation:

$$\frac{d^2[T(x)-T_\infty]}{dx^2} - \left(\frac{hp}{kA_c}\right)\frac{d[T(x)-T_\infty]}{dx} + \frac{\dot{Q}_{TEC}}{k} = 0 \qquad (7)$$

To simplify Eq. (7), define the following parameters:

$$\theta(x) = T(x) - T_\infty \qquad m^2 = \frac{hp}{kA_c} \qquad (8)$$

$$\frac{d^2\theta(x)}{dx^2} - m^2\frac{d\theta(x)}{dx} + \frac{\dot{Q}_{TEC}}{k} = 0 \qquad (9)$$

Eq. (9) is a linear second order ordinary differential equation with constant coefficients and its solution can be obtained by inserting the Eigenvalue function for boundary value problem as follows:

$$\theta(x) = e^{\lambda x} \qquad (10)$$

Inserting Eq. (10) into Eq. (9), one gets:

$$\lambda^2 - m^2 = 0 \Rightarrow \lambda = \pm m \tag{11}$$

Then the general temperature distribution of Eq. (9) assumes the form of:

$$\theta(x) = e^{\lambda x} = Ce^{\pm mx} = c_1 e^{mx} + c_2 e^{-mx} \tag{12}$$

A case with finite length:

The constant multipliers of Eq. (12) can be determined by considering appropriate boundary conditions at the base and tip of the fin as:

$$\begin{aligned} &\theta(x=0) = \theta_b = T_b - T_\infty = c_1 e^{m*0} + c_2 e^{-m*0} = c_1 + c_2 \\ &\theta(x=l) = \theta_l = T_l - T_\infty = c_1 e^{ml} + c_2 e^{-ml} \end{aligned} \tag{13}$$

Solving the simultaneous equations for the constant multipliers, one gets:

$$\begin{aligned} &c_1 = \theta_b \frac{e^{-ml}}{e^{-ml} + e^{ml}} \\ &c_2 = \theta_b \frac{e^{ml}}{e^{-ml} + e^{ml}} \end{aligned} \tag{14}$$

Inserting Eq. 14 into Eq. (12),

$$\theta(x) = \theta_b \frac{\dfrac{e^{m(l-x)} + e^{-m(l-x)}}{2}}{\dfrac{e^{ml} + e^{-ml}}{2}} = \theta_b \frac{\cosh[m(l-x)]}{\cosh(ml)} \tag{15}$$

The case with infinite fin length:

$$\begin{aligned} &\theta(x=0) = \theta_b = T_b - T_\infty = c_1 e^{m*0} + c_2 e^{-m*0} = c_1 + c_2 \\ &\theta(x=\infty) = \theta_\infty = T_\infty - T_\infty = 0 = c_1 e^{m\infty} + c_2 e^{-m\infty} = c_1 e^{m\infty} + 0 \Rightarrow c_1 = 0 \end{aligned} \tag{16a}$$

Then,

$$\begin{aligned} &c_2 = \theta_b \\ &and \\ &\theta(x \to \infty) = \theta_b e^{-mx} \end{aligned} \tag{16b}$$

Optimum fin length determination:

The rate of heat flow of a fin with infinite length is:

$$\dot{Q}_{x\to\infty} = -kA_c \left.\frac{d\theta(x)}{dx}\right|_{x=0} = \sqrt{khpA_c}\,\theta_b \tag{17}$$

The optimum length of any thin fin can be determined by taking the ration between the rate of heat transfer of any considered boundary condition and Eq. (17).

Considered Fin Boundary conditions

In order to get a simplified expression for Eq. (9) as applied to fins, we need to determine three boundary conditions dictated by the flow field physics, namely at base and tip of the fin.

The **first boundary** condition will be the temperature at that base cross-sectional area of the fin.

$$\theta(x)\big|_{x=0} = \theta_b = T_b - T_\infty \tag{18}$$

The **second boundary** condition we consider is the **adiabatic** conditions at the fin tip. This condition assumes that the convective heat transfer at the fin's tip is negligible due to the fact that the tip area extremely small and hence it can be considered as if there is no rate of heat transfer from the fin tip:

$$temperature\ \ gradient = \left.\frac{d\theta(x)}{dx}\right|_{x=l} = 0 \tag{19}$$

Applying Eqs. (18) and (19) to the heat distribution given by eq. (15), the **adiabatic rate of heat conduction** at the base of the fin will assume the form of:

$$q_{adiabatic\_fin\_tip} = \dot{Q}_{adiabatic\_fin\_tip} = -kA_c \left.\frac{d\theta(x)}{dx}\right|_{x=0} = \sqrt{khpA_c}\,\theta_b \tanh(ml) \tag{20}$$

Where θ_b is provided by Eq. (18).

The **third boundary** we are considering is the **convective heat transfer** from the fin's tip.

In this scenario, the rate of heat conduction from the fin's body has to be equal to the rate of heat convected the fin tip, hence:

$$-kA_c \left.\frac{d[T(x)-T_\infty]}{dx}\right|_{x=l} = hA_c[T(x)-T_\infty]_{x=l} \tag{21a}$$

From Eq. 18a and using Eq. 11, one gets:

$$km\left[c_2e^{-ml} - c_1e^{ml}\right] = h\left[c_1e^{ml} + c_2e^{-ml}\right] \tag{21b}$$

Then, to determine the coefficients, one forms a simultaneous equation using Eqs. (13) and (21b) to get:

$$\begin{cases} \theta_b = c_1 + c_2 \\ km\left[c_1e^{ml} - c_2e^{ml}\right] = h\left[c_1e^{ml} + c_2e^{-ml}\right] \end{cases} \tag{22a}$$

Define: $\alpha = \dfrac{km}{h}$ (22b)

Rearranging for the coefficients in Eq. (22a):

$$\begin{cases} c_1 + c_2 = \theta_b \\ c_1\left(1+\dfrac{1}{\alpha}\right)e^{ml} + c_2\left(\dfrac{1}{\alpha}-1\right)e^{-ml} = 0 \end{cases} \tag{22c}$$

Solving Eq. (22c) for the constants, one gets:

$$c_1 = \theta_b \frac{\left(1-\dfrac{1}{\alpha}\right)e^{-ml}}{\left(\dfrac{1}{\alpha}+1\right)e^{ml} - \left(\dfrac{1}{\alpha}-1\right)e^{-ml}} \tag{23a}$$

$$c_2 = \theta_b \frac{\left(\dfrac{1}{\alpha}+1\right)e^{ml}}{\left(\dfrac{1}{\alpha}+1\right)e^{ml} - \left(\dfrac{1}{\alpha}-1\right)e^{-ml}} \tag{23b}$$

Then, plugging Eq. (23) into Eq. (12) and rearranging it, one obtains:

$$\theta(x) = \theta_b \left\{ \frac{\left(e^{m(l-x)} + e^{-m(l-x)}\right) + \dfrac{1}{\alpha}\left(e^{m(l-x)} - e^{-m(l-x)}\right)}{\left(e^{ml} + e^{-ml}\right) + \dfrac{1}{\alpha}\left(e^{ml} - e^{-ml}\right)} \right\} \tag{24a}$$

Or in hyperbolic function form:

a) Temperature distribution:

$$\frac{\theta(x)}{\theta_b} = \frac{\cosh[m(l-x)] + \dfrac{1}{\alpha}\sinh[m(l-x)]}{\cosh(ml) + \dfrac{1}{\alpha}\sinh(ml)} \tag{24b}$$

b) Using (24c), the rate of heat transfer for a fin with convective tip is:

$$\dot{Q}_{convective\_tip} = -kA_c \left.\frac{d[T(x)-T_\infty]}{dx}\right|_{x=0} = \sqrt{khpA_{c\_fin}}\,\theta_b \frac{\dfrac{1}{\alpha}\cosh(ml) + \sinh(ml)}{\cosh(ml) + \dfrac{1}{\alpha}\sinh(ml)} \tag{24c}$$

In order to compensate for the negligible convection from the fin tip, the length for very thin fin thickness will be corrected as:

$$l_c = l + \frac{A_c}{p_c} = l + \frac{wt}{2(w+t)} = l + \left.\frac{wt}{2w\left(1+\frac{t}{w}\right)}\right|_{\frac{t}{w}\to very\ small} = l + \left.\frac{t}{2\left(1+\frac{t}{w}\right)}\right|_{\frac{t}{w}\to very\ small} = l + \frac{t}{2} \quad (25a)$$

$$\frac{p_c}{A_c} = \frac{2(w+t)}{wt} = \frac{2w\left(1+\frac{t}{w}\right)}{wt} \cong \frac{2}{t} \quad (25b)$$

$$m_c = \sqrt{\frac{h}{k}\frac{p}{A_{c\_fin}}} = \sqrt{\frac{2}{t}\frac{h}{k}} \quad (25c)$$

Determination of an optimum fin length for adiabatic boundary condition:

The proper length of a fin for an adiabatic fin tip is determined by comparing the adiabatic rate of heat transfer, Eq. (20), with the rate of heat transfer for an infinite length, Eq. (17). Hence:

$$\frac{\dot{Q}_{adiabatic}}{\dot{Q}_{l\to\infty}} = \frac{\sqrt{khpA_c} * \theta_b * \tanh(ml)}{\sqrt{khpA_c} * \theta_b} = \tanh(ml) \quad (26a)$$

Therefore, the optimum length of a fin with an adiabatic fin tip condition reaches when the hyperbolic function attains it maximum value, which is equal to 0.999 or 1.00 and it nears to this value when ml is equal to 5. Then, one can determine the fin optimum length as:

$$l_{opt} = \frac{5}{m} \quad (26b)$$

With the reduction of ml by 50%, one will just compromise a 1% lose of heat transfer and the optimum length can be further approximated by:

$$l_{opt\_\mathrm{mod}} = \frac{2.5}{m} \quad (26c)$$

Fin Performance:

With corrected dimensions as provided by Eqs. (25) and (26), the performance of a fin is evaluated by determining its efficiency and effectiveness as follows:

Fin efficiency:

a) Using Eq. (20) for adiabatic flow condition:

$$\eta_{adiabatic} = \frac{\dot{Q}_{fin}}{\dot{Q}_{\max}} = \frac{\dot{Q}_{fin}}{hA_{fin\_surface\_area}\left(T_b - T_\infty\right)} = \frac{\sqrt{khpA_{fin\_surface\_area}}\,\theta_b \tanh\left(m_c l_c\right)}{hA_{fin\_surface\_area}\theta_b} = \frac{\tanh\left(m_c l_c\right)}{m_c l_c} \qquad (27a)$$

b) Using Eq. (24c) for convective low condition:

$$\eta_{convective} = \frac{\dot{Q}_{fin}}{\dot{Q}_{\max}} = \frac{\dot{Q}_{fin}}{hA_{fin\_surface\_area}\left(T_b - T_\infty\right)} = \frac{\sqrt{khpA_{fin\_surface}}\,\theta_b \dfrac{\frac{1}{\alpha}\cosh\left(m_c l_c\right) + \sinh\left(m_c l_c\right)}{\cosh\left(m_c l_c\right) + \frac{1}{\alpha}\sinh\left(m_c l_c\right)}}{hA_{fin\_surface}\theta_b} = \frac{\dfrac{\frac{1}{\alpha}\cosh\left(m_c l_c\right) + \sinh\left(m_c l_c\right)}{\cosh\left(m_c l_c\right) + \frac{1}{\alpha}\sinh\left(m_c l_c\right)}}{m_c l_c} \qquad (27b)$$

Fin Effectiveness:

The main purpose of fins is to accelerate heat removal from the body they are attached to. If the fins are not properly dimensioned, numbered and positioned, they may contribute more resistance to heat conduction and may hamper to the heat removal effort from the heat pump. Therefore, the effectiveness, $\left(\varepsilon_{fin}\right)$, of fins for the better removal of heat or cooling and thereby condensing it water is decided by finding the ratio of convective heat removed by the fin surface area to the convective heat removed from the heat pump surface without the fin attached to it. It is generally accepted to incorporate fins if the efficiency ratio is greater or equal to 2. Hence:

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin\_surf}}{\dot{Q}_{no\_fin}} = \frac{\dot{Q}_{fin\_surf}}{hA_{fin\_base}\left(T_b - T_\infty\right)} = \frac{\dot{Q}_{fin\_sur}}{hA_{fin\_base}\theta_b} = \frac{\eta_{fin}\dot{Q}_{fin\_\max}}{hA_{fin\_base}\theta_b} = \frac{\eta_{fin} hA_{fin\_surf}\theta_b}{hA_{fin\_base}\theta_b} = \eta_{fin}\frac{A_{fin\_surf}}{A_{fin\_base}} \qquad (28a)$$

$$\varepsilon_{fin\_tot} = \frac{\dot{Q}_{tot\_fin\_surf}}{\dot{Q}_{no\_fin}} = \frac{\dot{Q}_{tot\_fin\_surf}}{hA_{fin\_base}\left(T_b - T_\infty\right)} = \frac{h\left[A_{no\_fin} + \eta_{fin}A_{fin\_surf}\right]\theta_b}{hA_{fin\_base}\theta_b} = \frac{A_{no\_fin} + \eta_{fin}A_{fin\_surf}}{A_{fin\_base}} \qquad (28b)$$

Where the various areas are provided by Eq. (1). $\dot{Q}_{fin\_surf}$ is the fin's heat transfer rate associated with one of the boundary condition provided either by Eq. (20) or (24c) and $\dot{Q}_{no\_fin}$ is the rate of associated with the fin's base area. In our case, the fin's base and cross-sectional areas at any location are identical since the fin is a unform rectangular fin.

a) Using Eq. (20), the fin's effectiveness with adiabatic fin tip with corrected dimensions can be shown to be:

$$\varepsilon_{fin\_adiabatic} = \frac{\sqrt{khpA_{c\_fin\_base}}\,\theta_b \tanh\left(m_c l_c\right)}{hA_{c\_fin\_base}\theta_b} = \sqrt{\left(\frac{k}{h}\right)\left(\frac{p}{A_{c\_fin\_base}}\right)_{corrected}}\tanh\left(m_c l_c\right) \qquad (29a)$$

b) Using Eq. (24c), the fin effectiveness with convective fin tip and corrected fin length will be:

$$\varepsilon_{fin\_convective} = \frac{\sqrt{khpA_{c\_fin\_base}}\theta_b \dfrac{\frac{1}{\alpha}\cosh(m_c l_c)+\sinh(m_c l_c)}{\cosh(m_c l_c)+\frac{1}{\alpha}\sinh(m_c l_c)}}{hA_{c\_fin\_base}\theta_b} = \sqrt{\left(\frac{k}{h}\right)\left(\frac{p}{A_{c\_fin\_base}}\right)_{corrected}} \frac{\frac{1}{\alpha}\cosh(m_c l_c)+\sinh(m_c l_c)}{\cosh(m_c l_c)+\frac{1}{\alpha}\sinh(m_c l_c)} \qquad (29b)$$

Determination of condensed liquid water on the cold side of the element:

The rate at which the liquid mass on the cold side condenses is given by:

$$d\dot{m}_l = \rho h_D\left(\omega - \omega_{SC}\right)dA \qquad (30)$$

If a unit depth of the thermo couple is considered, the area, dA, will be expressed as $1*dx = dx$:

$$d\dot{m}_l = \rho h_D\left(\omega - \omega_{SC}\right)dx \qquad (31)$$

The amount of air condensed on the wet side is equal to the amount of vapor extracted from the incoming/ambient humid air. This is expressed as:

$$d\dot{m}_l = -\dot{m}_{air\_humid}\, d\omega \qquad (32)$$

Equating Eqs. (31) and (32) are rearranged to yield:

$$\frac{d\omega}{dx} = -\frac{\rho h_D}{\dot{m}_{air\_humid}}\left[\omega - \omega_{SC}\right] \qquad (33)$$

For this analysis, we are assuming that the flow is laminar with a Reynolds number equal to:

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu} = 5*10^5 \qquad (34)$$

The analogy or equivalence between the mass and heat transfers are expressed in terms of the Nusselt and Sherwood numbers. The Nusselt, Nu_x, and Sherwood, Sh_x, numbers describe the heat and mass transfer respectively and are given as follows:

$$Nu_x = \frac{hx}{k} = 0.332*\mathrm{Re}_x^{\frac{1}{2}}*\mathrm{Pr}^{\frac{1}{3}} \qquad (35)$$

and

$$Sh_x = \frac{h_D x}{D_{va}} = 0.332 * \mathrm{Re}_x^{\frac{1}{2}} * Sc^{\frac{1}{3}} \tag{36}$$

Where the Prandtl and Schmidt numbers are defined as:

$$Sc_{T=25^0 C} = \frac{\mu}{\rho D_{va}} \approx 0.68 \tag{37}$$

Since the Prandtl and Schmidt number are very close to each other, then the Nusselt and Sherwood numbers are approximately the same. Hence, the analogy that the heat and mass transfers are equivalent is justified.

From Eq. (36), the expression for the coefficient for the mass transfer is:

$$h_D = 0.332 * \mathrm{Re}_x^{\frac{1}{2}} * Sc^{\frac{1}{3}} * \frac{D_{va}}{x} \tag{38}$$

Inserting Eq. (38) into Eq. (33) and integrating the resulting equation yields:

$$\ln\left[\frac{\omega - \omega_{SC}}{\omega_i - \omega_{SC}}\right] = \frac{-0.664 * \rho_{air\_humid} * D_{va} Sc^{\frac{1}{3}}}{\dot{m}_{air\_humid}} \sqrt{\frac{u_\infty x}{\nu}} \tag{39}$$

For the free stream velocity in Eq. (39), we can compute the air flow speed from the Reynolds number as:

$$u_\infty = \frac{\nu * \mathrm{Re}_L}{L} [m / \sec] \tag{40}$$

and the mass flow is:

$$\dot{m}_{air} = \rho_{air\_humid} u_\infty A = \rho_{air\_humid} u_\infty w L [kg / \sec] \tag{41}$$

Then Eq. (39) becomes:

$$\frac{\omega - \omega_{SC}}{\omega_i - \omega_{SC}} = e^{-a\sqrt{x}} \tag{42}$$

The humidity ratio at the exit (x=L) can be expressed as:

$$\frac{\omega_L - \omega_{SC}}{\omega_i - \omega_{SC}} = e^{-a\sqrt{L}} \quad (43)$$

From equation (43), we can derive the relationship between the ambient and exit humidity quantities as:

$$\omega_i - \omega_L = \left(\omega_i - \omega_{SC}\right)\left[1 - e^{-a\sqrt{L}}\right] \quad (44)$$

After integrating Eq. (39) and letting

$$a = \frac{0.664 * \rho * D_{va} Sc^{\frac{1}{3}}}{\dot{m}_{air\_humid}} \sqrt{\frac{u_\infty}{\nu}} \quad (45a)$$

To take into account the flow analysis with fins, the parameter a, must be modified as:

$$a_{fin} = a + a\left(\eta_{fin} * \frac{A_{fin}}{A_{b=c=no\_fin}}\right) \quad (45b)$$

Depending on the boundary condition considered for the analysis, η_{fin} is calculated by either using Eq. (27a) or (27b).

From Eq. (32), the condensed mass of liquid is equivalent to the mass of water vapor lost by the air due to condensation. Then, integrating Eq. (32) the over length, L, of the Thermo-Electric cell and using Eq. (44), the liquid mass is:

$$\dot{m}_l = -\dot{m}_{air\_humid}\left(\omega_L - \omega_i\right) = \dot{m}_{air\_humid}\left(\omega_i - \omega_L\right) \quad (46)$$

Determination of the temperatures on cold and hot sides of the Thermo-Electric element:

The sensible heat transferred from the cold side is equal to the heat conducted across the channel to the hot side of the thermo-Element.

$$d\dot{q} = Ah(T - T_{SC}) = \dot{m}_{air\_humid} c_p dT \quad (47)$$

Using the expression for h from the Nusselt number:

$$h = 0.332 * \frac{k * \text{Re}_x^{\frac{1}{2}} * \text{Pr}^{\frac{1}{3}}}{x} \quad (48)$$

$$\frac{dT}{T - T_{SC}} = 0.332 * \frac{k * \text{Pr}^{\frac{1}{3}}}{x} \sqrt{\frac{u_\infty x}{\nu}} \quad (49)$$

Then the temperature distribution, Eq. (49), on the cold face of the couple becomes:

$$\frac{T_{x=L} - T_{SC}}{T_i - T_{SC}} = e^{-b\sqrt{L}} \Rightarrow \quad (50a)$$

or

$$(T_{x=L} - T_i)_{cold\_face} = [T_i - T_{SC}][e^{-b\sqrt{L}} - 1] \quad (50b)$$

In the same manner, we can show that the temperature distribution on the hot face is:

$$\frac{T_{x=L} - T_{SH}}{T_i - T_{SH}} = e^{b\sqrt{L}} \Rightarrow \quad (51a)$$

or

$$(T_{x=L} - T_i)_{ho\_face} = [T_{SH} - T_i][1 - e^{b\sqrt{L}}] \quad (51b)$$

After integrating Eq. (49) and letting:

$$b = \frac{-0.664 * k}{\dot{m}_{air} c_p} * \text{Pr}^{\frac{1}{3}} * \sqrt{\frac{u_\infty}{\nu}} \quad (52a)$$

In the same manner done for condensation computation with fins, the parameter b, in the heat and temperature computations must be modified as:

$$b_{fin} = b + b\left(\eta_{fin} * \frac{A_{fin}}{A_{b=c=no\_fin}}\right) \quad (52b)$$

It is also known that the amount heat absorbed by the cold face and respectively the heat extracted from the hot face of the thermoelectric unit are:

$$\dot{Q}_L = \dot{m}_a c_p \left(T_{x=L} - T_i\right)_{cold\_face} + \dot{m}_a \left(\omega_i - \omega_L\right) h_{fg} \tag{53}$$

$$\dot{Q}_H = \dot{m}_a c_p \left(T_{x=L} - T_i\right)_{hot\_face} \tag{54}$$

We insert equation (51) into equations (46) and (53), respectively, to compute the amount of condensed water and absorbed heat on the cold face of the thermoelectric couple. In the same manner, equation (52) is inserted into Equation (54) to compute the rate of heat extracted from the hot face of the thermo couple.

The performance data of the thermoelectric pump is provided by the manufacturer [Ref. 5] by figure 2 as:

$$\dot{Q}_{H\_TEC} = A + B * \left(T_{SH} - T_{SC}\right) \tag{55}$$

$$\dot{Q}_{L\_TEC} = C + D * \left(T_{SH} - T_{SC}\right) \tag{56}$$

where A, B, C and D are constants that are determined from the removed and absorbed heat graphs [Fig. 2].

Numerical Optimization Procedure:

As provided by equations (55) and (56), the salient characteristics of the thermoelectric module [Fig. 1] were computed by linear interpolation from the graphs [fig. 2]. The heat pump is considered to be one meter in height and 0.02 meter deep. To initiate the numerical optimization, the incoming ambient temperature and humidity values are predicted. One must be cognizant that, for the condensation to take place on the cold face, the ambient temperature and humidity need to be higher than the temperature and humidity on the cold surface. Since the temperatures on the hot and cold sides of the thermo couple are unknown apriori, iterative method is adopted as presented in reference 1.

Numerical optimization results:

Using the converged values, the liquid mass rate, Eq. (46), is computed to be 29.387 kg/hour. Along with the condensed mass of condensed liquid condensed, the converged values of humidity and temperatures are shown on Tables 2 and 3.

The analysis for the TEC with fins indicates that once the condensation is completed and the heat is removed from the hot face of the couple, the temperature on the hot side subsides from 47.2401 degrees to 43.4707 degrees Celsius. As reported in reference 1, the temperature on hot side without fins was 85.6599 degrees Celsius and reached to 48.9629 degrees Celsius after the condensation is completed. The numerical experimentation shows that inclusion of fins speeds up the de-humidification process thereby increasing the amount of water collected from 2.53 kg/hr.to 29.387kg/hr. [Tables 2 & 4]. In Tables 4 and 5, the temperatures and rate of heat both on the hot and cold sides produced are presented. As the result of the fins' inclusion, one can observe from these tables that the temperature quickly draws down while the amount of heat removed or condensed increases likewise. Consequently, the process of de-humidification ensues fast and the amount of water accumulation increases dramatically.

Figs. 1-3 describe the feature, characteristics and arrangement of the Thermo-Electric Couple (TEC) for the dehumidification process. Figs. 4-6 describe the fin's geometric characteristics and fin array arrangement. The number of fins used in this analysis is 40 and their dimensions are provided in table 1.

Conclusion:

A thermoelectric device was used to serve dual purposes, namely to cool the incoming air and produce water. In doing so, the air is dehumidified resulting in comfort that can be useful in places where humidity extremely high. To increase the efficiency and the amount of potable water collected by the thermocouple, fins were incorporated. The analysis with the incorporation of fins to the Thermo-Electric couple shows that the temperature on both sides of the gets lwater accumulation get higher quickly. To get a full scope of the analysis, we recommend that Computational Fluid Dynamics needs to be used so that one can fully analyze the interaction of TEC and fins with the flow field.

TEC height	1.0 [m]
TEC width	0.02 [m]
Fin length	0.05 [m]
Fin width	0.02 [m]
Fin thickness	0.005 [m]
Fin gap	0.05 [m]
Total number of fins	40 (calculated using fin gap and TEC length)

Table 1: TEC and fin dimension specification

$\dot{m}_{liquid}\left[\frac{kg}{hr}\right]$	$\omega_{x=L}\left[\frac{kg_{H_2O}}{kg_{dry\_air}}\right]$	$T_{SH}\left[{}^0C\right]$	$\left[{}^0C\right]T_{SC}$	$T_{x=L}\left[{}^0C\right]$
2.53152	0.056222	48.9629	33.5555	42.7058

Table 2: Optimum values at x=L without fins[Ref. 1].

$\dot{m}_{liquid}\left[\frac{kg}{hr}\right]$	$\omega_{x=L}\left[\frac{kg_{H_2O}}{kg_{dry\_air}}\right]$	$T_{SH}\left[{}^0C\right]$	$\left[{}^0C\right]T_{SC}$	$T_{x=L}\left[{}^0C\right]$
29.387	0.0158525	43.4707	30.8292	34.0976

Table 3: Optimum values at x=L with fins.

X[m]	TSH[Celsius]	TSC[Celsius]	Q_dot_H[kwatts]	Q_dot_L[kwatts]
0.005	85.6599	37.3002	0.0367654	0.00617589
0.1	60.139	38.5276	0.0658227	0.0306745
0.2	55.5834	36.5921	0.0681536	0.0326397
0.3	53.4543	35.642	0.0691974	0.0335198
0.4	52.1523	35.0482	0.0698233	0.0340474
0.5	51.2497	34.6311	0.070252	0.0344089
0.6	50.576	34.317	0.0705693	0.0346764
0.7	50.0481	34.0692	0.0708164	0.0348847
0.8	49.6198	33.8672	0.0710159	0.0350529
0.9	49.2632	33.6982	0.0711813	0.0351924
0.999	48.9629	33.5555	0.0713201	0.0353094

Table 4**:** Temperature and heat distributions along the cold and hot faces of the thermo-couple without fins [Ref. 1].

X[m]	TSH[Celsius]	TSC[Celsius]	Q_dot_H[kwatt]	Q_dot_L[kwatt]
0.005	47.2401	32.7275	0.0722536	0.0360964
0.1	44.0791	31.1634	0.0735122	0.0371576
0.2	43.8123	31.0237	0.0736254	0.037253

0.3	43.6956	30.9606	0.0736728	0.0736728
0.4	43.627	30.9224	0.0736997	0.0373157
0.5	43.5809	30.896	0.0737171	0.0373304
0.6	43.5473	30.8763	0.0737294	0.0373407
0.7	43.5216	30.8609	0.0737385	0.0373484
0.8	43.5012	30.8484	0.0737455	0.0373543
0.9	43.4845	30.838	0.073751	0.0373589
0.999	43.4707	30.8292	0.0737554	0.0373626

Table 5**:** Temperature and heat distributions along the cold and hot faces of the thermo-couple With fins.

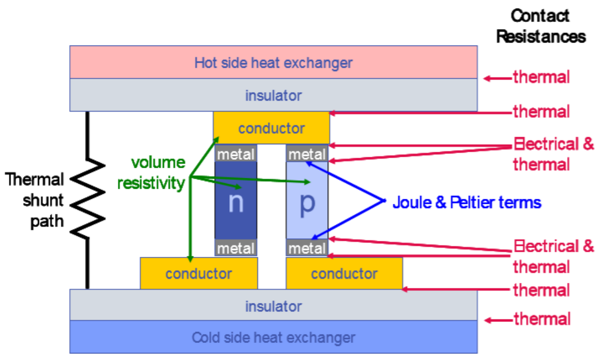

Fig. 1: General Feature of Thermoelectric Module [Ref. 5] (www.tetech.com)

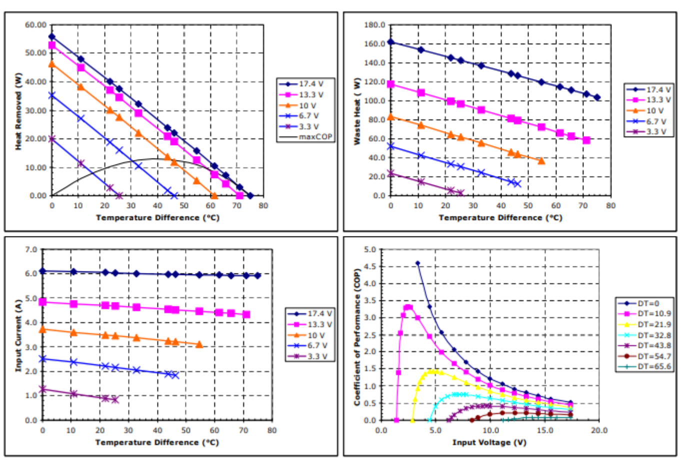

Fig. 2: Peltier-Thermoelectric Module chart [Ref. 5] (www.tetech.com)

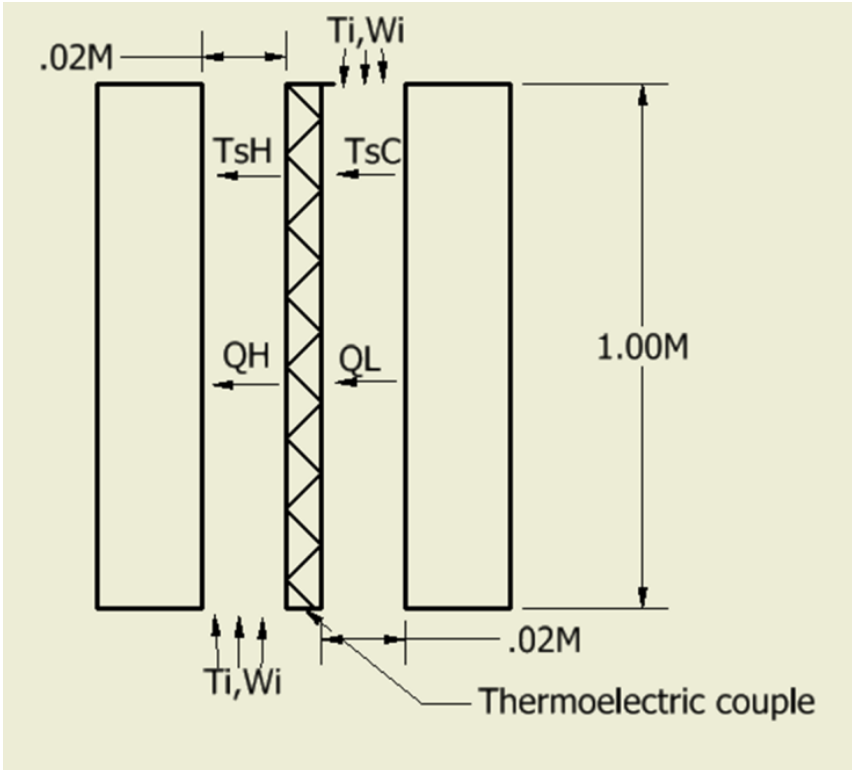

Fig. 3: Counter flow mechanism in Thermo-electric couple.

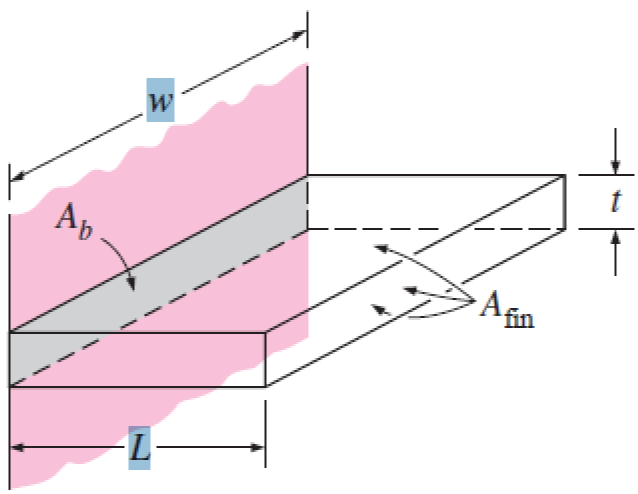

Fig.4: Fin geometry specification [Refs. 3 & 4]

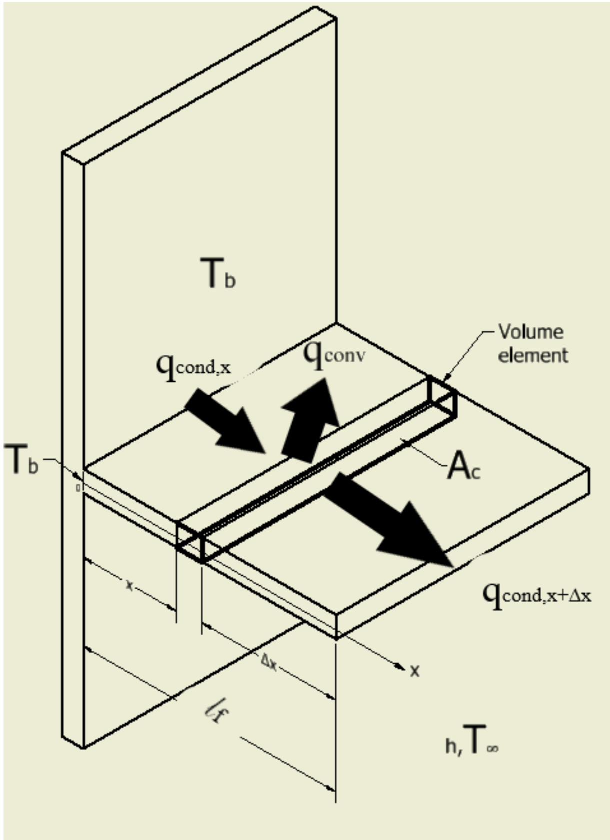

Fig. 5: Fin conductive and convective heat flow mechanism.

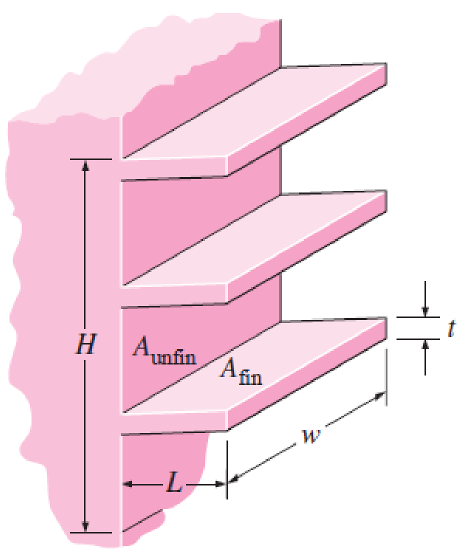

Fig. 6: Fin distribution along a thermo-electric height and fin area definition [Refs. 3 & 4]

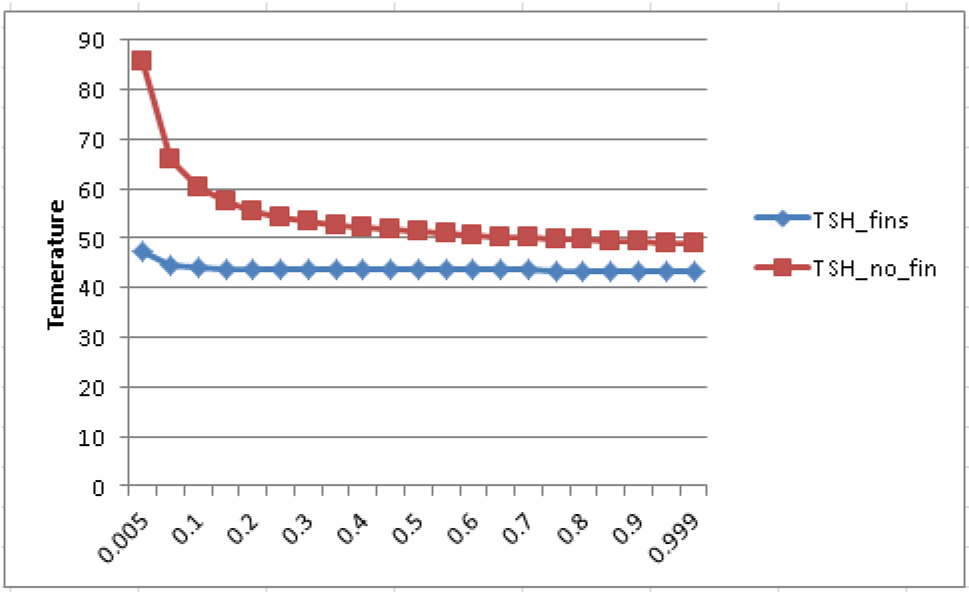

Fig. 7: Comparison of temperature distribution on the hot side of TEC.

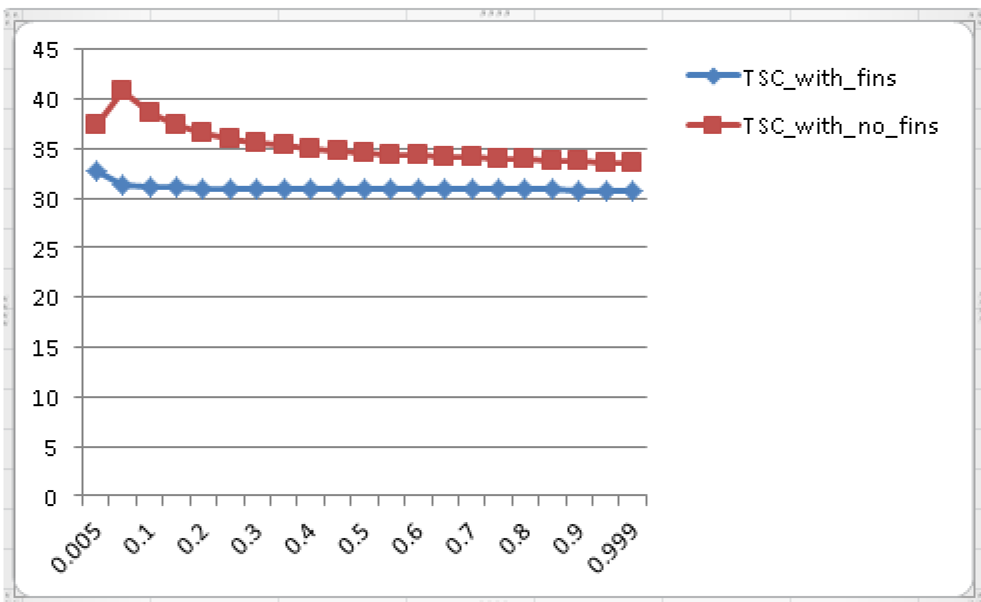

Fig. 8: Comparison of temperature distribution on the cold side of TEC.

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