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EFFECT OF DERIVATIONS ON ANALYTIC FUNCTIONS

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ABSTRACT

To study derivations satisfying certain analytic function of region R of domain D. By Cauchy's Integral and Riemann Equations of an analytic in the region R at any point z=a. Taylor's Series expansion of an analytic function determine.

KEYWORD

Analytic function, Taylor Series expansion, Region R, Cauchy's.

INTRODUCTION

In the complex plane, let boundary behaviour of functions be as $(1-z^2)f^{(n)}(z)$ Where f is an analytic function defined on the open unit disk and $n > 0$ .By taking an example let $(1-z^2)f'(z)$

be bounded on D for Block Space B, set of analytic function f on D.

Also, $(1-z^2)f'(z) \to 0$ as for Block Space B0. $z \to 1$ be the set of analytic function f on D

Let Lebesgue area denoted by dA measure on the complex plane.

By the Bergman Space ^{p}

$$\int_D |f|^p \, dA < \infty \qquad \text{for } p \in [1, \infty),$$

Where ^p_a is the set of analytic function f on D.

Next, if $f \in L^1$ then $f(z) = \frac{1}{\pi} \int_D \frac{f(w)}{(1-zw)} dA(w)$, for every $z \in D$

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Let us define an analytic function $P(f)$ on D then,

$$P(f)(z) = \frac{1}{\pi} \int_D \frac{f(w)}{(1-zw)} dA(w) \quad for \quad f \in L^1(D, dA)$$

Let us suppose P restricted to $L^2(D, dA)$ then,

Again, $L^2(D, dA)$ is the orthogonal project of $L^2(D, dA)$ onto L^2.

Let us suppose P restricted to $L^p(D, dA)$ then,

$L^p(D, dA)$ is a bounded projection of $L^p(D, dA)$ onto L^p.

By the, Analytic Function following lemma exist:

Lemma 1: Let us suppose f be an analytic function on D then by an analytic function as an area integral of its equivalents are as follow:

$(a)\ f \in B$

$(b) \sup\{(1-z^2)^n f^{(n)}(z) : z \in D\} < \infty, \quad for\ every\ \forall\ n > 0;$

$(c) \sup\{(1-z^2)^n f^{(n)}(z) : z \in D\} < \infty, \quad for\ some\ \forall\ n > 0;$

Lemma 2: Let us suppose $f \in B$ such that, f has a zero of order at least 2n at 0 then

$$f(z) = \frac{1}{n!\pi} \int \frac{(1-w^2)^n f^{(n)}(w)}{(1-zw)^2 (w)^n} dA(w), \quad \forall\ n > 0\ z \in D$$

Lemma 3: Let us suppose $h \in u$ and $t > 0$ then $\exists$ $b > 0\ st$

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$h(b_z(\lambda)) - h(b_z(\lambda')) < \in \quad \forall\ z \in D$ $L^{\infty}(D, dA)$ generated

For the need of

$\lambda, \lambda' \in D$ and $d(\lambda, \lambda') < \delta$ Where u is the closed subalgebra

by the Complex Conjugate $H^{\infty}(D)$.

Lemma 4: Let us suppose f be an analytic function on D. then its equivalents are:

$(a)\ f \in P(u)$

$(b)(1-z^2)^n f^{(n)}(z) \in u,$ for every $\forall\ n > 0;$

$(c)(1-z^2)^n f^{(n)}(z) \in u,$, $for\ some\ \forall\ n > 0;$

Proposition (1) $p(u)$ is properly contained in the Block Shape B.

Proposition (2) Let u∈ C(D then u is bounded on D .

Lemma 5: Let u be a bounded, continuous, complex-valued function on D in such a way that

$$\sup \left\{ (1-z^2 \limsup_{w \to z} \frac{u(w)-u(z)}{z} w - : z \in D \right\} < \infty$$

for u∈ C(D

Cauchy's Integral Theorem:

Let $f(z)$ is an analytic function on D by supposing D as a bounded domain with piecewise smooth boundary, then

$$\int_D f(z) dz = 0$$

Cauchy's Integral Formula:

Let $f(z)$ is an analytic function on D over C at a is then,

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$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$$

Cauchy's Riemann Equations:

Let $f(z)$ is an analytic function on D over C at a over a complex plane satisfies Cauchy's Riemann Equations throughout D.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

To check weather f has a complex derivative and to compute that derivative. Cauchy's Riemann Equations uses the partial derivatives of u and v.

Taylor series :

The Taylor Series of a function is an infinite sum of terms also known as Maclaurin Series if zero is the point of derivative that are expressed in terms of the function's derivatives at a single point.

A real or complex-valued function $f(x)$ of Taylor Series for complex or real number a is the power series of n! is

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \text{............}$$

Then it can be written as $\sum_{n=0} \frac{f^{(n)}(a)}{n!}(x-a)^n$

where derivative of f is $f^{(n)}(a)$.

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METHODOLOGY

If a function is having complex derivative $f'(z)$ then a function

$f(z)$ is analytic. Real derivative of a Real function is too much similar as Complex derivative of Complex function.

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$\therefore \quad \exists \quad f$ *is analytic and differentiable at* z_0

By Cauchy's Integral Formula

Let $f(z)$ is an analytic in region R then its derivation at any point z = a is also analytic in R

$$F'(a) = \frac{1}{2\pi i} \int \frac{f(z)}{(z-a)^2} dz \qquad (1)$$

By Analytic functions and the necessary Cauchy's Riemann.

A function of z defined a single valued function of z

i.e. $w = f(z)$ is same domain D, then the function is

differentiable at $z = z_0$

if $\lim_{z \to z_0} f(z) - f(z_0)$ (2)

By using equation (1) and (2) in Taylor Series expansion

of an analytic function.

$$\Rightarrow f(z) = \frac{1}{2\pi i} \int_c \frac{f(z')}{z' - z} dz'$$

$$\Rightarrow \frac{1}{2\pi i} \int_c \frac{f(z')}{(z' - z_0) - (z - z_0)} dz'$$

Thus,

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f is analytic in R

$$\Rightarrow \frac{1}{2\pi i}\int_c \frac{f(z')}{(z'-z)} dz' \quad -(z-z_0) \quad 0 \quad 1 \quad (z'-z_0)$$

where $(z'-z_0) > (z-z_0)$

By an analytic function Cauchy's Riemann, $f(z)$ is a complex

By the theorem of complex line integral if analytic function then,

$$\int_c f'(z)dz = f(z_1) - f(z_0)$$

The Cauchy-Riemann equations

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)$$

Taylor's Series expansion $f(z)$ about z_0

Derivative of all order exist if $f(z)$ is analytic function.

$$\text{Thus} \quad \lim_{n\to\infty} \frac{h^n}{n!} f^n(x) = 0$$

A Taylor-series expansion is available for functions which are analytic within a restricted domain.

CONCLUTION

In analytic function $f(z)$ of bounded domain D over a complex plane. $\frac{du}{dx}\cdot\frac{du}{dy} = \frac{dv}{dy}\cdot\frac{dv}{dx}$ over C at a point a i.e ($z-a)dz$ then real and complex valued function $f(x)$ is a power series of (n!). Series expansion $f(z)$ about z_0 of analytic series of expansion by Taylor-series are Shower.

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