International Research Journal of Mathematics, Engineering and IT

ISSN: (2349-0322)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 7.132 Volume 8, Issue 9, September 2021
Website- www.aarf.asia, **Email** : editoraarf@gmail.com

DERIVATION IN NON-ASSOCIATE ALGEBRAIC

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ABSTRACT

In this paper we will explain about the basic of non-associative algebra using triality group and some were matric and graph representation.A Non-associative algebra structure can be studied with other associative algebraic as K-endomorphisms of A just like K-Vector Space. Whereas as some of the researchers we have also consider non-associative algebraic as a multiplicative operation of a commutative Ring R.

KEY WORDS

Non-associative, Triality, multiplicative operation, associator, vector space.

INTRODUCTION

Non-associative algebra: A Non-associative algebra also known as distributive algebra.Non-associative algebra is a vector space over a field because its vector space provides a bilinear product. An algebra known as an algebraic structure which consists of a set with an additive, multiplicative and scalar operations of the elements of a field of vector space. Thus, an algebraic structure A, is a non-associative algebra with binary multiplicative operation over a field K. An algebraic structure $A \times A \to A$ may or may not be associative.

R-module in mathematics is one of the fundamental algebraic structures. The notation of vector space over a field is a module over a ring R with operation of a scalar multiplication known as R-binary multiplication operation. Some of the researcher also stated that Z-algebra as a non-ass0ciative rings. Where Z-algebra is an algebra which obeys all the properties and laws of ring for example as R-algebra. An algebraic structure ring act with two binary operations. Associative is a part of one of the binary operations in which rearranging of the paratheses is done .

The linear algebra multiple mapping is a function $f : V_1 \times V_2 ...... V_n \to W$ known as K-multiple linear maps. Therefore associator of A is a K-multiple linear map with the operation as $[\square,\square]: A\times A\to A$ and with three operations are as $[\square,\square,\square]: A\times A\times A\to A$. Thus, the associator on A is given by $[x,y,z]=(xy)z-x(yz)$ where x,y,z is an arbitrary element of the algebra.

Derivative algebra is equipped with number of derivations as differential rings, differential fields which satisfied Leibniz's product rules. A K-derivation is also a K-linear map for an algebraic A over a ring or a field is such that $D: A\to A$ which satisfies Leibniz's product rules.

$D[xy]=xD(y)+D(x)y$

Triality Group:Let A be an algebra with a bi-linear product over a field F denoted by x y for all x and y an element of A. Let us suppose **a first triple** $h=(h_1,h_2,h_3)\in(E_{pi}A)^3$

Equipped with $$h_j(x+y)=h_jx+h_jy$$ $$and \quad h_j(\alpha x)=\alpha h_j(x)$$

$E_{pi}A$ represents the set of epimorphisms of A, for $\alpha\in F$

By the Global Triality relation we have,

$h_j(x,y)=(h_{j+1}x)(h_{j+2}y)$……………………………………(1)

And $h_{j+3}=h_j$ ………………………………………………………

(2)

Where j=1,2,3 is a module3 , $\forall x,y\in A$

Similarly **for second triple**

$h'=(h_1',h_2',h_3')\in(E_{pi}A)^3$

Equipped with $hh'=(h_1h_1',h_2h_2',h_3h_3')$

…………………………………(3)

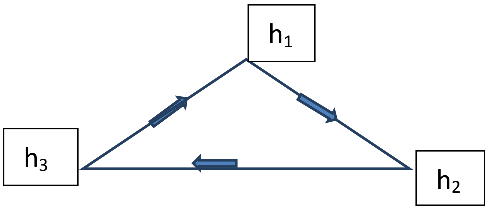
$\therefore$ $\text{Trig}(A)=\left\{h=(h_1,h_2,h_3)\in(E_{pi}A)^3 : h_i(xy)=(h_{j+1}x)(h_{j+2}y)\right\}$…………………………(4)

$Auto(A)=\left\{h\in E_{pi}A: h(xy)=(hx)(hy),\forall x,y\in A\right\}$………………………………. (5)

Let us assume a function $f\in End(Trig(A))$

So as $f: h_1\to h_2\to h_3\to h_1$

……………………………………………………………………………….. (6)



It is a closed walk with cyclic graph and satisfying $f^3 = id$

Trig(A) is invariant for the cyclic group Z_3 by the generation function f.

Similarly let there be three functions $\gamma_1, \gamma_2, \gamma_3$ by *for* μ=1,2,3 $\gamma_\mu \in End\left(Trig\left(A\right)\right)$

i.e ,
$$\left.\begin{array}{l}\gamma_1 : h_1 \to h_1, h_2 \to -h_2, h_3 \to -h_3 \\ \gamma_2 : h_1 \to -h_1, h_2 \to h_2, h_3 \to -h_3 \\ \gamma_3 : h_1 \to -h_1, h_2 \to -h_2, h_3 \to h_3\end{array}\right\}$$

…………………………………………………………..(7)

In Matrix form it is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ unit matrix of order 3

i.e I_3

Which satisfies and is isomorphic to K_4
$$\gamma_\mu \gamma_\nu = \gamma_\nu \gamma_\mu, \gamma_\mu{}^2 = id$$
$$\therefore \quad \gamma_1 \gamma_2 \gamma_3 = id$$

………………………………………(8)

For $\gamma_4 = \gamma_1 \qquad f \gamma_\mu f^{-1} = \gamma_{\mu+1}$

…………………………………………………………….(9)

For $x \to x'$ which fulfil $\left(x'\right)' = x$ & $\left(xy\right)' = y'x'$

…………………………………………………(10)

We define $\overline{R} \in End(A)$ *for* $R \in End(A)$

$\overline{Rx} = \overline{R}\ \overline{x}$

…………………………………………………………………………………………(11)

Take $x \leftrightarrow \overline{y}$

$\overline{h_j}\left(xy\right) = \left(\overline{h}_{j+2}x\right)\left(\overline{h}_{j+2}y\right)$ ………………………………………………………………….(12)

For $\theta \in End\left(Trig\left(A\right)\right)$

$\theta : h_1 \to \overline{h_2}, h_2 \to \overline{h_1}, h_3 \to \overline{h_3}$ ………………………………………(13)

$$f\theta f=\theta,\theta^2=id,$$

Now, $$\left.\begin{array}{l}\theta_{\gamma_1}\theta^{-1}=\gamma_2\\ \theta_{\gamma_2}\theta^{-1}=\gamma_1\\ \theta_{\gamma_3}\theta^{-1}=\gamma_3\end{array}\right)$$

……………………………………………………………………………(14)

METHODOLOGY

If $D(x,y)=d_0(x,y)+d_1(x,y)+d_2(x,y)$

Then it holds a derivative $D(x,y)\,\&\,D(x,yz)+D(z,zx)*D(z,xy)=0$

Therefore, a model global triality is $\overline{h_j}(x*y)=(h_{j+1}x)*(h_{j+2}y)$

CONCLUTION

Thus, a researcher stated that automorphism of Cayley algebra can be constructed. Also, Triality algebra (Trig(A) is an important concept of lie algebra.

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