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SPECTRAL ANALYSIS OF GLOBAL WARMING USING WAVELET TRANSFORMS IN CONTEXT OF INDIA

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Abstract: Global warming led to the climate imbalance and has negative impact on humans, plants, and animals' life. Global warming is measured in terms of average temperature of a place. Wavelet is a new tool to analyze non-stationary and transient signal/data. We have taken average monthly temperature record of India from year 1901 to 2017 as raw data. This data has been decomposed into approximation and detail with help of Haar wavelet transforms at level 9. The approximation provides average behavior of the data while detail to the fluctuations in the data at each level. The average behavior the given data reveals a gradual increase in the temperate throughout the time with fast rate in the recent years.

Keywords: Approximation, detail, global warming, temperature, wavelet.

1. Introduction

Global warming is the phenomenon of a gradual increase in the temperature near the earth surface. This change disturbs the climatic pattern of the earth. There are several causes of global warming, which have a negative effect on humans, plants and animals. These causes may be natural or might be the outcome of human activities. In order to curb the issues, it is very important to understand the negative impacts of global warming. The main reasonbehind the global warming is the greenhouse effect caused by increased levels of carbon dioxide, chlorofluorocarbons (CFCs) and other pollutants [1]. Global warming has affected the coral reefs that can lead to a loss of plant and animal lives. Increase in global temperatures has made the fragility of coral reefs even worse. Global warming has led to a change in climatic conditions. There are droughts at some places and floods at some. This climatic imbalance is the result of global warming. Global warming leads to a change in the patterns of heat and humidity. This has led to the movement of mosquitoes that carry and spread diseases. Due to an increase in floods, tsunamis and other natural calamities, the average death toll usually increases. Also, such events can bring about the spread of diseases that can hamper human life. A global shift in the climate leads to the loss of habitats ofseveral plants and animals. In this case, the animals need to migrate from their natural habitat and many of them even become extinct. This is yet another major impact of global warming on biodiversity. We have studied and analyzed the average monthly temperature data of India from 1901 to 2017.

To represent functions that have discontinuities and sharp peaks, wavelet transforms have advantages over classical Fourier transforms. It is also used for precise decomposition and reconstruction of finite non-periodic and / or non-stationary signals. There are many signals around us such as seismic tremors, human speech, medical pictures, financial data, music and many other types of signals that need to be analyzed. Wavelet analysis is a newand favorable set of tools and techniques for analyzing these signals. The scaled andtranslated copies of a finite length or fast decaying oscillating waveform are represented as wavelets. A wavelet is a mathematical function which is used to divide a given function into different scale components. The concept of wavelets is embedded in many areas, including mathematics, physics, and engineering. A zero average oscillating function which is well localized over a short period of time is called a wavelet. For analyzing and transforming discrete data, wavelets have been widely useful. Wavelets are powerful techniques for analyzing and processing digital signals. The family of wavelets is produced by wavelet functions that are translated (shifted) and dilated (stretched or compressed) versions of the original mother wavelet, so the wavelet function is known as the mother wavelet [2]. Wavelettransform, like Fourier transform, is a magical mathematical tool for many problems in science and engineering. Wavelet transform is a hierarchical set of wavelet functions. All wavelet transforms are related to harmonic analysis because we can assume, wavelet transforms form of time-frequency representation for continuous-time (analogue) signals. Discrete time filter banks are used by all discrete wavelet transforms. In waveletnomenclature, these filter banks are named as wavelet and scaling coefficients. Either finite impulse response (FIR) or infinite impulse response (IIR) filters may be involved in these filter banks [3].

2. Wavelet Transforms and Multiresolution Analysis (MRA)

Wavelets exhibit oscillatory behavior for a short period of time and then die out. A wavelet represents a small wave that can be dilated and translated.

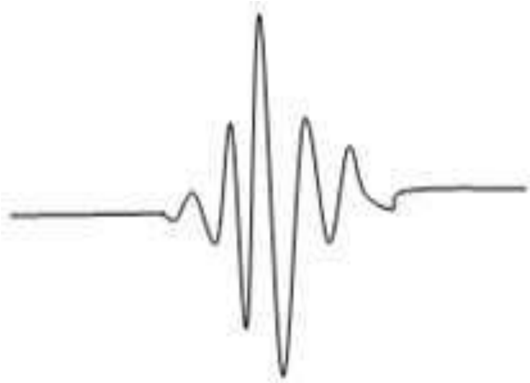

Figure 1: A wavelet

A whole family of wavelets can be developed by translating and scaling the mother wavelet with the help of the mother function [4].

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) = T_b D_a \psi \qquad (1.1)$$

Here b is the translation parameter and a is the dilation or scaling parameter. Continuous wavelet transform is a function of two parameters a and b. It contains a high amount of extra information due to two parameters when analyzing a function and described as: -

$$W_{(a,b)} = \int_t f(t) \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) dt \quad (1.2)$$

The continuous wavelet transform (CWT) coefficient does not affect by the scale and position values only, but the value of coefficient is also affected by the choice of wavelet. An important sampling of the continuous wavelet transform is expressed as: -

$$W_{a,b} = \int f(t)\, \phi_{a,b}(t) dt \quad (1.3)$$

The discrete wavelet transform is expressed as: -

$$W_{j,k} = \int f(t) 2^{\frac{j}{2}} \phi(2^j t - k)\ dt \quad (1.4)$$

It is obtained via a = 2^{-j}, and $\frac{b}{a} = k$ where j and k are integers.

The wavelet transform of a signal captures the localized time frequency information of the signal. Discrete wavelet analysis is performed by multiresolution analysis, which is a radically new recursive method. An MRA is introduced by Mallat and extended by other researchers [5-8] consists of a sequence V_j, $j \in \mathbb{Z}$ of closed subspaces of $L^2(\mathbb{R})$, a space of square integrable functions, satisfying the following properties: -

1) $V_{j+1} \subset V_j : j \in \mathbb{Z}$ (1.5)

2) $\cap_{j \in \mathbb{Z}} V_j = \{0\}, \ \cup_{j \in \mathbb{Z}} = L^2(\mathbb{R})$ (1.6)

3) For every $L^2(\mathbb{R}), f(t) \in V_j \Rightarrow f(2t) \in V_{j+1}, \forall j \in \mathbb{Z}$ (1.7)

4) There exists a function $\phi(t) \in V_0$, such that $\{\phi(t-k) : k \in \mathbb{Z}\}$ is orthonormal basis of V_0.

……..(1.8)

The function $\phi(t)$ is called scaling function of given MRA and property 3 implies a dilation equation as following: -

$$\phi(t) = \sum_{k \in \mathbb{Z}} \alpha_k\ \phi(2t-k) \quad (1.9)$$

where h_k is low pass filter and is defined as follows: -

$$\alpha_k = \left(\frac{1}{\sqrt{2}}\right) \int_{-\infty}^{\infty} \phi(t)\phi(2t-k) dt \quad (1.10)$$

Now we consider W_1 be the orthogonal complement of V_1 in V_0 i.e.

$$V_0 = V_1 \oplus W_1$$

If $\psi \in W_0$ be any function then,

$$\psi(t) = \sum_{k \in Z} \beta_k\ \phi(2t-k) \quad (1.11)$$

Where, $\beta_k = (-1)^{k+1} \alpha_{1-k}$ are high pass filters. We can express a signal in terms of bases of V_0 space. If we combine the bases of V_1 and W_1 space, we can express any signal in V_0 space. Mathematically, we can write,

$$V_0 = V_1 \oplus W_1$$

In general,

$$V_j = V_{j+1} \oplus W_{j+1}$$

But,

$$V_{j+1} = V_{j+2} \oplus W_{j+2}$$

Therefore

$$V_j = W_{j+1} \oplus W_{j+2} \oplus V_{j+2}$$

$$\vdots$$

$$V_j = W_{j+1} \oplus W_{j+2} \oplus W_{j+3} \oplus \ldots\ldots\ldots\ldots W_{j_0} \oplus V_{j_0} \qquad (1.12)$$

By MRA, the orthogonal decomposition of space $L^2(\mathbb{R})$ is as following [9]: -

$$L^2(\mathbb{R}) = \sum_j V_j = \sum_j \left(V_{j_0} \oplus \sum_{p=j+1}^{j_0} W_p\right)$$

3. Research Methodology

We can approximate the data in space of square summable sequences $\ell^2(\mathbb{Z})$ as follows [10]: -

$$f[n] = \frac{1}{\sqrt{M}} \sum_k c[j_0, k] \phi_{j_0,k}[n] + \frac{1}{\sqrt{M}} \sum_{p=j+1}^{j_0} \sum_k w[p,k] \psi_{p,k}[n] \qquad (1.13)$$

where f[n], $\phi_{j,k}[n]$ and $\psi_{j,k}[n]$ are discrete functions defined in $[0, M-1]$, totally M points. We can simply take the inner product to obtain the wavelet coefficients,

$$c[p,k] = \frac{1}{\sqrt{M}} \sum_n f[n]\, \phi_{p,k}[n] \qquad (1.14)$$

$$w[p,k] = \frac{1}{\sqrt{M}} \sum_n f[n]\, \psi_{p,k}[n] \qquad (1.15)$$

where $c[p,k]$ and $w[p,k]$ are called approximation and detail coefficients respectively. From property of scaling and wavelet functions, other coefficients can be derived as follows: -

$$c[p+1,k] = h[n'] * c[j,n] \qquad (1.16)$$

where $n \geq 0$. Similarly, for the detail coefficients, we can write,

$$w[p+1,k] = g[n'] * c[j,n] \qquad (1.17)$$

After N iteration a signal S= $\{S_n : n \in \mathbb{Z}\}$ can be reconstructed as S = $a_N + d_1 + d_2 + d_3 + d_4 + d_5 + \ldots\ldots\ldots\ldots + d_N$. The high- scale, low frequency components of the signals are known as the approximations. The low-scale, high-frequency components are known as the details. There are two complementary filters from which the original signal S passes, namely low pass filter and high pass filter as shown in figure 2.

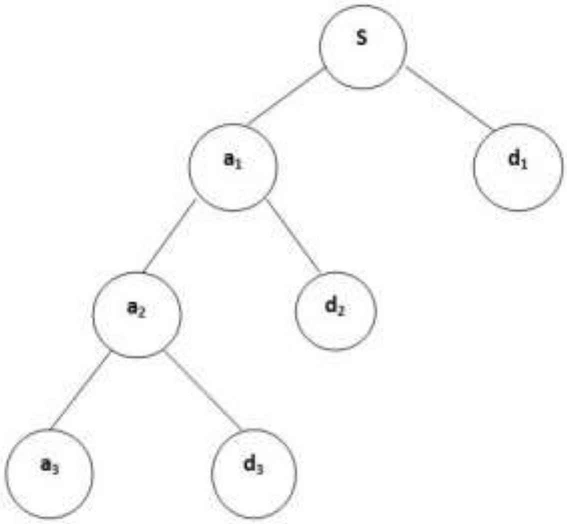

Figure 2: Wavelet pyramidal algorithm for data decomposition

where,

$$\begin{aligned} S &= a_1 + d_1 \\ &= a_2 + d_2 + d_1 \\ &= a_3 + d_3 + d_2 + d_1 \\ &= a_4 + d_4 + d_3 + d_2 + d_1 \qquad (1.18) \end{aligned}$$

and so on [11].

4. Results and Discussion

The average monthly temperature record of India from year 1901 to 2017 has been taken as the raw data (Figure-3).

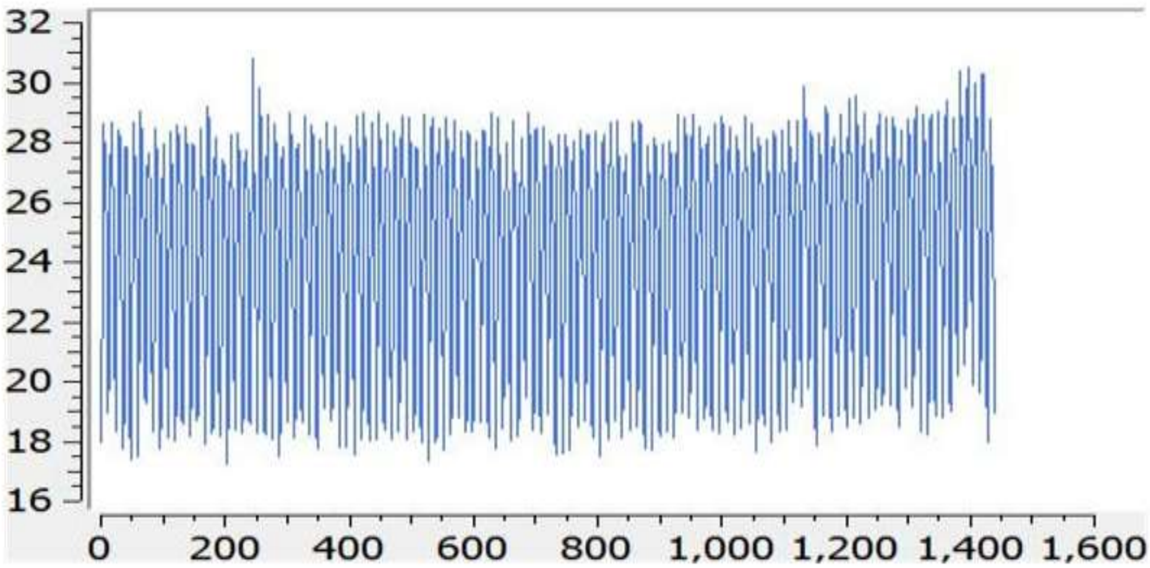

Figure 3: Average monthly temperature of India from 1901-2017

The data is decomposed through wavelet pyramidal decomposition scheme using Haar wavelet transforms at level-9 (Figure-4).

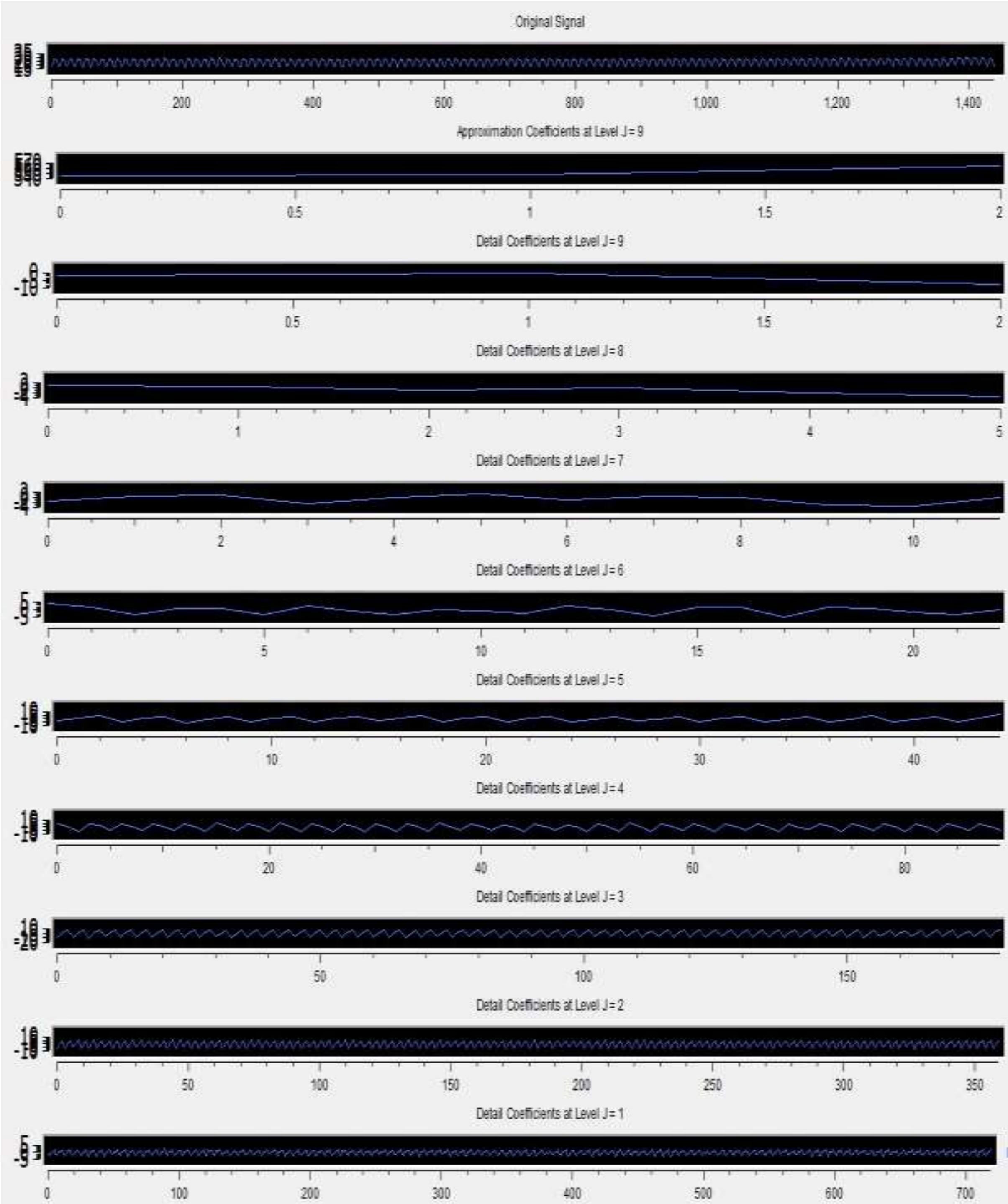

Figure 4: Wavelet decomposition of temperature data (At level 9)

The approximation coefficients at level 9 provide average behavior of the temperature data, while detail coefficients provide differential behavior or fluctuations of given data at different decomposition levels. The approximation of the data represents its average behavior throughout the time (Figure-5).

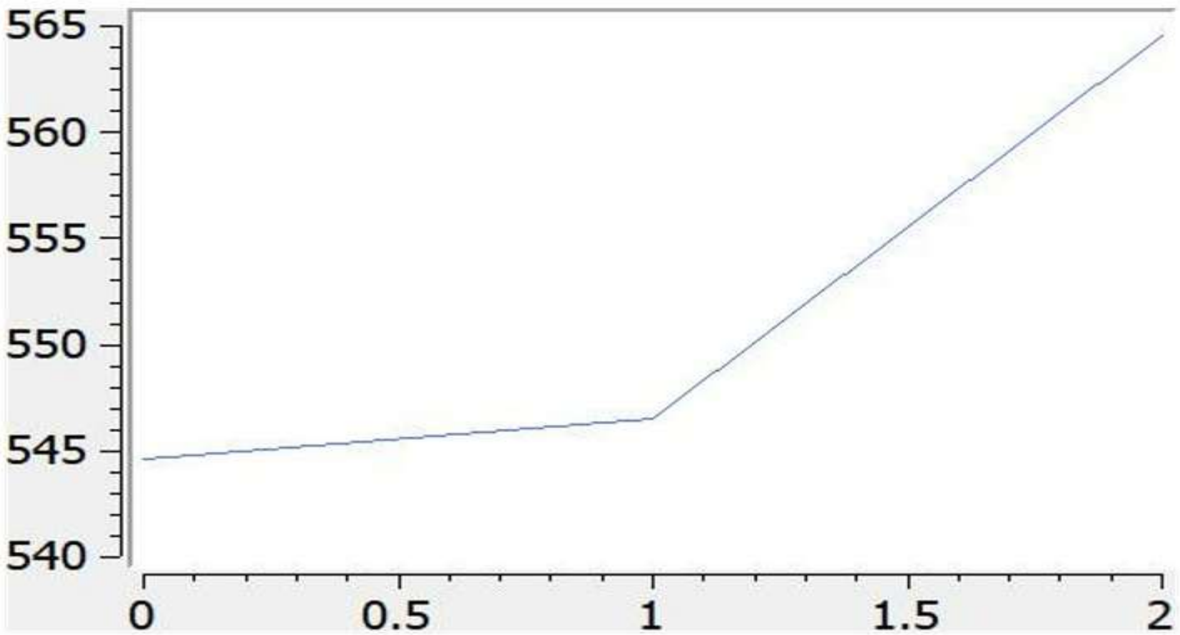

Figure 5: Approximation of temperature

The average behavior of temperature in India shows a is gradual increase throughout the given time interval. Some statistical parameters of the given data are as follows: -

S. No.	Statistical Parameter	Value
1	Skewness	-1.218267522
2	Kurtosis	-0.454495434
3	Standard Deviation	3.516446048

The skewness measures the asymmetry of the probability distribution of a real-valued random variable about its mean. The small and negative value of skewness represents that data are slightly skewed left. Kurtosis parameter is a measure of the tail character of the distribution. The small and negative value of kurtosis indicates that the given data has a light tail than that of normal distribution [12]. The high value of standard deviation indicates a wide spreading of the data points over mean value.

5. Conclusion

The wavelet approximation provides average behavior of the given data, while detail provides its differential behavior or fluctuations at different levels. The average behavior of average monthly temperature data of India from year 1901 to 2017 shows a gradual increase with fast rate in recent years. The wavelet analytical results are consistent with the statistical results of the given data of the temperature record. The spectral and statistical analysis provide a strong consistency of the global warming in context of India. On basis of the results, it is possible to conjecture that the wavelet analytical study provides a simple and accurate framework for modelling the spectral analysis of global warming.

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