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## MATHEMATICAL MODELING OF COMPLEX BLOOD FLOW

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*Mathematical modelling of blood flow in different vessels under various physiological conditions has drawn the attention of several workers. Since Poiseuille's empirical equation of finding out the apparent viscosity of viscous fluid in fine glass capillaries, many workers attempted to find out the apparent viscosity of blood, an important parameter affecting the blood rheology. Theoretical modeling on different blood flow situations has drawn an increasing interest and keen attention of many researchers. There could be noticed a great many attempts which are motivated to the viscometric studies and theoretical modeling on blood flow as well as in the investigation of flow variables, such as velocity profile, flow rate, pressure drop, apparent viscosity, pressure-flow relationship and resistance to flow, in a vascular channel. The main aim of this investigation is to Study of Mathematical Models in Complex Blood Flow Analysis. In this thesis the flowing blood is modeled as Herschel-Bulkley non-Newtonian fluid considering the axial variation of viscosity. This model is divided into two sections: In section- A, the stenosis is of cosine shaped type where as in Section-B, the stenosis is taken to be overlapping. In both the sections, the blood flow through an atherosclerotic artery is assumed to be laminar, ax symmetric, incompressible, fully developed, one-dimensional with no-slip condition at the wall. In this model, the constitutive equations of the model are solved analytically using the boundary conditions and the graphs have been plotted for flow parameters such as flow rate, flow resistance and wall shear stress versus the stenosis parameters. The variations of these parameters with stenosis parameters have been studied with the help of the graphs.*

**Keywords:** *Mathematical Models, Blood Flow, Blood Circulatory, Newtonian Model.*

### 1. Introduction

The term biomathematics alludes to the utilization of quantitative and different numerical techniques, in tackling the different genuine circumstances, overall and organic issues. The significant regions which might be remembered for this field, are synovial joints, physiological liquid elements, computational liquid elements (cfD), natural contamination, investigation of intricate liquids, liquids with microstructure, chemotherapy, bug control technique, clinical finding and abstaining from excessive food intake, cardiovascular framework, trade of O<sub>2</sub> and CO<sub>2</sub>, energy necessity of the body and so on.

Science is a part of information that arrangements with living life forms and their essential cycles. Biomathematics is the term which is likewise utilized for the numerical science. It incorporates something like four significant subfields: natural numerical displaying, social

science or, complex frameworks science, bioinformatics and computational bio demonstrating or, bio processing. It is an interdisciplinary scholarly examination field, with a wide extent of uses in science, medication and biotechnology. Numerical science focuses on the numerical portrayal, treatment and demonstrating of natural cycles, by utilizing an assortment of applied numerical strategies and instruments. It has both hypothetical and useful applications in natural, biomedical and biotechnological explores.

The liquid is a substance which is characterized to be a conglomeration of particles. It is an isotropic substance which is fit for streaming. The liquid goes through constant disfigurement under the activity powers is appeared in the propensity of liquids to stream. as such, the liquids misshapes ceaselessly when exposed to a shear pressure, regardless of how little that shear pressure might be. A liquid might be considered to comprises of limited particles, every a lot bigger than an atom yet tiny in size contrasted with the all out volume of liquid. The shear pressure presents distortions which set up inward obstruction. The deformity develops until the obstruction shear approaches the applied shear and afterward stops.

## **2. An Overview of The Blood Circulatory System In General**

A creature is believed to be a complex of barely particular and interrelated organs concerning mathematical shape and fluid motion circulation: heart, frontal cortex, lungs, kidneys, gastrointestinal framework, skeletal muscles, skin, etc. These organs are connected by blood stream, which transports oxygen and other fundamental supplements. Thus, blood course plays a basic capacity in aiding the defeating of the significant diffusive opposition forced by the body's gigantic size. The quantity of organs in the framework viable is enormous, and blood stream is convoluted. Subsequently, zeroing in on a more modest gathering of organs, for example, the circulatory system is much of the time best. The numbers at the organ names on the plan address the worth of blood dissemination (in percent of the moment volume); the numbers further down address the volume of contained blood (in percent of the overall volume) in different vascular framework areas; the upper Roman numerals compare to the names of these districts.

## **3. Numerical Modeling of Blood Flow**

A numerical model is a numerical idea and language depiction of a framework. Mathematic demonstrating is the most common way of making a numerical model. In innate science and designing, yet additionally in the sociologies, numerical models are utilized. A model can support the clarification of a framework, the investigation of the effects of different parts, and the expectation of conduct. The consistent course of blood in the circulatory framework is known as blood stream. The cardiovascular framework in the human body is comprised of three sections: blood, heart, and veins. The cardiovascular framework ensures the section of supplements, chemicals, metabolic squanders, oxygen, and carbon dioxide all through the body to keep up with cell, level digestion, pH, osmotic tension, and temperature guideline, as well as security from microbiological and mechanical harms. Hemodynamic is a part of science that concentrates on the physical science of blood stream. Having a simple comprehension of the cardiovascular framework's life structures and hydrodynamics is important. It is critical to note, in any case, that blood is a non-Newtonian liquid and that blood corridors are not unbending cylinders.

For late concentrate on mass or intensity move in a separated non-Newtonian liquid with changing liquid properties under Fourier-Fick is utilized. The nonlinear speed profile of pulsatile blood course through the dropping aorta has been sufficiently explored in numerous past logical examinations in idea with blood stream in the new past, who regarded the supply

route as a consistently tightened thick barrel shaped container of isotropic and incompressible material. Likewise, some exploration has been finished on estimating electrical action, misshapening, stream, heart displaying, and computational plan of cardiovascular movement in the body. The focal point of this examination will be on numerical displaying of blood flow hardships. In spite of the fact that blood is a non-Newtonian liquid, for the reasons for this review, it will be treated as a Newtonian liquid (blood properties will become direct) represented by the Navier-Stokes condition and the coherence condition.

#### **4. Numerical Models In complex Blood Flow Analysis**

By contrasting Newtonian and non-Newtonian blood models, the shaky entry blood stream in a 90° bended cylinder is mathematically and exactly investigated. The non-Newtonian nature of blood stream is considered for the end goal of demonstrating. The mathematical and trial results are basically the same. Angiogram information was utilized to research the consistent blood stream through the four particular right coronary corridors. The dissemination of divider shear pressure in the conduits is researched utilizing five non-Newtonian models and one Newtonian model. Coming up next is an overall correlation of non-Newtonian models: Viscosity diminishes as strain expands up to  $226.5 \text{ s}^{-1}$  in the Power Law and Walburn - Schneck models. However, in the Casson and Carreau model, at high shear rates, consistency keeps an eye on a restricting worth. Power Law, Newtonian, and Casson models can be in every way utilized in a few unique conditions of Generalized Power Law models. Blood is thought to be an incompressible liquid with a thickness of  $1050 \text{ kg/m}^3$  and is represented by Navier-Stokes conditions and coherence conditions in the models as a whole. A course is considered a firm cylinder. The conditions are tackled utilizing the limited volume approach, which is remembered for the CFD-ACE programming. Limit conditions are utilized to tackle the stream conditions. Calm circumstances are utilized at the outpouring, and no slide is expected at the limit divider. The middle line speed values are 0.02, 0.05, 0.1, 0.2, 0.5, and 1 m/s, and the speed profile is viewed as paraboloidal. The right coronary corridor divider shear pressure dissemination is introduced for various models for determined inflow speeds of 0.02, 0.2, and 1 m/s, as well concerning the Generalized power regulation model. For all non-Newtonian and Newtonian models, the example of divider shear pressure in a specific conduit for a given inflow speed is something very similar.

All through the model results, there is all a predictable example. The size of divider shear pressure differs with varieties in consumption speed. At low flow speed, the size distinction in divider shear pressure is enormous. At low flow speeds, the power regulation models misjudge divider shear pressure while underrating it at high flow speeds. At high admission speeds, the Walburn-Schneck model misjudges divider shear pressure, while the Newtonian model underrates divider shear pressure at low flow speeds. In contrast with different models, the power regulation and Walburn-Schneck are not appropriate for blood thickness displaying for the reasons expressed previously. In the mid-to-high shear rate range, the Newtonian model is more suitable. The Generalized model (non-Newtonian model) performs better compared to the Newtonian model as far as divider shear pressure, which is especially essential at low flow speeds.

#### **5. Objectives And Scope**

Bifurcating stream stand out enough to be noticed in the area of biorheology and befouled mechanics. Hemodynamic factors, for example, change of speed, strain and transmission of volume play significant part being developed of complicated blood stream designs. In the neck, the normal carotid supply route partitions into the inner and outer carotid courses. The inner carotid conduit supplies the blood to the mind. Writing overview has been finished in

Liepsch(1986), Lou and Young(1993) concentrated on the course through bifurcation for Newtonian and Non-Newtonian liquids for unbending as well as adaptable cylinders.

The motivation behind the proposed work is:

- To plan precisely proportional and silicon models, wooden station lastly silicon model to make consistence model like genuine supply route.
- Tension and stream estimation are completed with pressure transducers fitted in the framework.
- Numerical demonstrating, examination and its approval will be finished in well way.

In applied science, biomathematics is to be sure an arising and a powerful field. It gives conceivable solution to countless issues which can advance math training in science and innovation. In this field, there can be viewed as various issues in medication, hereditary qualities, nature, the study of disease transmission, inner and outer bio liquid elements and, neurophysiology Some of the issues might be obliged as: drug dispersion and assurance of the volume of blood, in the human body, clinical demonstrative issues, emergency clinic diet control issues, bug control procedure, ecological contamination issue, bug control issues and so on.

## **6. Newtonian Model of Blood Flow**

### **6.1. Two-Layer Newtonian Model of Asymmetric Stenosed Artery Blood Flow**

Hemodynamic factors have recently been displayed to assume a part in the beginning of atherosclerosis. The confounded state of supply routes (i.e., uniform, bowing, bifurcating, spreading, tightening, broken, etc) is another significant element that influences neighborhood hemodynamic. Albeit the specific reasons of stenosis are obscure, it has been proposed that cholesterol stores on the corridor divider and connective tissue expansion might be at fault. Circulatory issues can be brought about by stenosis in the cardiovascular framework. Hydrodynamic factors play a critical impact in the creation, improvement, and movement of vascular stenosis, as per a significant group of examination. Viscometric exploration and hypothetical displaying of blood stream, as well as the examination of clear thickness, pressure-stream relationship, divider shear pressure dissemination, and protection from stream in a vascular channel, have seen a ton of interest and excitement lately. Throughout the long term, various logical assessments into blood stream have been completed according to different viewpoints. The most serious ramifications of stenosis in a corridor are expanded obstruction and the subsequent decrease in blood stream to the vascular bed provided by the vein. Therefore, quantitatively breaking down blood stream in stenosed veins is very important. Moreover, with the revelation that numerous cardiovascular illnesses are firmly connected with the stream conditions in veins, which is obviously connected with cardiovascular sicknesses, legitimate consideration and quickly developing excitement of specialists in this specific space of Biomechanics has expanded a lot. The conduits in a human body, then again, are not overall a similar shape. There is a distinction in the strain slope in blood stream because of their non-consistency.

### **7.2. Basic Equations**

The fundamental equations regulating fluid flow are mass and momentum conservation equations, as well as fluid constitutive equations. The mass and momentum conservation equations are as follows:

Continuity formula:

$$\dot{\rho} + \rho \nabla \cdot \vec{V} = 0 \quad (3.1)$$

Equation of motion:

$$\vec{\nabla} \cdot \hat{T} + \rho \vec{b} = \rho \dot{\vec{V}} \quad (3.2)$$

Where  $\rho$  represents the fluid's density, and the overlaid dot denotes the material's time derivative.  $\vec{\nabla}$  (nebla) the vector differential operator, the typical velocity field  $\vec{V}$ , the stress tensor (T), and the body force per unit mass (b).

A Newtonian fluid's constitutive equation is

$$\begin{aligned} T_{ij} &= -p\delta_{ij} + D_{ijkl} V_{kl} \\ \Rightarrow T_{ij} &= -p\delta_{ij} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad \delta_{ij} = 1 \text{ for } i = j \\ \Rightarrow T &= -pI + 2\mu D \end{aligned} \quad (3.3)$$

where D is the rate of strain tensor, which is the symmetrical component of the velocity gradient in this case, and  $\mu$  is the shear viscosity. The equation of continuity (3.1) is changed for an incompressible fluid

### 7.3.Flow Geometry

A two-layered model for blood flow has been constructed with an asymmetric stenosis at the vessel wall. The model is made up of two layers: a central layer with a red blood cell suspension core and an outer layer with a peripheral plasma layer (PPL) (as shown in fig.3.1). Both the core and peripheral plasma layers are assumed to be represented by Newtonian fluids with viscosities of  $\mu_1$  and  $\mu_2$ , respectively.

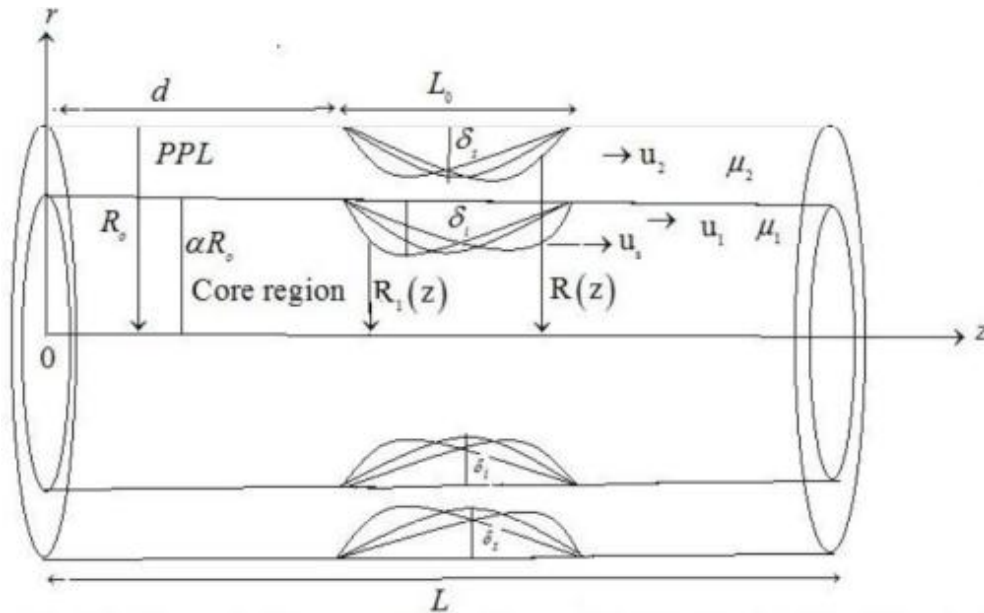
The stenosis geometry, which develops in an unequal manner in the artery wall, is mathematically described in dimensionless form as -

For PPL

$$\begin{aligned} \frac{R(z)}{R_0} &= 1 - A[L_0^{n-1}(z-d) - (z-d)^n], \quad d \leq z \leq d + L_0 \\ &= 1, \quad \text{Otherwise} \end{aligned} \quad (3.4)$$

The shape of the core layer has been believed to be represented by the function  $R_1(z)$ :

$$\begin{aligned} \frac{R_1(z)}{R_0} &= \alpha - A_1[L_0^{n-1}(z-d) - (z-d)^n], \quad d \leq z \leq d + L_0 \\ &= \alpha, \quad \text{Otherwise} \end{aligned} \quad (3.5)$$



**Figure 1.** this diagram depicts a two-layered Newtonian model of asymmetrically stenosed arterial blood flow.

where  $R(z)$  is the radius of the stenosed tube,  $R_0$  is the tube's constant radius, and  $R_1$  is the radius of the artery in the core region, and  $a = \frac{R_1(z)}{R_0} L_0$ . The stenosis length is  $s$ , the tube length is  $L$ , and the stenosis location is  $d$ .  $\delta_s$  and  $\delta_i$  are the maximal stenosis heights in the PPL and the Core regions, respectively, at  $z = d + \frac{L_0}{1-n}$  such that the stenotic height to artery radius ratio is significantly smaller than unity, i.e.  $\frac{\delta}{R_0} \ll 1$

$$A = \frac{\delta s}{R_0 L_0^n} \cdot \frac{1}{n^{n-1} (n-1)}$$

$$A_1 = \frac{\delta i}{R_0 L_0^n} \cdot \frac{1}{n^{n-1} (n-1)}$$

Here,  $n \geq 2$  is a parameter that determines the stenosis' shape.

- **Boundary Conditions**

The current problems' boundary conditions are as follows:-

- $u_2 = 0$ , at  $r = R(z)$  (no slip at the stenotic wall) (3.6a)

- $u_1 - u_2 = us$ , at  $r = R(z)$  (slip at the interface) (3.6.b)

- $\frac{\partial u_1}{\partial r} = 0$ , at  $r = 0$  (symmetric condition) (3.6.c)

- $\frac{\partial u_2}{\partial r} = 0$ , at  $r = 0$  (symmetric condition) (3.6.d)

Where  $u_s$ , is the velocity slip at the stenotic region's contact

We acquire the velocity function for the core region by integrating Equation (3.6) twice-

$$u_1(r) = -\frac{cr^2}{4\mu_1} + A_1 \ln r + B_1, 0 \leq r \leq R_1(z) \quad (3.7)$$

In addition, the PPL (peripheral plasma layer) is-

$$u_2(r) = -\frac{cr^2}{4\mu_2} + A_2 \ln r + B_2, R_1 \leq r \leq R_1(z) \quad (3.8)$$

Using the aforesaid boundary conditions (3.5.a-d) in equations (3.16-3.17), we get the following axial velocity expressions for the core and PPL regions:

$$u_1(r) = u_s + \frac{c}{4\mu_1}[R_1^2(z) - r^2] + \frac{c}{4\mu_2}[R^2(z) - R_1^2(z)], 0 \leq r \leq R_1(z) \quad (3.9)$$

$$u_2(r) = \frac{c}{4\mu_2}[R^2(z) - r^2], R_1 \leq r \leq R(z), \quad (3.10)$$

The volumetric flow rates  $Q_1$  and  $Q_2$  for the core and peripheral regions can be calculated using the following formulas (3.9-3.10) for velocities.

## 8. Axial Viscosity Variation in A Stenosed Artery: A Non-Newtonian Model

Demonstrating the way of behaving of the genuine world with science is a discipline of math that is utilized to depict and foresee the way of behaving of this present reality. As of late, the investigation of blood stream displaying in an assortment of circumstances has gotten forward momentum and ignited a lot of revenue among researchers around the world. It has been found by Blair and Spanner that the Herschel-Bulkley model is a better model than the Casson model and that when the yield pressure is high; the Herschel-Bulkley model can be utilized at exceptionally low shear rates, though the Casson liquid model must be utilized at moderate shear rates in containers of little measurements. A review directed by Chakravarthy and Mandal investigated the issue of stream in a course under the suspicion of covering stenosis and simply a variety in strain all through the length of the cylinder. The impacts of covering stenosis on blood stream attributes have been researched, steering into account the variety in strain in the two headings (spiral and pivotal) of the blood vessel area being scrutinized. It's worth noting that increasing the value of  $n$  causes the stenosis shape to alter. When  $n=2$ , the stenosis geometry becomes symmetrical at  $z = d + \frac{L_0}{2}$  and  $a = \frac{\delta_i}{\delta_s}$ .

### • Boundary Conditions

In order to better understand the behaviour of the fluid, the following boundary conditions are introduced.

$$\frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0 \quad (3.11a)$$

$$u = 0 \quad \text{at } r = R(z) \quad (3.11b)$$

$$\tau \text{ is finite at } r = 0 \quad (3.11c)$$

$$P = P_0 \text{ at } z = 0 \text{ and } P = PL \text{ at } z = L \quad (3.11d)$$

$$\tau \text{Is finite at } r = 0 \quad (3.11e)$$

$$P = P_0 \text{ at } z = 0 \text{ and } P = PL \text{ at } z = L \quad (3.11f)$$

The Navier-Stokes equation for blood is expressed in a cylindrical coordinate system.

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r} \frac{\partial(r\tau)}{\partial r} \quad (3.12)$$

The Herschel-Bulkley fluid's governing equations are as follows:

$$\left(-\frac{\partial u}{\partial r}\right) = \frac{1}{\mu(z)}, \quad \tau \geq \tau_0$$

$$= 0, \quad \tau < \tau_0 \quad (3.13a)$$

Where  $u$  is the axial velocity of blood in the sagittal plane,  $\tau_0$  the yield stress is the amount of force applied,

$\mu(z)$  is the fluid viscosity measured in the axial direction, and  $n$  is the fluid behavior index.

Viscosity  $\mu(z)$  is given in the axial direction by,

$$\mu(z) = \mu_0 \left(\frac{R(z)}{R_0}\right)^{-\alpha}; \quad d \leq z \leq d + L_0$$

$$\mu_0; \text{ Elsewhere} \quad (3.13b)$$

Where  $\mu_0$  is the plasma viscosity, and  $\alpha$  is a parameter for axial viscosity variation.

## 9. Conclusion

These days, liquid elements assumes a fundamental part in liquid stream with applications inside the human body, and blood stream demonstrating is a vital and testing component of cardiovascular physical science. In any case, the models that have been built up to this point are very convoluted with regards to three-layered examination. A numerical model can be utilized to portray an assortment of true peculiarities. Atherosclerosis is a cardiovascular illness that happens when plaque develops in blood corridors and river, thickens, solidifies, and rebuilds them.

The circulatory framework is one of the main frameworks in the human body, as it is answerable for giving food all through the body, managing internal heat level, and fighting illnesses. It is likewise powerless against an assortment of illnesses. Recreating the framework with a reasonable model can support early recognizable proof and the board of issues that emerge in the framework. Blood move through blood corridors has been concentrated broadly, as confirmed by the monograph. As a result of the multifaceted plan of flexible tissues that make up a vein, definitively displaying the communication of blood stream with the vessel divider is troublesome. The vessel dividers are comprised of an overlay structure comprised of three layers of tissues (adventitia, media, and intima) with fluctuating creations and flexible characteristics.



Blood is a non-Newtonian liquid that contains molecule materials suspended in it. Red platelets (RBCs), white platelets (WBCs), and platelets are the molecule solids. Plasma, which is a mind boggling combination of proteins and other intergradient's in a watery base, is the liquid. The investigation of blood stream involves estimating pulse and deciding the stream through veins. The significance of this work to human wellbeing couldn't possibly be more significant.

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