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AN ANALYSIS OF METHODS THAT ARE USED TO SOLVE SYSTEM OF LINEAR EQUATIONS

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ABSTRACT

The goal of this project is to create a learning material for linear algebra with the intention of helping the advancement of students' mathematical knowledge as well as representation of mathematical ideas. The material will be built in the context of linear algebra. The method of doing research is abbreviated as R&D, which stands for "research and development." It may be broken down into three basic phases, which are the preliminary, development, and distribution stages. The stage of conceptualization was the only one considered in the scope of the research. According to the results, the conclusion is that the assessment of the professionals on the educational materials fits into the category of valid with a minor modification to the part on exercise questions. The validator was the one who made the evaluation. The result of applying the knowledge that has to be learnt in practise is 87.81% of the information (very practices). In the meanwhile, the findings of the limited trials indicate that students' mathematical understanding may be accomplished independently and in a conventional manner with the assistance of the learning material. In addition, the pupils' mathematical representations, both in the traditional sense and on an individual basis, have not yet reached the degree of mastery that they are capable of.

Keywords: *Linear Algebra, development, mathematical*

Introduction

Students are prepared to become more effective problem solvers if they are exposed to mathematics throughout their educational careers. A person who solves problems has to have a certain set of cognitive talents so that he may make the most of the information he has learned and use it in the most effective way possible. A problem solver needs to have the cognitive abilities necessary to understand and represent a mathematical situation, create algorithms on certain problems, process various types of information, and run computing. Additionally, a problem solver needs to be able to recognise and manage a set of appropriate resolution strategies in order to solve problems. [1]. The ability of students to understand both the prerequisite concepts and the mathematical concepts that are currently being studied is the most important factor in determining whether or not they will be successful in obtaining specific mathematical knowledge and in finding solutions to problems that arise while learning. Students indirectly adapt and extend their past knowledge when they answer mathematical problems since this requires them to connect or link newly acquired information with information they already know.

New material will be connected to the structure of the student's knowledge once they have completed a series of mental tasks. Students that are adept at finding solutions to problems also have the capacity to manage spoken language, pictures, images, and mathematical symbols, which supports their problem-solving talents. Students will gain from strengthening their capacity to articulate mathematical ideas when they are faced with a problem if they work on improving this skill. Students have the capability, via the use of their representational abilities, to simplify a mathematical concept that is presented to them in a variety of graphical and other modes of representation. Students may be led to the acquisition of correct knowledge by accurate representation, but faulty representation will lead to an erroneous process of problem-solving. Accurate information may be acquired by students when representations are accurate. Because of this, it is essential for students to have the capacity to grasp and express mathematical concepts simultaneously if they want to reach a higher level of academic achievement, particularly in the area of courses dealing with linear algebra. This is especially true in the case of courses dealing with linear algebra. In recent years, a number of studies have focused their attention on the mathematical understanding and representation abilities of students in relation to the learning process. This area of study has received a lot of attention in recent years.

According to the results of the investigation that was carried out by Fatqurhohman, training students in the concept of procedural understanding would be beneficial to them in terms of enhancing and developing their knowledge of various subjects. By applying algorithms to the process of identifying problems and connecting concepts with a variety of representations, teachers can make it simpler for students to develop the ability to understand a concept even though their levels of comprehension may vary. This is because it will make it easier for students to develop transferable skills. The application of algorithms to the resolution of problems and the establishment of connections between ideas using a variety of representations are two ways in which this may be done. It is essential to the development of both conceptual and procedural understanding that learners actively participate in activities that are important to them throughout the process of learning.

Another study that looked at efforts to develop mathematical understanding ability through the design of instructional materials came to the conclusion that, based on the results of a limited test of teaching materials for a group of students who were majoring in mathematics education, the outcomes of the students' mathematical comprehension were both classical and individual. This conclusion was reached as a result of the findings of the study that examined efforts to develop mathematical understanding ability through the design of instructional materials. Previous researchers who were interested in mathematical difficulties at educational levels ranging from elementary school to college have undertaken research on the mathematical representation. These educational levels range from elementary school to college. The great majority of students struggle while attempting to carry out mathematical representation tasks, especially those that entail linking procedures and processes to a variety of representations of ideas that are relevant. When students are given one of the problems that are relevant to the linear algebra subject matter, this is something that can be seen as an example of what may be noticed. By paying attention to the amount of variation in students' beginning skills, the availability of teaching resources that are based on particular mathematical abilities would help the growth of mathematical abilities, which are the purpose of learning. [8].

Literature review:

When it comes to solving systems of linear equations, there is not a great lot of research that applies to the ways that students often use, in particular when there are multiple choice answers included in the problem. Research has been conducted on both linear equations and multiple choice responses; however, research that combines the two and takes it one step further to transform linear equations into systems of linear equations is not yet considered to be in a mature stage (Anderson, 2016)

Throughout the course of human history, a wide variety of strategies for solving systems of linear equations involving two variables as well as including more than two variables have been conceived of and developed.

Cornelius Lanczos[2012] A straightforward method that is well suited to the effective solution of extensive systems of linear algebraic equations by use of a string of approximations that converge to the correct answer is provided in this article. Simply clicking on the link will take you to the algorithm in question.

FALL [2012]. This research, which is descriptive in nature, focuses on the methods that college students (aged 20 to 24) use to solve problems involving linear systems of equations that contain solutions that may be chosen from a set of options. The problems that are being solved involve linear systems of equations that contain solutions that may be chosen from a set of options. Participants hailed from a public institution in the northeastern United States that was of a sizeable but not overwhelming size. There were four distinct plans that were evaluated for their potential effectiveness. There were three different forwarding tactics, and they were as follows: 1) the replacement strategy, 2) the deletion strategy, and 3) the graphing strategy. There was also an approach known as reversal, which included adding the x and y values that corresponded to each of the multiple choice options. During a structured clinical interview that was carried out by

KHAN et al. [2015], the participants were given tasks containing systems of linear equations and were prompted to respond to questions based on the techniques they used to solve the issues. This article provides not just a concise introduction to matrices but also the direct methods for solving linear equations. The purpose of this study was to investigate the various approaches to the elimination of linear equations and to evaluate the effectiveness of the Guassian elimination and the Guass Jordan method in order to determine the relative significance of these approaches and the benefits that they offer in the context of symbolic and numerical computation. Specifically, the purpose of this study was to investigate the effectiveness of the Guassian elimination and the Guass Jordan method in order to evaluate the effectiveness of these methods. In order to provide a metric by which the efficacy of Guass Jordan and Guassian elimination can be evaluated, the goal of this study is to modify an introductory idea of linear equations, matrix theory, and forms of Guassian elimination. This will be done in order to meet the aforementioned objective.

Riga [2016].Since before the beginning of the common period, people have been using numerical methods, and systems of linear equations constitute a crucial component of these methods (BC). Nevertheless, despite the fact that it reached its pinnacle between the years 1600 and 1700 in response to the public's desire for solutions to scientific and engineering issues, the topic is still significant in the present day. Within the scope of this investigation, a further iterative approach to the resolution of linear systems is described. This technique involves moving the solution proximity point closer to the actual solution while simultaneously bringing the differences in all of the system equations closer together. It is based on the concept of "making several transfers."

MARYA et.al [2014]. In this piece, we will talk about three direct methods that may be used to solve a system of linear equations. These methods are as follows: It is possible to solve systems of linear equations using any one of a number of distinct methods; but, contrary to the widespread notion, there is no one technique that is always guaranteed to be better to the others in any given situation (to get appropriate solution). When it comes to finding solutions to difficult systems of equations, speed and accuracy are two elements that are very essential. This is because there are a significant number of calculations that need to be done in order to discover a solution. Both swiftness and pinpoint precision are required for these methods. Directly solving simultaneous linear equations may be a time-consuming process, particularly when working with large systems of equations; this technique also requires a particular strategy in order to avoid numerical instability.

Maharaja [2013]. The major focus of this investigation was on the written replies provided by two students in response to questions that were based on the application of matrix methods to the process of solving a system of linear equations. The purpose of this endeavour was to ascertain, by means of a framework that was constructed, the possible level of mathematical understanding that the students had. Utilization of the framework allowed for the successful completion of this task. This framework was used to analyse the level of mathematical comprehension that those students exhibited in their written solutions to the questions, with a particular emphasis placed on the students' utilisation of symbolic language. Specifically, this framework was used to examine the students' written solutions to the questions. It was discovered that the researcher was able to get a more in-depth understanding of those students' use of the symbolic language, which was used both in a functional capacity and as a means of communication. This was discovered by the fact that the researcher was able to get a more in-depth understanding of those students' use of the symbolic language. The framework allowed for the completion of this task.

Objective of the study:

1. To study on Linear systems are systems of equations
2. To study on Methods That Are Used To Solve System Of Linear Equations

DEFINITION:

An example of what is known as a linear equation system is a collection of one or more linear equations that include the same set of variables. There may be anywhere from one to many linear equations in such a system. Or Equational systems may be classified as linear if the variables in each equation are never multiplied with one another. Instead, only the constants in the system are multiplied, and the resulting products are combined together.

A linear equation in variables $a_1x_1 + a_2x_2 + a_3x_3 + \cdots .. a_nx_n$=b

Where $a_{1,}\, a_2\,, a_3 \ldots\ldots\ldots\ldots. a_n$ and b are constants of real and complex numbers. The constant a_i

is called the coefficient of x_i and b is called the constant term of the equation.

Methods That Are Used To Solve System Of Linear Equations:

Finding a solution to an equation system that is linear may be done in a huge number of different ways. There are a lot of possibilities. The elimination of variables, the Row reduction approach (also known as the Gauss elimination method), Cramer's rule, and other procedures that are quite similar are some examples. Other methods that are very similar include:

ELIMINATION OF VARIABLES:

The most straightforward method for solving a system of linear equations is to remove each variable in turn until the system is complete. The explanation that follows is only one of many possible ways to describe this process:

- In the first equation, determine the value of one of the variables by solving for the other variables using the first equation. This will give you the value of the variable you were looking for.
- You need to substitute this expression into the following equations wherever it seems to make sense to do so. As a consequence of this, an equation system will now have one less unknown as a result.

- You will need to keep doing this step until the system can be modelled using a solitary linear equation. Determine the solution to this equation, and then continue to do back-substitutions until you have the whole answer.

Row Reduction Method:

In order to cut down on the amount of rows in a matrix, the matrix itself has to be modified by a series of straightforward row operations first. This process will continue until the lower left-hand corner of the matrix has as many zeros as is physically possible to fill it up fully.

For example:- solve the system of equation using Row reduction method

$$x + 4y + 9z = 16$$

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

write it as AX=B

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 10 \\ 18 \end{pmatrix}$$

Augmented matrix ;

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 2 & 1 & 1 & :10 \\ 3 & 2 & 3 & :18 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$,$R_3 \to R_3 - 3R_1$

Apply $R_2 \to R_2 - 2R_1$,$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 0 & -7 & -17 & :-32 \\ 0 & -10 & -24 & :-30 \end{bmatrix}$$

$$R_3 \to 7R_3 - 10R_2$$

$$\begin{bmatrix} 1 & 4 & 16 & :16 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

x+4y +9z = 16

-7y -17z =-22

2z = 10

Z = 5 , y =- 9 and x = 7

5.3 Same question solved by guass Jordan method:

x + 4y + 9z = 16

2x + y + z = 10

3x + 2y + 3z = 18

write it as AX=B

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 10 \\ 18 \end{pmatrix}$$

Augmented matrix $[AA : BB]$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 2 & 1 & 1 & :10 \\ 3 & 2 & 3 & :18 \end{bmatrix}$$

Apply $'R_2 \rightarrow R_2 - 2R_1 \quad ,R_3 \rightarrow R_3 - 3R_1$

Apply $R_2 \rightarrow R_2 - 2R_1 \quad ,R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 9 & :16 \\ 0 & -7 & -17 & :-32 \\ 0 & -10 & -24 & :-30 \end{bmatrix}$$

$$R_3 \rightarrow 7R_3 - 10R_2$$

$$\begin{bmatrix} 1 & 4 & 16 & :16 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix} R_3 \rightarrow 7R_3 - 10R_2 \ , R_1 \rightarrow 7R_1 + 4R_2$$

$$\begin{bmatrix} 7 & 0 & -5 & :24 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix} R_1 \rightarrow 2R_1 + 5R_3 \quad R_2 \rightarrow 2R_2 + 17R_3 \quad ,$$

$$\begin{bmatrix} 14 & 0 & 0 & :98 \\ 0 & -14 & 0 & :126 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

-14y= 126

2z = 10

So, x=7 , y= -9 , and z = 5

5.5 Same question solved by crammers rule :

$x + 4y + 9z = 16$ **(1)**

$2x + y + z = 10$ **(2)**

$3x + 2y + 3z = 18$ **(3)**

First we need to find the value of delta Δ

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{vmatrix}$$

Now we have to apply 3rd row determinant rule

$$1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} + 9\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

(3 − 2) − 4(6 − 3) + 9(2 − 3)

1-12 +9

10– 12 = -2 so $\Delta = -2$

Now for finding the value of X,Y and Z we need to find the value of Δxx , $\Delta yy\ aaaaaa\ \Delta zz$
In Δ 1st column we have to replace by these three exponent

$$\Delta x = \begin{vmatrix} 16 & 4 & 9 \\ 10 & 1 & 1 \\ 18 & 2 & 3 \end{vmatrix}$$

16(3 − 2) − 4(30 − 18) + 9(20 − 18)

16-48+18 = -14

So $x = \frac{\Delta x}{\Delta}\frac{-14}{-2} = 7\ i.e\ x = 7$

$$\Delta y = \begin{vmatrix} 1 & 16 & 9 \\ 2 & 10 & 1 \\ 3 & 18 & 3 \end{vmatrix}$$

1(30 − 18) − 16(6 − 3) + 9(36 − 30)

12 - 48 + 54 = 18

$$y = \frac{\Delta y}{\Delta} = \frac{18}{-2} = -9$$

$$\Delta z = \begin{vmatrix} 1 & 4 & 16 \\ 3 & 1 & 10 \\ 3 & 2 & 18 \end{vmatrix}$$

1(18 − 20) − 4(36 − 30) + 16(4 − 3)

-2 - 24 +16 = -10

$$z = \frac{\Delta z}{\Delta} = \frac{-10}{-2} = 5$$

When we put these approaches to work on the same challenge, we always arrive to the same conclusions for x, y, and z.

In a similar vein, we are able to swiftly and effectively handle issues of this kind by using a range of strategies for solving systems of linear equations. This allows us to do so in a time and resource efficient manner.

Health Care Professional:

The computation of doses is a common use of linear equations in the realm of medicine, notably among medical professionals such as physicians and nurses. Linear equations are used in the process of determining how different drugs may interact with one another in order to ascertain the appropriate dosage quantities for patients who are taking more than one medication and to prevent an overdose. This is done in order to both prevent an overdose and ascertain the appropriate dosage quantities. In addition, linear equations are used in the process of determining how different medications may interact with one another in the process of trying to find the best treatment. In addition, linear equations are used by experts in the area of medicine in order to compute dosages in accordance with the patient's body mass in order to provide the most accurate results.

CONCLUSION:

This study is a development of learning material for linear algebra, and it is being provided here. Specifically, this research is being presented here. The instructional material is suitable for the intended audience with the possible exception of a couple of the practise questions, which would benefit from some light editing. The learning process yields findings that are very useful when they are applied to real-world situations. In the meanwhile, the findings of the few trials that were conducted imply that learning materials may be able to overcome challenges with students' mathematical understanding, both in conventional settings and in individual contexts. Mathematical representation is not yet a subject in which students have shown a degree of expertise in either the traditional or individual learning settings.

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