International Research Journal of Human Resource and Social Sciences
ISSN(O): (2349-4085) ISSN(P): (2394-4218)
Impact Factor 6.924 Volume 9, Issue 12, December 2022
Website- www.aarf.asia, Email : editoraarf@gmail.com

FUNDAMENTALS AND APPLICATIONS OF GRAPH THEORY

SYEDA FARRA FATIMA NAHRI
RESEARCH SCHOLAR SUNRISE UNIVERSITY ALWAR
DR. MANISH KUMAR SINGH
ASSISTANT PROFESSOR SUNRISE UNIVERSITY ALWAR

ABSTRACT

This text has been carefully designed for flexible use. It is primarily designed to provide an introduction to some fundamental concepts in Graph Theory, for under-graduate and post-graduate students. Each topic is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks. All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematical sciences. Proofs are motivated and developed slowly; their steps are all carefully justified

Keywords: - Graph Theory, Applications, Fundamentals, Diagram, Tree

I. INTRODUCTION

A linear graph (or simply a graph) G = (V, E) consists of a set of objects V = {v1 v2 , . . .} called vertices, and another set E = {e1' e2 ,. . .}, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i , v_j) of vertices. The vertices $v_i v_j$associated with edge e_k are called the end vertices of e_k.

The most common representation of a graph is by means of a diagram, in which the vertices are represented as points and each edge as a line segment joining its end vertices. Often this diagram itself is referred to as the graph. The object shown in Fig. 1-1, for instance, is a graph.

Observe that this definition permits an edge to be associated with a vertex pair (v_i , vi). Such an edge having the same vertex as both its end vertices is called a self-loop (or simply a loop.

The word loop, however, has a different meaning in electrical network theory; we shall therefore use the term self-loop to avoid confusion). Edge e1 in Fig.1 is a self-loop. Also note that the definition allows more than one edge associated with a given pair of vertices, for example, edges e4 and e5 in Fig. 1. Such edges are referred to as parallel edges.

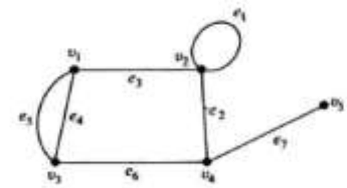


Figure.1 Graph with five vertices and seven edge

A graph that has neither self-loops nor parallel edges is called a simple graph. In some graph-theory literature, a graph is defined to be only a simple graph, but in most engineering applications it is necessary that parallel edges and self-loops be allowed; this is why our definition includes graphs with self-loops and/or parallel edges. Some authors use the term general graph to emphasize that parallel edges and self-loops are allowed. It should also be noted that, in drawing

It should also be noted that, in drawing a graph, it is immaterial whether the lines are drawn straight or curved, long or short: what is important is the incidence between the edges and vertices. For example, the two graphs drawn in Figs. 2(a) and (b) are the same, because incidence between edges and vertices is the same in both cases.

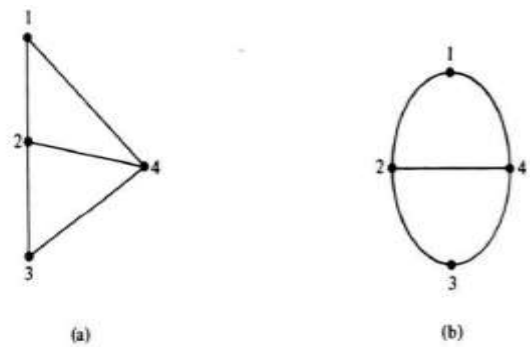


Figure. 2 Same graph drawn differently.

In a diagram of a graph, sometimes two edges may seem to intersect at a point that does not represent a vertex, for example, edges e and f in Fig.3. Such edges should be thought of as being in different planes and thus having no common point. (Some authors break one of the two edges at such a crossing to emphasize this fact.)

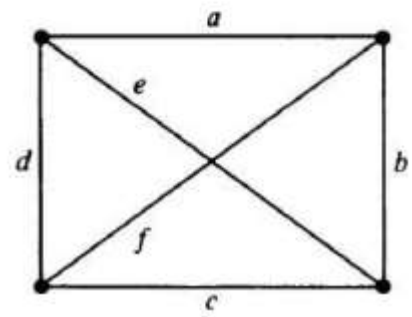
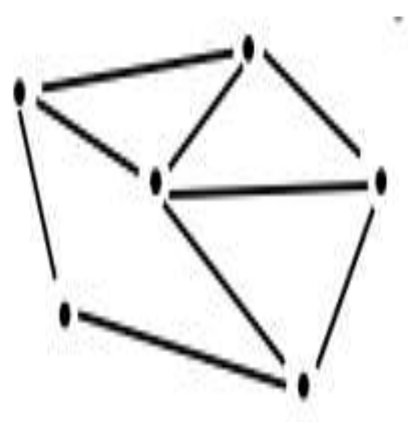


Figure. 3 Edges e and f have no common point.

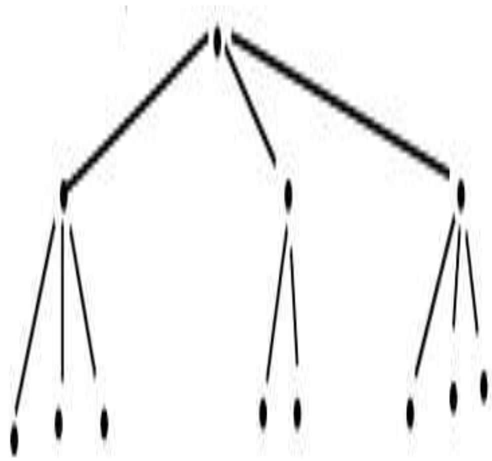
A graph is also called a linear complex, a 1-complex, or a one-dimensional complex. A vertex is also referred to as a node, a junction, a point, 0-cell, or an 0-simplex. Other terms used for an edge are a branch, a line, an element, a 1-cell, an arc, and a 1-simplex. In this book we shall generally use the terms graph, vertex, and edge.

II. FUNDAMENTALS OF GRAPH THEORY

Graphs: Structures on sets. Computer science and software engineering can fundamentally be regarded as applied discrete mathematics. One of the best illustrations of this is that CS & SE utilize the concept of a graph in many situations. Now we have a graph whenever we have a situation where several entities, known as vertices, or nodes, that are related to each other in some way. We then say that the relation between two nodes or vertices constitute an edge and we can give this a pictorial representation by drawing the whole situation:



This could represent a range of different situations: each dot could be a router in a wide-area network, and each edge between the nodes could represent that there is a connection between the routers (they know of each other's IP-addresses). We will now use the words vertex and edge to denote the building blocks of a graph. Another instance is state-diagrams of digital automatons, each vertex would then represent a state that the automaton can assume and an edge between two vertices/states would then denote that there is a state-transition present between two states/vertices. If we draw a graph in the form of a tree (which has no closed circuits) we could draw it in the following manner:



Here the vertices could represent folders or files in a file-system and an edge being present between two vertices/files/folders could represent that one folder/file is contained in the other, we can talk about parent vertices. The same graph could be used to represent hierarchies of executing processes in an a model of a running operating system. In another instance, the same model could be used to describe the architecture of a software systems built of components, where each component can be made up of other components (such as function or libraries of functions).

The possibilities are endless and studying graphs are of utmost importance for any student of discrete amatics and/or computer science and software engineering.

III. APPLICATIONS OF GRAPH THEORY

In this section we list out the various application of graph theory in different disciplines

- In chemistry, graph theory and its concepts are used to study and represent the structure, properties and construction of the bonds in atoms/molecules by means of diagraphs and parallel graphs in which the atoms are represented as vertices and the bonds between atoms as edges. Thus the molecular representation of a molecule or atom using graph theory helps in representing any complex structure in its simplified form.

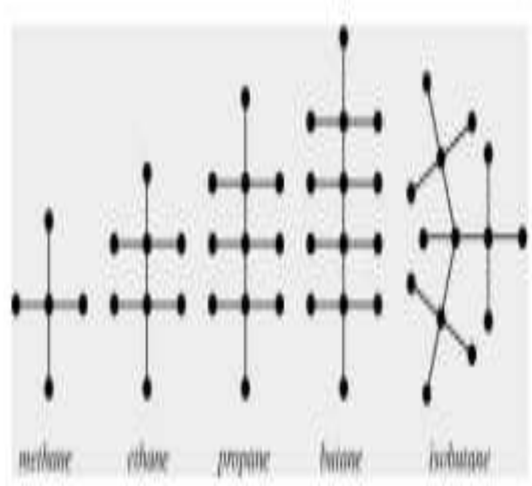


Figure. 4 Graphical representation of hydrocarbons.

- Graph theory is extensively employed in biology to study the conservation effort of certain species (treated as vertices) and their migration path / movement (treated as edges) between the territories (regions). This information helps for tracking the breeding patterns, spread of diseases or parasites and to study the migration impact that affects other species.
- In biology, graph theoretical concept of tree (acyclic graph) is used to represent the evolutionary relationships between the existing biological species.
- Such a graphical representation is called a "Phylogenetic tree or phylogeny". Following is an example of a phylogenetic tree of life. Hence the concept of trees is used to represent any hierarchal order in the biological system.

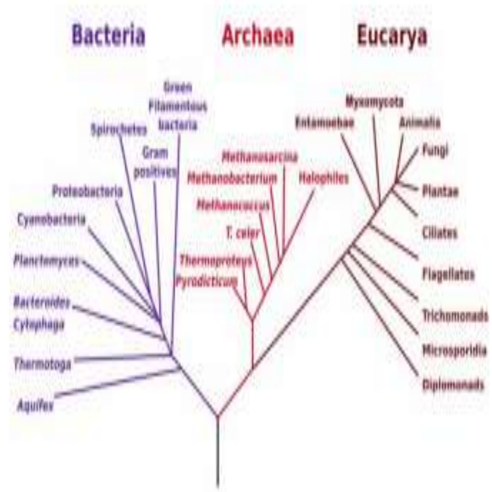


Figure. 5 Phylogenetic tree of life

- Graph theory and its concepts find its vast applications in Operational Research."The Travelling salesman Problem" is the best example exhibiting the application of graph theory in the field of operation research. This problem is used to obtain an optimal match of men and job and trace the shortest spanning tree for the given weighted graph. This property of graphs to obtain the shortest spanning tree for the given weighted graph finds its application in Network modeling (network maps).

Figure. 6 Network maps

- Another important application of graphs is to the field of Game theory. Game theory is a mathematical method of decision-making used for the analyses of a competitive situation determining the optimal action course for an interested party, often used in political, economic, and war sciences. Here the concepts of directed graph are used to represent the method of finite games where the positions represent vertices and the moves represented by edges.

- Though graph theory find its vast applications in the fields of sciences it also find its application in the field of social sciences too. One of the important applications of Graph theory in sociology is to explore diffusion mechanisms. The sociological theory of diffusion is the study of the diffusion of innovations throughout social groups and organizations. Thus using graph theory the behavioral patterns of these social groups and organizations are analyzed and studied.

- Out of all the application of graph theory, its application to electrical network/circuit is the most natural one. The graph representing any electrical network/circuit is one in which the terminals are represented as vertices and the electrical components equipped between the terminals are represented as edges. Thus an electrical circuit can be modeled and the its working can be studied easily using graph theory.

In the section above we listed the various application of graph theory to different fields. In the following section we give few examples of modeling a real world situation/problem exclusively using graph theory.

Because of its inherent simplicity, graph theory has a very wide range of applications in engineering, in physical, social, and biological sciences, in linguistics, and in numerous other areas. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. The following are four examples from among hundreds of such applications.

Königsberg Bridge Problem: The Königsberg bridge problem is perhaps the best-known example in graph theory. It was a long-standing problem until solved by Leonhard Euler (1707-1783) in 1736, by means of a graph. Euler wrote the first paper ever in graph theory and thus became the originator of the theory of graphs as well as of the rest of topology. The problem is depicted in Fig 7.

Two islands, C and D, formed by the Pregel River in Königsberg (then the capital of East Prussia but now renamed Kaliningrad and in West Soviet Russia) were connected to each other and to the banks A and B with seven bridges, as shown in Fig.7. The problem was to start at any of the four land areas of the city, A, B, C, or D, walk over each of the seven bridges exactly once, and return to the starting point (without swimming across the river, of course).

Euler represented this situation by means of a graph, as shown in Fig. 7. The vertices represent the land areas and the edges represent the bridges.

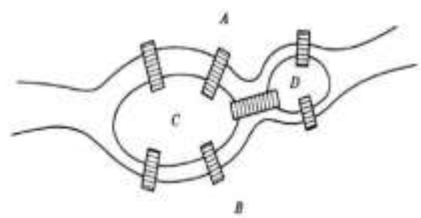


Figure.7 Königsberg bridge problem.

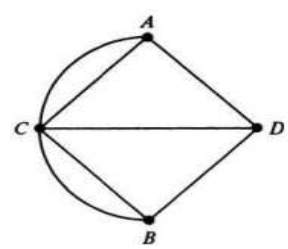


Figure.8 Graph of Königsberg bridge problem

IV. HISTORY OF GRAPH THEORY

Graph theory is an emerging mathematical branch used for structural modeling and geometric approach for studying objects. Thus the principle objective of this theory is a graph and its generalization. Graph theory's journey started from the famous problem of Konigsberg bridge in 1736. Euler provided a mathematical solution to the problem, now known as Euler's Puzzle, to find a path by which a person could traverse each of the seven bridges connecting the city of Konigsberg exactly once and return to the starting point.

Figure 9 Konigsberg bridge problem

He not only showed that such a path does exist but gave a solution that was applicable to any arbitrarily arranged bridges and landmasses. Euler pointed out the physical distances and geographical positions are not being considered as important factors solving this problem. Thus this problem led to the notion of Graph, hence the birth of graph theory. Hundred years later, the second important contribution to graph theory was made by A.F Mobius in the year 1840. He introduced the idea of complete and bipartite graphs and with the help of Kuratowski proved that these graphs were planar by means of recreational problems. Another important contribution was by Gustav Kirchhoff in the year1845 in the field of electronics. He introduced the concept of acyclic connected graphs termed as ‘tree’ and employed these concepts to calculate the flow of currents in an electrical circuits. In 1852, Thomas Gutherie stated the famous four color problem (conjecture) that set the bench mark in map coloring problems.

Graph theory was once considered an unimportant branch of topology (some even called it the ”slum of topology”), graph theory has long since, justified its existence through many important contributions to a wide range of fields. Rich in interesting problems and applications, it is one of the most studied and fastest growing areas within discrete mathematics and computer 2 science. Countless problems involving a collection of discrete objects can be phrased and solved by graph-theoretic methods. Many such methods have become standard and are considered a natural part of every curriculum in discrete mathematics and computer science.

V. CONCLUSION

The purpose of a brief excursion into the world of modeling and graph theory indicating their utility as a practical problem solving tool in areas of natural and social sciences, networking and engineering with special focus to electrical engineering. In our research, the primary goal was to further explore graph properties and vertex colorings. Although we completed this, due to time constraints, we were unable to investigate more aspects of graph colorings and infinite graphs. A possible direction within this would be to be able to look at algorithms that determine chromatic number. As of right now, there are no possible algorithms that determine the minimum number of colors on a graph. In addition, if allowed more time, we would have liked to work with variations of infinite graphs to determine the properties that result from them. Another possibility in this research would also be to look at labelings such as harmonious and graceful labelings

REFERENCES:-

1. S G Shrinivas, S G Shrinivas&Vetrivel, Santhakumaran&Elango, N. (2010). APPLICATIONS OF GRAPH THEORY IN COMPUTER SCIENCE AN OVERVIEW. International Journal of Engineering Science and Technology. 2.

2. Riaz, Ferozuddin& Ali, Khidir. (2011). Applications of Graph Theory in Computer Science. 142 - 145. 10.1109/CICSyN.2011.40.

3. Basener, William & Narayan, Darren. (2007). Modern applications of graph theory. CongressusNumerantium.

4. Nadoos, Wessam&Alsaedi, Rusul&Kadhim, Qasim. (2019). The application of the graph theory for developing and testing the software. International Journal of Civil Engineering and Technology. 10. 2026-2031.

5. Sarma, Salaka. (2012). APPLICATIONS OF GRAPH THEORY IN HUMAN LIFE.

6. Gazda, Vladimir & Horvath, Denis &Rešovský, M.. (2015). An Application of Graph Theory in the Process of Mutual Debt Compensation. ActaPolytechnicaHungarica. 12. 7-24. 10.12700/APH.12.3.2015.3.1.

7. VijayaChandra, J. &NarasimhamChalla, &SaiKiranPasupuleti, & Komati, ThirupathiRao& Reddy, Vuyyuru. (2015). Numerical formulation and simulation of social networks using graph theory on social cloud platform. 11. 1253-1261.

8. J. M. Vasak, The thickness of the complete graph, ProQuest LLC, Ann Arbor, MI, 1976. Thesis (Ph.D.)–University of Illinois at Urbana-Champaign. MR2626173 [2013]

9. S. Toida, Properties of a Euler graph, J. Franklin Inst. 295 (1973), 343–345, DOI 10.1016/00160032(73)90046-X. MR332580 [2010]