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UNDERSTANDING THE CONCEPT OF IMPULSIVE DIFFERENTIAL EQUATIONS WITH SOME EXAMPLES

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ABSTRACT

Some real-world processes look amenable to mathematical modeling, and this has led to the theory of impulsive differential equations being recognized as a promising new area of study. Solutions to impulsive differential equations have been studied extensively to characterize their qualitative behavior. We present an impulsive differential equations-inspired inverse framework, impulse extension equations, that can be utilized to test the veracity of impulsive models.

Keywords: Impulsive, Differential equations, Mathematical, Perturbations,Assumption

I. INTRODUCTION

The dynamics of a wide variety of physical processes may be modeled by use of differential equations. There is often a sudden transition between stages in evolutionary processes. Short-term fluctuations, on the other hand, have a small impact when compared to the full course of these processes. Because of this, it is reasonable to suppose that the effects of these disturbances take the shape of sudden impulses. It seems, then, that impulsive differential equations, or differential equations including impulse effects, provide a suitable way to describe the known evolutionary patterns of a number of practical situations.

The sudden transition from one state to another is a hallmark of many evolution processes at critical junctures. This is because to transient fluctuations, which have a minor effect over the long run of the operation. Naturally, we suppose that the effects of these disturbances take the form of sudden jolts, or impulses. Consequently, differential equations with impulse effects (or "impulsive" differential equations) are viewed as a natural representation of the known evolution phenomena of a number of real-world issues. Impact in mechanical systems, thresholds in biology, bursting rhythms in medicine and biology, optimum control models in economics and pharmacology and industrial robotics, and many more all demonstrate impulsive effects. As impulsive differential equations will likely become more useful in many different contexts, it is prudent to devote time and energy to learning about this subject as a legitimate academic topic. Both A. D. Myshkis and V. D. Mil'man published seminal articles in the 1960s that laid the groundwork for this hypothesis.

Despite its significance, many solutions to impulsive differential equations are performed analytically. V. Lakshmikantham, D. Bainov, P. Simeonov, and many more are only a few of the well-known scholars who have given conclusions of statistical significance. Yet, solving many impulsive differential equations analytically is either not possible or involves a great deal of complexity. So, it is necessary to investigate the numerical solution of impulsive differential equations and enhance the obtained findings. Using the Euler technique, this work seeks numerical solutions to impulsive differential equations. The proposed algorithm is understood in light of the theory of impulsive differential equations developed by V. Lakshmikantham and colleagues.

II. IMPULSIVE DIFFERENTIAL EQUATIONS

Differential equations with impulse effect, or impulsive differential equations, appear to be a suitable way to describe the known evolution processes of a number of practical situations. The area of impulsive differential equations has been the subject of numerous excellent monographs. In the field of applied sciences, differential equations are used to represent a wide variety of phenomena. Many physical phenomena, however, such as mechanical systems with impact, biological systems like heart beats, blood flows, population dynamics, theoretical physics, radiophysics, pharmacokinetics, mathematical economy, chemical technology, electric technology, metallurgy, ecology, industrial robotics, biotechnology processes, chemistry, engineering, control theory, medicine, and so on, present a very different picture. Systems of differential equations with impulses are suitable mathematical models for such phenomena.

The study of impulsive differential equations is a relatively recent but crucial development in the field of differential equations. The hypothesis was initially published in 1960 and 1963 by A. D. Mishkis and V. D. Mil'man. Major advancements in this idea have been made in recent decades. In spite of its significance, the theory has progressed slowly because of the unique properties of impulsive differential equations in general, such as pulse phenomena, confluence, and loss of autonomy. Many books and articles deal with impulses in first and second order ordinary differential equations.

There are three parts to defining an impulsive differential equation. a continuous-time differential equation that regulates the system's state between impulses; an impulse equation that models an impulsive jump defined by a jump function at the instant an impulse occurs; and a jump criterion that specifies the set of jump events for which the impulse equation applies. This equation may be written out as

$$x'(t) = f\big(t, x(t)\big), t \neq \tau_k, t \in J,$$

$$\Delta x(\tau_k) = I_k(x(\tau_k)), k = 1,2, \ldots, m,$$

III. HISTORY OF IMPULSIVE DIFFERENTIAL EQUATIONS

Differential equations (DEs) having discontinuous jumps in state, known as impulsive differential equations (IDEs), have found use in a wide variety of scientific disciplines. These kinds of systems were originally used in the engineering field in the early 20th century, along with the Dirac delta distribution. As the impulse effect is now often treated synthetically rather than analytically, the delta distribution is not mentioned in detail in most current monographs on impulsive differential equations (i.e. in the distributional sense). With his focus on dynamical explanations of pulse-frequency modulation, Pavlidis surely provided the impetus for the theory's current (synthetic) form. For the current topic, he appears to have been the first to abstractly examine systems with discontinuities, defining the trajectories in terms of transition operators or jump maps rather than analytical formulations like the Dirac delta function. Discontinuous flows, which may be understood as flows (in the sense of dynamical systems) that exhibit discontinuities in their paths, now encompass Pavlidis's work. You can learn the basics of differential equations with impulse effects by reading the monographs.

In our opinion, impulsive differential equations are typically used in mathematical modelling to simplify a more involved model. The following is how Agarwaal and Leela, in their assessment of V. Lakshmikantham's (one of the pioneers of impulsive system theory) publications, explain the rationale behind these equations.

This sudden transition from one state to another is a hallmark of many evolution processes. This is because of temporary fluctuations that have a miniscule impact throughout the course of the procedure. Thus, it is reasonable to suppose that these disturbances take the form of instantaneous impulses.

Samoilenko and Perestyuk provide still another citation for the assumptions: "to represent mathematically a development of a genuine process with a short-term disturbance, it is sometimes beneficial to overlook the length of the perturbation and to assume these perturbations to be 'instantaneous.'"

Of course, one of the presumptions of modelling using impulsive differential equations is that the time scale of the perturbation is short relative to the time scale of the underlying dynamics. Yet, this raises the issue of how much space there should be between them.

Nonetheless, it may be claimed that in certain cases, modelling disturbances by discontinuities more faithfully represents reality. As an example, Liu and Chen take into account a predator-prey model with a Holling type II functional response and periodic impulsive introduction of the predator. For two reasons, they prefer to add predator all at once rather than adding at regular intervals. To begin, the analysis is made more difficult by the presence of constant periodic forcing. The opposing theory posits that a discontinuous rather than a continuous introduction of predators is more in line with reality. Given that, in practice, it is impossible to release a negligible number of a given creature into a new ecosystem, we agree that this holds true at least for low predator densities. If the number of additional predators to be introduced is substantial, however, a realistic model of predator introduction would require a series of precisely timed shocks. An argument might be made that a continuous perturbation of predator ingress at this stage is a good approximation, especially if it takes a significant amount of time for the population to become well-mixed.

As the above scenario suggests, there may be a modeling option to consider. A discontinuity provides a more realistic modeling of the introduction of a negligible amount of predator in the context given. A continuous time-dependent perturbation may be a better approximation for a more advanced predator introduction. Nonetheless, unlike continuous time-dependent systems, impulsive systems may be more amenable to the application of certain analytic approaches. In particular, determining the local stability of periodic orbits for autonomous impulsive differential equations is as simple as finding the eigenvalues associated with the simplest solution of a linear time-dependent system. In contrast, stability is a major issue for continuous hybrid systems subject to time-dependent disturbances (which, in certain cases, can be reduced to autonomous impulsive DEs given acceptable timeframe separation assumptions).

Differential equations with discontinuous solutions may be preferred over those with continuous solutions and discontinuous perturbations for a number of reasons. Nonetheless, it may not be obvious which approach provides the most accurate description of reality. As was previously indicated, a necessary condition for the employment of impulses to simplify perturbative behavior is that the perturbations in question occur on a relatively short time scale. We seek to identify the circumstances under which impulsive systems and continuous systems that are similar to them (in a way that will be specified later) display the same qualitative behavior.

IV. COMMON FORM OF IMPULSIVE SYSTEMS DIFFERENTIAL EQUATIONS

To simplify, the impulsive equations can be broken down into three sections:

- The differentiable portion of a solution is typically described by a differential equation, which has the form

$$\frac{dx}{dt} = f(t, x),$$

In the case when f is a continuous function across some set of values for t and x

- Need for reliably pinpointing the instantaneous onset of impulse influence on the solution. An ideal scenario would have the spontaneous moments predetermined. The impulsive moments are successive solutions of the equation in the general and more difficult case.

$$g(t, x(t)) = 0,$$

In this case, x= x(t) is a solution of a system of differential equations, and the corresponding function is referred to as switching, which is defined and continuous in the extended phase (or solely in the phase) space of differential equations.

- The impulsive function specifies the size and direction of the impulsive effect:

$$x(t_i + 0) = x(t_i) + I\big(x(t_i)\big), i = 1,2, \dots$$

Function In the phase space of the considered impulsive system, I is well-defined and continuous, earning the name "impulsive." The impulsive moments $t_i, i = 1,2, \dots,$ are solutions of the previous equation.

V. CASE STUDIES OF APPLICABILITY OF IMPULSIVE DIFFERENTIAL EQUATIONS

We point out some of the problems with using impulsive differential equations as models and show how our goal, even if achieved only partially, can improve the theory.

Example 1

The SIR model with pulse vaccination and permanent immunity is one of the traditional models of infectious illness at the population level that integrate impulse effects. Pulse vaccination involves vaccinating a subset of the population against a disease at staggered intervals, with p ∈ (0, 1). To clarify, let's say that S represents the percentage of people who are vulnerable.

$$\dot{S} = m - (\beta I + m)S, \quad t \neq \tau_k$$

$$\Delta S = -pS, \quad t = \tau_k,$$

When I is the infected population percentage, 1/m is the average lifespan, and τk is the time interval between successive vaccination pulses, depicts the dynamics of this proportion through time. Pulse vaccination effectively "eliminates" those who are vulnerable from the population. Effective rescaling of the birth rate is used to model ongoing immunisation, in contrast to.

$$\dot{S} = (1-p)m - (\beta I + m)S.$$

In practise, it is not practical to vaccinate a non-trivial fraction of the vulnerable population all at once, making pulsed vaccination an approximation. If there are fewer vaccine administrators than there are individuals who need to be vaccinated, we clearly have a problem. The impulsive pulse vaccination technique, on the other hand, may be understood as a time-scale separation approximation of a system with rapid, perturbative vaccination.

$$\dot{S} = m - (\beta I + m)S. \quad t \notin (\tau_k, \tau_k + d)$$

$$\dot{S} = m - (\beta I + m)S - f_k(t, S(\tau_k)), \quad t \in (\tau_k, \tau_k + d)$$

where d is the length of time the disturbance lasts and fk(t, S) is the vaccination rate for a given input fraction S of unvaccinated susceptible, satisfying

$$\int_{\tau_k}^{\tau_k+d} f_k(t, S(\tau_k))dt = pS(\tau_k).$$

Example 2

The ordinary differential equation may simulate harvesting of a single population displaying logistic growth.

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - Ex$$

where r is the natural rate of increase, K is the sustainable yield, and E is the (constant) harvesting input. Fishermen don't fish for twenty-four hours straight, and they fish net by net, therefore the assumption of continuous harvesting may be unrealistic, as pointed out by Zhang et al. After much deliberation, they decide that it is reasonable to mimic the harvesting process using brief, incidental changes. They go on to use impulsive differential equations to characterize the harvesting phase of a logistic growth model they develop.

Yet, this sacrifices reality as well. A continuous, continual harvesting E is impossible because fishermen do not fish constantly; similarly, impulsive harvesting is unrealistic because fishermen do not always fish at the same moment. Even in the extremely unlikely case when there are few harvesting agents (as in a small farm), the impulse assumption still fails to take into account the population dynamics that occur during harvesting. This might not be too much of a challenge with the help of the logistic equation (which is well known and understood). However this is not always the case, especially if the underlying dynamics are chaotic, as in forced predator-prey models, the Duffing oscillator, and the Lorentz attractor. It may be impossible to ignore these short-term dynamics if there is competition for the harvested population (as in a predator-prey paradigm with external harvesting by humans, for instance).

As a result, it is prudent to think about the potential outcomes of switching from impulsive to rapid, perturbative harvesting. Any formulation that yields analytically tractable findings may be utilised if the dynamics are analogous to the impulsive case. We may use any formulation we choose, and the only variation will be in the quantitative parts of the results, but this corresponds to a decrease of bias in the qualitative elements.

VI. CONCLUSION

Examining the findings of alternative numerical approaches can help enhance the precision of the results. Specifically, we suggested a generic numerical approach for handling the impulsive differential equations at discrete times. Several studies have attempted to numerically resolve the impulsive differential equations. Because higher accuracy may be gained if nodes are more frequent on those sections of t-axis on which the state of the process changes more rapidly, it is interesting, for example, to explore the effect of nodes (i.e. the technique of splitting the segments to the sub-segments) to accuracy of results. Hence, further investigations are needed to improve and validate the current findings.

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