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THE INTEGRO-EXPONENTIAL FUNCTION IN ITS GENERALIZED FORM

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ABSTRACT

Exponent integral (incomplete Gamma) and order derivatives define General integrated function exponential. A section contains a collection of analytical results. Sufficient rational minimum approximations are provided in another section to calculate the first six first order functions.This article presents an extension of the distribution of power mouth (PM) that models positive Kurtosis data. The distribution is higher than the PM distribution in the resultant distribution. It can be shown using the incomplete generalized whole-exponential feature that the density can be expressed. We examine some of its characteristics and moments, asymmetry and kurtosis coefficients. We use the moments and the maximum probability techniques to estimate the parameter recovery and provide a simulation analysis. Application results for two real data sets show that in the presence of outliers the new model is extremely good.

Keywords: Generalized Integro-exponential function; Exponential function; Generalization; Laplace Transform.

INTRODUCTION

The universal integral Exponential function is of crucial importance in transportation systems and fluid flow theory, but in the literature, there is not a comprehensive description of its characteristics. First presented by Van de HülstChandrasekhar has summarized the function and recently examined it by Van de Hülst, who has provided power and numerical taboos for specific situations.

The slash distribution is a symmetrical extension of the normal distribution. The quotient is a uniform distributional power between two independent random variables, one normal and one. Thus, if its representation is, X has a slash distribution:

$X = \frac{Y}{U^{\frac{1}{q}}}$,

Where Y ∼N(0, 1), U ∼ U(0, 1) and Y is independent of U and q > 0.

It has heavier tails, i.e. a bigger Kurtose than the regular distribution. The properties are explored in Rogers and Tukey, Mosteller and Tukey. Kafadar explores the maximum probability (ML) estimates. Gómez et al. and Gómez and Venegas increased the distribution by adding the elliptical slash family. To extend Birnbaum – Saunders distribution, Gómez et al. used the slash elliptical family. Olmos et al. used the slash method in semi-normal and usually semi-normal distributions; Reyes et al. in the Birnbaum–Saunders distribution; Gomez et al. in the Gunbel distribution; and Segovia et al. in the Maxwell power distribution, among others.

Muth suggested continuous probability distribution via the idea of reliability; and Jodrá et al. A random variable Y is called an α-form distribution when the function of probability density is specified by

$$f(y; \alpha) = (e^{\alpha y} - \alpha) \exp\{\alpha y - \frac{1}{\alpha}(e^{\alpha y} - 1)\},\ y > 0,$$

which we denote by Y ∼ M(α).

The generalized integral-exponential function is a key function in the M distribution, expressed in the following integral representation:

$$E_S{}^{m}(t) = \frac{1}{T(m+1)} \int_1^{\infty} (\log u)^{m} e^{-tu} u^{-s}\, du,\ t \in (-\infty, \infty),$$

Where, s ∈ (−∞, ∞), m > −1 and Γ(·) is the gamma function.

Jodrá et al. have created a M model modification dubbed PM that fixes the form parameter alpha = 1 in the M model. In this research they add an α-form parameter, making the PM model more flexible than the α-parameter. They generate a model with two parameters which we take into account following. A random X variable has a β-and-shape PM-distribution when its pdf is given

$$f(x; \beta, \gamma) = \frac{\Upsilon}{\beta^{\Upsilon}} x^{\Upsilon-1} \left(e^{\left(\frac{x}{\beta}\right)^{\Upsilon}} - 1\right) \exp\left\{\left(\frac{x}{\beta}\right)^{\Upsilon} - e^{\left(\frac{x}{\beta}\right)^{\Upsilon}} + 1\right\},\ x > 0 \quad \textbf{(1)}$$

Which we denote by X ~PM(β, γ)

Let X ~PM(β, γ), some properties of this distribution are:

1. $F_X(x; \beta, \gamma) = 1 - \exp\left\{\left(\frac{x}{\beta}\right)^{\gamma} - e^{\left(\frac{x}{\beta}\right)^{\gamma}} + 1\right\}$, x > 0

2. $Q(p) = \beta\left(\log\left\{-W_{-1}\left(\frac{u-1}{e}\right)\right\}\right)^{1/\gamma}$, 0 < u < 1,

3. $E(X^r) = e\beta^r\, T\left(\frac{r}{\gamma} + 1\right) E_0^{\frac{r}{\gamma}-1}(1)$, r = 1, 2, . . .

Where F(•) is a cumulative function for X distribution, Q(•) is a quantile function, and W−1 indicates the Lambert W function's negative branch and is described above as the generic integral-exponential function.

The incomplete generalized integral exponential function is defined for future advancements

$$E_S{}^m(t;y) = \frac{1}{T(m+1)} \int_1^y (\log u)^m e^{-tu} u^{-s}\, du, \quad t \in (-\infty, \infty), \qquad (2)$$

Where $\lim_{y\to\infty} E_S{}^m(t; y) = E_S{}^m(t)$.

The main aim of this paper is to examine a larger range of the PM model in its Kurtosis coefficient so that this new distribution may be used in the models and atypical observations of data sets.

LITERATURE REVIEW

Serdar Beji (2021) A fast-converging power series initially designed to solve Grandi's dilemma is used to assess the exponential component of real arguments. The historical solution of Laguerre is first summarized and then the new solution technique is detailed. The numerical results from the current series solution are compared to the accurate tabular values at nine decimal places. Finally, remarks are given on the further application of this technique to integrals with certain functions in the denominator.

SupapornKaewta ,SeksonSirisubtawee and SurattanaSungnul (2021) The main aim of this article is to provide precise route wavelength solutions for Kadomtsev–Petviashvili (KP) exp-function compatible Time 2 integrodifferential hierarchy and (2 + 1) compatible time partial integro-differential Evolution of Jaulent–Miodek (JM) with the Kudryashov technique in general. Both of these issues require the compatible time partial derivative. Initially, a fractional complex transformation allows conforming partial temporal differential equations to be Transformed into non-linear common equations of differences. The resultant equations are then analytically resolved using the appropriate techniques.

Juan M. Astorga, Jimmy Reyes, Karol I. Santoro, Osvaldo Venegas, Orcid andHéctor W. Gómez (2020) this paper presents extended power mouth distribution(PM) for the modeling of high kurtosis value positive data sets. Kurtosis is greater than PM in the resulting distribution. We show that a general integral-exponentially incomplete function may be used as a means to characterize the density. We analyze some of its features, moments and coefficients for asymmetry and Kurtosis. We use moments and maximum probability techniques and provide a simulation exercise to demonstrate the recovery of the parameters. The results of the application in two real data sets indicate that the new model is very good for outlines.

Yudhveer Singh et al. (2020) In this paper we use a comprehensive transforming method to resolve the integrodifferential (FVIDE) Volterra-type fractional order involving the generalized in terms of complex kernel variables, Lorenzo-Hartely function and the generalized Lauricella hypergeometric confluent function. The Lorenzo-hardly generalized helpers' derivatives and the hypergeometric confluent function of Lauricelle is also studied and presented. Three discoveries in this Article were recognized as lemmas which provide us new results in the three functions referred to above, and utilizing these results we have derived our major conclusions in the form of theorems. Our primary findings are extremely broad, giving some fresh and existing findings here as a particular example of the results.

Tibor K. Pogány, Gauss M. Cordeiro, M. H. Tahir, Hari Mohan Srivastava (2017) In 2000 Chen developed a two-parameter model and provided just moments, quantities, and functions for producing mathematical properties, among other things. In this paper, we provide an extension to the power series for the newly presented generalized integral/exponential function E ps (z), which extends some Milgram results. By our new findings, Moments, function generation, Renyi entropy and Chen quantile function power series are expressed in a closed form.

EXPONENTIAL FUNCTION

Exponential functions in mathematics are the relationship of the shape $y = a^x$, and the independent Variable x is an exponent for a positive integer across the whole real number line. Probably y = ex, commonly written y = exp(x), is the most important exponential function; e (2,7182818...) is the foundation of natural logarithmic systems (ln). x is definitely the logarithm and thus the logarithmic function differs from the exponential function (Figure 1).

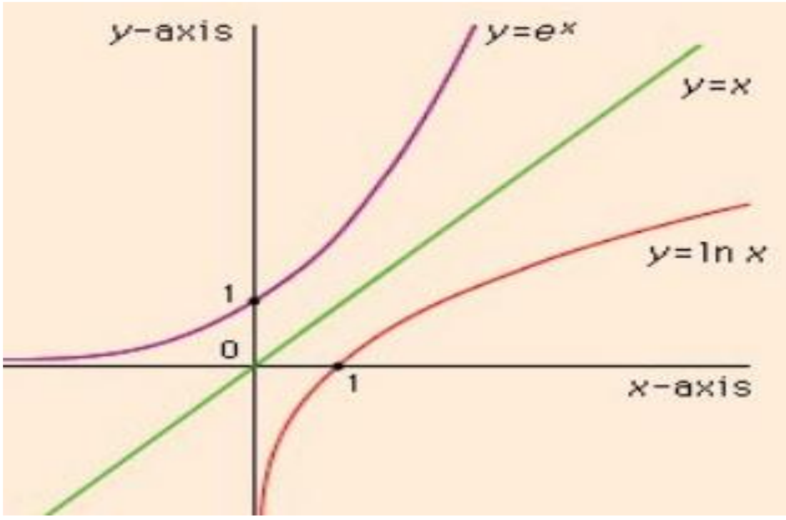

Figure 1 The exponential and natural logarithm functions

If y = e^x, then x = ln y, in particular. Also, the exponential feature is the sum of the infinite series

$e^{\pm x} = 1 \pm x + \frac{x^2}{2!} \pm \frac{x^3}{3!} + \frac{x^4}{4!} \pm \frac{x^5}{5!} + \ldots\ldots ,$

This is a product of the first n positive integer for every x and n! So the constant in particular

$e^1 = 2.7182818$

$= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!} + \ldots\ldots ,$

Exhibited functions contain examples of non-algebraic or transcendental functions – functions not representable as a Product, sum and variable differences which are raised to specified non-negative integral power. Also common transcendental functions are logarithmic and trigonometric functions. Exponential functions often occur and quantitatively explain some physical processes, such as radioactive decay where the rate of change is directly reliant on its current value in a process or substance.

The exponential integral Ei(x) is defined as true nonzero values of x

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t}\, dt = \int_{-x}^{\infty} \frac{e^{t}}{t}\, dt.$$

The Risch method shows that Ei is not a fundamental function. For positive values of x, the following formula may be applied, however due to the uniqueness of the integer at nil, the integral must be interpreted in the Cauchy main value.

For complex values, because of branch points at 0 and ∞, the definition becomes confusing.The notation below is used instead of Ei,

$$E_1(z) = \int_{z}^{\infty} \frac{e^{-t}}{t} dt, \quad |Arg(z)| < \pi$$

For positive values of x, we have $-E_1(x) = \mathrm{Ei}(-x)$

In general, a branch is severed across the negative real axis and E1 somewhere on the complex plane may be determined by analytical continuation.

For real portion positive values of z, this can be written

$$E_1(z) = \int_{1}^{\infty} \frac{e^{-tz}}{t}\, dt = \int_{0}^{1} \frac{e^{-z/u}}{u} du, \quad \Re(z) \geq 0.$$

The following relationship may be observed in E1's branch cutting behavior:

$$\lim_{\delta \to 0+} E_1(-x \pm i\delta) = -\mathrm{Ei}(x) \mp i\pi, \quad x<0.$$

GENERALIZATION

It may also be the exponential integral widespread $\boldsymbol{E_n}(x) = \int_{1}^{\infty} \frac{e^{-xt}}{\boldsymbol{t^n}} dt$

that may be represented as an incomplete gamma function as a particular instance:

$\boldsymbol{E_n}(x) = x^{n-1}\Gamma\,(1- n, x)$.

The wide-spread Misra Function is frequently termed form $\varphi_{m(x)}$,

$$\varphi_{m(x)} = E_{-m(x)}.$$

The NIST mathematical digital library has many characteristics of this extended form.

The generalized integral-exponential function is defined by a logarithm

$$E^j{}_s(z) = \frac{1}{T(j+1)} \int_1^{\infty} (log\ t^j) \frac{e^{-zt}}{t^s} dt.$$

The indefinite component:

Ei (a.b) = $\iint e^{ab}\ da\ db$

is like the usual generating function, the number of dividers of n:

$$\sum_{n=1}^{\infty} d(n)x^n = \sum_{a=1}^{\infty}\sum_{b=1}^{\infty} x^{ab}$$

GENERALIZED INTEGRO–EXPONENTIAL FUNCTION

Our primary objective in this section is to take full account of the generic integrated exponential function with higher p parameter

$$E_S{}^p\ (z) = \frac{1}{T(p+1)} \int_1^{\infty} (\ln x)\ ^p x^{-s} e^{-zx}\ \ dx, \Re(p) > -1; s, z \in C(2)$$

The resulting power series form of that integral, which meets the upper parameter, provides the key instrument to provide Chen's distributed RV T with moments and incomplete moments of positive real order. By this finding we significantly expand the results of the current Milgram for p / N (for which compare and the references there in).

$$\Phi_{?,\nu}{}^{(\rho,\sigma)}(z,s,u) = \sum_{n\geq 0} \frac{(\mu)\rho n}{(\nu)\sigma n} \frac{z^n}{(n+u)^s},$$

Where $\mu \in C$, ν, $u \in C \setminus Z_0^-$ $\rho, \sigma \in R_+$ and $\rho < \sigma$ when $z, s \in C$

$$(\lambda)_n = \frac{\Gamma(\lambda + \eta)}{\Gamma(\lambda)}, \lambda \in C \setminus \{0\}$$

represents the generalized symbol of Pochhammer by convention $(0)_0 = 1$. Thus,]

$$\Phi_{?,1}{}^{(0,1)}(-a, p + 1, 1) = \lim_{y\to\infty} \Phi_{?,1}{}^{(\rho,1)} (-a, p + 1, 1) = \sum_{n\geq 0} \frac{(-a)^n}{n!(n+1)^{p+1}} (4)$$

Next, we determine the power series of $E_S{}^p$ (z). By setting $(1 - x)^{-1} \to x$ in the integrand of (3), we obtain

$$E_S{}^p\,(z)= \frac{(-1)^p}{\Gamma(p+1)} \int_0^1 \left(1 - x\right)^{-s-2} \ln^p(1-x).\exp\left\{-\frac{z}{1-x}\right\} dx.$$

By expanding the exponential and the binomial terms and interchange the sums and the integral it follows

$$E_S{}^p\,(z)= = \frac{(-1)^p}{\Gamma(p+1)} \sum_{n,k\geq 0} \frac{(-1)^k\,(-Z)^n}{n!} \binom{n-s-2}{k} \underbrace{\int_0^1 \ln^p(1-x)\,dx.}_{J_k(p)}$$

The inner integral Jk(p) can be handled by substituting $1 - e^{-x} \leftrightarrow x$:

$$J_k(p) = \int_0^\infty \left(1 - e^{-x}\right)^k e^{-x}(-x^p)dx = (-1)^p \sum_{m=0}^k (-1)^m \binom{k}{m} \int_0^\infty e^{-(m+1)x}\, x^p\, dx$$

$$= (-1)^p \sum_{m=0}^k \binom{k}{m} \frac{(-1)^m \mathrm{T}(p+1)}{(m+1)^{p+1}} = (-1)^p \sum_{m=0}^k \binom{k}{m} \frac{\mathrm{T}(p+1)(-k)_m}{m!(m+1)^{p+1}}, \quad \textbf{(5)}$$

When both amounts in (5) can be utilized for accurate numerical assessment. We also arrive at the more compact textual form in the final equation.

$$J_k(p) = (-1)^p\ \mathrm{T}(p+1)\Phi_{?,1}{}^{(0,1)}(-k, p+1, 1), \qquad ? \in \mathrm{C},$$

Where $\Phi_{?,1}{}^{(0,1)}(-k, p+1, 1)$ is the k–partial sum of the HL Zeta function (6).

Remark 1. The situation of a non-negative integer p for $J_k(p)$ is well established. The formula (converted into our environment)

$$\left(\frac{d}{dx}\right)^p \frac{1}{(x+1)_{k-1}} = \frac{(-1)^p p!}{k!} \sum_{m=0}^k \frac{(-k)_m}{m!(m+x+1)^{p+1}} \ , \ p \in N_0$$

The progenitor of the Hungarian Probabilistic School Charles (K'aroly) Jordan appears in the monograph.

The relation (6) leads to

$$E_S{}^p\,(z)= \sum_{n,k\geq 0} \frac{(-1)^k\,(-Z)^n}{n!} \binom{-n-s-2}{k} \Phi_{?,1}{}^{(0,1)}(-k, p+1, 1)$$

$$= \sum_{n,k\geq 0} \frac{(s+2)_{n+k}}{k!\,(s+2)_k} \frac{(-Z)^n}{n!} \Phi_{?,1}{}^{(0,1)}(-k, p+1, 1)$$

$$= \sum_{k\geq 0} \frac{(s+2)_k}{k!} \Phi_{?,1}{}^{(0,1)}(-k, p+1, 1) \sum_{n,\geq 0} \frac{(s+2+k)_n}{(s+2)_n} \frac{(-Z)^n}{n!}$$

$$= \sum_{k\geq 0} \frac{(s+2)_k}{k!} \Phi_{?,1}{}^{(0,1)}(-k, p+1, 1),\ {}_1F_1(s+k+2; s+2; -z),$$

Where the hypergeometric confluent function - Kummer function.

$${}_1F_1(a; b; x) = \sum_{n,\geq 0} \frac{(a)_n}{(b)_n} \frac{x^n}{n!},\ x, a \in C; b \in C \setminus Z - 0$$

is employed. Hence the desired result.]

Theorem 1. Forall $\Re(p) > -1$, s, z ∈ C, The expansion equations of the generalized integral-exponential function in three power series remain valid.

$$E_S{}^p(z) = \sum_{k\geq 0} \frac{(s+2)_k}{k!} \Phi_{?,1}{}^{(0,1)}(-k, p+1, 1)\, {}_1F_1(s+k+2; s+2; -z).$$

Remark 2. We point out that the closed form expression of $E_S{}^p$ (z), for a non-negative integer p the Meijer G function is given by Milgram; see also the appropriate formulas in. However, findings for general real positive p have not been reported to the best of our knowledge.

LAPLACE TRANSFORM OF GENERALIZED EXPONENTIAL INTEGRALS

The transformation of the E_v(x) Laplace is defined in this section

$$Na\ \{E_v\ (x)\} \equiv a \int_0^\infty e^{-ax}\ E_v(x)dx = a \int_0^\infty dx \int_0^\infty dt\ t^{-v}\ e^{-(t+a)x}. \quad \textbf{(7)}$$

According to Milgram's paper, in the case $|a| < 1$, one obtains:

$$Na\ \{E_v\ (x)\} = -\sum_{n=0}^\infty \frac{(-a)^{n+1}}{v+n}, \qquad v \in R, |a| \leq 1. \quad \textbf{(8)}$$

If $a > -1$, the integration order in (6) may be changed and the results can be achieved:

$$Na\ \{E_v\ (x)\} = a \int_0^\infty \frac{dt}{t^v} \int_0^\infty e^{-(t+a)x}\, dx = a \int_0^\infty \frac{dt}{t^v\,(a+t)}, \quad a > -1. \quad \textbf{(9)}$$

Using the basic identity:

$$\frac{1}{t\,(t+a)} = \frac{1}{a}\left(\frac{1}{t} - \frac{1}{t+a}\right), \quad \textbf{(10)}$$

The Laplace transformation of E_v leads in a recurrence relation (x):

Na $\{E_{v+1}$ (x)$\}=\frac{1}{v}-\frac{1}{a}$ Na $\{E_v$ (x)$\}$, v ≥1, a> -1, **(11)**

Extending the finding to v = n integer

It is also possible to get equation (11) in the transformation of both recurrence members. Besides Equation (8), the following equation may be easily derived.

Na $\{E_v$ (x)$\}$, v=m+$\boldsymbol{\alpha}$, m ∈ N_0,0≤ a < 1:

Na $\{E_v$ (x)$\}$ = $\left(-\frac{1}{a}\right)^{m-1}$ In (1+ a) + $\left(-\frac{1}{a}\right)^{m-1}\left[\sum_{k=1}^{m-1}\frac{(-a)^k}{k+\propto}+\propto\sum_{j=1}^{\infty}\frac{(-a)^j}{j\,(j+\propto)}\right]$, -1<$a$ ≤1, **(12)**

which, for v integer, reduces to the result. In particularNa $\{E_1$ (x)$\}$ = In (1+ a). Equation (12) Can be acquired accordingly. Putting v=m+$\boldsymbol{\alpha}$ into Equation (8), one gets

Na $\{E_v$ (x)$\}$ $=a\sum_{n=0}^{\infty}\frac{(-a)^n}{n+m+\alpha}=-\left(-\frac{1}{a}\right)^{m-1}\sum_{j=m}^{\infty}\frac{(-a)^j}{j+\propto},|a|\leq 1.$ **(13)**

where j = m + n.

Furthermore, considering:

In (1+ a) $=-\sum_{j=1}^{\infty}\frac{(-a)^j}{j},$ -1< a ≤1, (14)

It follows that using this term in equation (13):

Na $\{E_v$ (x)$\}$ = $\left(-\frac{1}{a}\right)^{m-1}\left[-\sum_{j=1}^{\infty}\frac{(-a)^j}{j+\propto}+\sum_{j=1}^{\infty}\frac{(-a)^j}{j}+\text{In}\,(1+a)+\sum_{j=1}^{m-1}\frac{(-a)^j}{j\,(j+\propto)}\right]$,-1< a ≤1 (15)

After the initial equation series (15), the identity at that point was introduced

$$\frac{1}{a+j}=\frac{1}{j}-\frac{\alpha}{j\,(j+a)},$$

You may recover easily equation (12).

The following relationship also applies to equation (12):

$Na\,\{E_{m+a}\ (\mathrm{x})\} \;=\; Na\,\{E_m\ (\mathrm{x})\}+\frac{\propto}{(-a)^{m-1}}\sum_{j=m}^{\infty}\frac{(-a)^j}{j\,(j+\propto)}$, -1< a ≤1, m ∈ N_0,0≤ a < 1(16)

Which the phrase might be derived$Na\{E_{m+a}$(x)}and for$Na\ \{E_m$ (x)}, equation (12) provided and equation considered (16).

Lastly, it should be noted that the phrase (12) is more computational than equation (8). In addition, it is totally convergent since the Abel series raises the suitable absolute value series. The applicable equation series is converging faster than the Equation series (12).

$$\psi\,(1) = \sum_{k=1}^{\infty} {}^{1}\!/_{k^2} = {}^{\pi^2}\!/_{6}$$

CONCLUSION

In this paper, we have deducted joint pdfs of a generalized order for the Generalized and Integro-Exponential functions in explicit form. The probability density function (pdf) of the conditions of the general order distribution generated from the distributions under discussion is also given.

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