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THE RELEVANCE OF THE BASIC FRACTIONAL CALCULUS WITH NUMERICAL METHODS

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Abstract

Fractional calculus is the study of different fractional orders-integral operators, and it is used in many engineering and scientific fields. A fractional difference makes up the newline. Only an operator with a comprehensive perspective on the common distinction. Fractional derivative newline equations, such as real or complex order differentiation, have not fully addressed the exceptional complexity of many components in some of the most diverse fields of engineering and research that depend on complex newline structures. In this paper, we present a unique numerical method for solving fractional differential equations. Given a fractional derivative of any real order, we provide an approximation method for the fractional operator using solely integer-order derivatives. As a result, we can rephrase FDEs in terms of a conventional model and then use any acceptable technique. With a few examples, we show how accurate the method.

Keywords:*Fractional Calculus,accuracy,fractional derivative*

1. INTRODUCTION

Since the inception of differential calculus, the question of what could be a derivative of a no integer order has been important, and even Leibniz considered the derivative of order = 1/2.

Following a thorough investigation, Lowville proposed the idea of a fractional integrator operator. Later, Riemann created a fractional integration and developed the science of fractional calculus by expanding on Cauchy's n-fold integral formula.In 1967, a new class of fractional operators appeared thanks to Michel Caputo, and it has since been demonstrated that these operators are effective in a variety of situations. It exhibits two advantages: when using this operator to solve fractional differential equations, the derivative of a constant is zero and ordinary initial conditions can be used instead of establishing fractional order initial conditions. FDEs have lately proven to simulate some real phenomena more successfully since these fractional operators have memory and some dynamics of trajectories are characterized by non-integer order derivatives from experimental data. As a result, they have several applications in numerous scientific domains as well as a wide range of technical uses (such as viscoelasticity, viscoelasticity, modeling polymers, transmission of ultrasonic waves, etc.). These differential equations can't be solved quickly and easily, though. As a result, the literature contains a wide variety of numerical methods that can be utilized to solve them.

In Section 2, examples from are used to briefly introduce fractional calculus. In Section 3, the study's main finding is presented and illustrated: under certain smoothness assumptions, we may approximate any real order fractional derivative by a sum that only contains integer-order derivatives. Additionally, a rough estimate of the error is given. By taking into account fractional derivatives of any real order, we further the main conclusions of. The examples in Section 4 compare the exact computation of the Caputo fractional derivative of a given function with a few numerical approximations as an early evaluation of the method's performance. We give an illustration of how solving fractional differential equations might be useful in the end.

1.1 Fractional Calculation

Nearly as old as calculus itself, fractional calculus has recently gained in popularity due to its practical applications as well as its strengths as pure mathematics. Recently, a number of textbooks on this topic were published, each of which addressed various difficulties in a unique manner. The idea of the non-integer differential and integral operators researched in the field of fractional calculus may be the most understandable, given that Cauchy's well-known definition of an n-fold integral as a convolution integral.

$$\boldsymbol{j^n y(x) = \int_0^x \int_0^{xn=1} \ldots \int_0^{x1} y(x_0)\, dx0..dx_n..2dxn - 1}$$

$$\boldsymbol{= \frac{1}{(n-1)} \int_0^x \frac{1}{(x-t)1-n} y(t)dt, n \in Nx \in R +}$$

Where J 0y(x) = y and J n is the n-fold integral operator (x). the discrete factorial (n 1) is being replaced! With Euler's continuous gamma function, (n), one can define a non-integer order integral by proving that (n 1)! = (n) for n N.

$$\boldsymbol{j^\alpha y(x) = \frac{1}{r(\alpha)} \int_0^x \frac{1}{(x-t)1-\alpha} y(t)dt, \quad \alpha x \epsilon R +}$$

Non-integer order derivatives, which are easiest characterized as the concatenation of integer order differentiation and fractional integration, are the source of several significant parts of fractional calculus.

$$D^{\alpha}y(x) = D^{n}j^{n-\alpha}y(x) or\ D^{\alpha}y(x) = j^{n-\alpha}D^{\alpha}y(x)$$

whereDn, n N, is the n-fold differential operator with D0y(x) = y and n is the number satisfying n + 1. (x). One of the challenging and gratifying parts of this mathematical discipline is that there are undoubtedly multiple ways to define non-integer order derivatives. The operator D is typically designated as Riemann-Liouville. It is clear that these derivatives are non-local operators because of the integral in the formulation of the non-integer order derivatives, which explains one of their most important applications: A non-integer derivative at a specific time or spatial location contains details about the function at previous times or locations, respectively. Thus, non-integer derivatives have a memory effect, which they share with a number of substances, including polymers or viscoelastic substances, as well as with principles used in applications like anomalous diffusion. One of the factors contributing to the increased interest in fractional calculus is also this fact: Fractional derivatives can be utilised to create straightforward material models and unifying principles due to their non-local nature. The textbook by Oldham and Spanier and the paper by Olmstead and Handelsman both provide prominent examples of diffusion processes. The classic papers by Bagley and Torvik , Caputo , and Caputo and Mainardi provide examples of modelling viscoelastic materials, and the publication by Marks and Hall discusses applications in the field of signal processing. The works of Chern ,Diethelm and Freed , Gaul, Klein, and Kemplfe, Unser and BluPodlubnyand Podlubny et al contain a number of fresh results. Several surveys that include collections of applications can also be found, for example in Gorenflo and Mainardi,Mainardi , or Podlubny.

The memory effect of fractional derivatives has a high cost in terms of numerical solvability when used to build simple material models or unified principles. Any method that uses a discretization of a non-integer derivative must, among other things, take into consideration the non-local structure of the derivative, which typically results in a large storage need and high algorithmic complexity. In the literature, there have been several attempts to resolve equations involving various kinds of non-integer order operators: Abel-Volterra integral equations can be solved using so-called collocation methods, according to several works by Brunner The non-integer order integral as previously defined serves as the integral portion in these equations. His book on the subject has these findings as well as others. For instance, Linz's book andOrsi's article both employ product integration strategies to resolve Abel-Volterra integral equations. The so-called fractional linear multistep methods are used in several works by Lubich as well as Hairer, Lubich, and Schlichte to numerically solve Abel-Volterra integral equations. Additionally, a number of works discuss numerical approaches to the solution of differential equations of fractional order. With the exception of the non-integer order of their derivatives, these equations resemble conventional differential equations. The publications by Diethelm, Ford and Simpson , Podlubny and Walzare just a few examples of approaches based on fractional formulation of backward difference methods. For instance, Diethelm et al. explore the fractional formulation of Adams-type approaches in their articles the majority of the cited concepts are introduced and developed in this thesis, with the exception of the collocation methods by Brunner and the product integration approaches by Linz and Orsi.

2. REVIEW OF LITREATURE

The introduction of a literature review process and the proper methodology required for carrying out the process chosen for evaluating the research papers are presented at the beginning of this work. This course will also include a categorical review of "Efficient Numerical Methods for Fractional Differential Equations and their Applications" by summarizing the research papers and organizing them according to the issues they address. It concentrates on the advantages and disadvantages of the discussed Area/Sub Area.**I. Podlubny et al. 1997** constructed Riemann-Lioville fractional differ-integrals of weighted Jacobi polynomials as special examples by extrapolating fractional Riesz potential connected to Jacobi polynomials. The Jacobi polynomials have certain relationships, according to authors.**Virginia Kiryakova et al. 2000**based on two sets of multiple indices, Mittag-Liffler analogues were investigated. With the introduction of generalised integral and differential operators known as the GelfondLeontiev-type, also known as the Borel-Laplace-type integral transform, the author worked on the fundamental properties as well as the relationships between the M-L functions and the operators of the fractional equations.

H. J. Glaeske et al. 2000 investigated the integral transform using a kernel that was based on a generalisation of either the Macdonald function or the modified Bessel function. With both the left and right sides Liouville fractional differ-integrals, authors looked at the integral transform's composition and kernel features.

H. M. Srivastava et al. 2010 introduced and studied a family of generalised M-L functions-related integral operator in fractional calculus. Special cases of Mittage-Leffler functions were discussed by the author. **DeshnaLoonker et al. 2012** ABoehmian space wavelet transform of the fractional integral operator. Additionally, the Gabor transform for integrableBoehmians was explored. The relationship between the Fourier and Wavelet transforms was used by the authors to generate the Gabor transform for the fractional integral operator.

Kazuhiro onodera et al. 2010 examined several sine and gamma functions' external values over a range of fundamental intervals. In order to demonstrate that all local maximum and minimum values were larger than one and less than zero, respectively, authors illustrated the quantity and location of external points.

N. Virchenko et al. 2010 Wrighthypergeometric function and the confluent hypergeometric function were introduced, along with the generalisation of Gauss hypergeometric functions. Some of its qualities were derived by him. Using Laplace and Mellin transforms, four new integral operators are defined after their derivation, and their inverse operators are also obtained.

MunmumKhanra et al. 2010 investigated a number of earlier approaches, including function Laplace, for rationally approximating fractional functions. The pros and disadvantages of variation approaches were discussed by the authors.

Jean Claude et al. 2011 contrasted the Grunewald derivative, a frequency distributed model, and a fractional integrator operator. After the author provided the frequency integrator and its integer order approximation connected to the frequency response, it was also discussed different integral and integrators that are equivalent and organised the Riemann Liouville integral as well as the corresponding fractional derivatives.

3. GENERALIZED FRACTIONAL INTEGRAL OPERATOR

The investigation of a novel contemplate fractional integral operator incorporating K4 mapping is the focus of this paper . Along with the research of boundedness and the construction of new composite features connected to the operator, the Mellin and Laplace transforms for the suggested generalised operator are also examined. To solve the fractional derivative equation using Hilfer differentiations and K4 mapping, the acquired findings are put to use. The K4 mapping is a generalisation of the M-series, and the accuracy of the conclusions depends on how precisely the earlier-mentioned findings of our inquiry were followed. Applications for generalised special functions in applied sciences, engineering, and technology have lately been discovered. To "confirm the results," many corollaries and lemmas are established.

3.1 Introduction and Preliminaries

Kilbas researched the characteristics and a number of adjunctions of various fractional integral and differential operators as a result of the widespread use of fractional calculus. As introduced by Sharma , the K4 function is as follows:

$$\sum_{n=0}^{\alpha} \frac{k^{(a,B,y)}(a_1)n \ldots (a_p)n(y)nc)n}{(b_1)n \ldots (b_q)n \quad (n)\ r(n+y)\alpha-\beta)}$$

Notation in Pochhammer. If any number variable I an is zero or a negative integer, the series transforms to a polynomial in variable x, and the following equation (3.1) is only true when none of the variables j b are zero or integer (negative). If q 1 p, the series is convergent.

If we change, 1, 1 1 c in (3.1), we get the following outcome:

$$\boldsymbol{K(a, a-\beta_1)(1,0)(p,q)(a_1 \ldots a_p . b_1 \ldots b_q : x)}$$
$$\boldsymbol{= x^{\beta-1} M(a_1, a_2, .., a_p b_1, b_{2,..} b_q x^a)}$$

where the well-known generalized M-series, studied by Sharma and Jain [97], is designated as a power series and 1 p Mq.

$$\boldsymbol{M(a_1, a_{2,\ldots,} a_p b_1, b_2, \ldots b_q) = M(z) = {}^{a}_{p} m(a_i)(b_j)^{q}_{1}, . z)}$$
$$= \sum_{n=0}^{\alpha} \frac{(a_1)n \ldots (a_p)n}{(b_1)n \ldots (b_p)n} \frac{Z^n}{r(\alpha n+\beta)} (Z, a, \beta, \varepsilon c R(a) > 0)$$

where the recognisedPochhammer symbols are n I n j a, b. A polynomial in z replaces the series given in (3.3) if any numerator variable, I a, is zero or a negative integer. The series given in (3.3) is characterised as only when none of the variables, b s j q j ', 1, 2,...., are. If p q, the equation (3.3) is confluent for all variables with z, concurrent for all variables with z, and divergent if p q. Equation will converge on conditions depending on variable values when p = q + 1 and z .

The generalisedMittag-Leffler mapping was first presented by Prabhakar [98] and can be found in (3.3) for ; 1 ; 1 a1 C p q b1, as

$$E^{y}_{a,\beta}(z) = \sum_{m=0}^{\infty} \frac{(\gamma)_m}{(1)_m} \frac{Z^m}{r(ma+\beta_1)} \frac{z^m}{m} = \sum_{m=0}^{\infty} \frac{(\gamma)_m}{(1)_m} \frac{Z^m}{r(ma+\beta_1)} = M(Y; 1; z)$$

It is possible to identify the induced M-series represented by (3.3) as a specific instance of the Wright generalized hyper geometric function and the Fox H-function as

$$M\left(\left(a_1\right)_1^p\left(b_j\right)_1^q; z\right) = k_{p+1}\psi_{q+1}\begin{bmatrix} (a_1 1) & (a_{p}, .1) \\ (b_1, .1) & (1, b_j 1) \end{bmatrix}$$

Let's go through some essential definitions related to fractional calculus. Let x: [a, b] R be a function, n N be such that (n 1, n), and R be a positive non-integer number. The following makes the assumption that x is good enough for the fractional operators to have a clear definition. The definition of the left and right Riemann-Liouville fractional integrals of order generalizes the Cauchy's formula to any real number.

$$a_{I_t^a} x(t) = \frac{1}{r(\propto)} \int_a^t (t-r)^{\alpha-1} x(r)\, dr$$

Respectively. We look at two different sorts of operators for fractional derivatives. The fractional derivatives of the left and right Riemann-Liouville sides are given by

$$D_t^a x(t) = \frac{1}{r(n-a)} \frac{d^n}{dt^n} \int_a^t (t-r)^{n-a-1} x(r) dr$$

4. EFFICIENT NUMERICAL METHODS AND THEIR APPLICATIONS

The goal of this chapter is to examine several effective numerical techniques and show how they may be used to solve a variety of time-fractional order partial differential equations that are both linear and nonlinear. The fractional differentiations are used in Caputo's interpretation. Several applications are used to show the efficacy of the suggested numerical methods. The outcomes exactly correspond to the outcomes for classical orders of derivatives found in literature. The paper introduction of numerical approaches is proven to be effective and simple to use.

4.1 Power Series Method

The solutions to fractional differential equations are in series form since using fractional operators in calculations can be challenging. Therefore, one of the effective methods to produce series solutions when fractional derivatives arise in an equation is the fractional power series approach. provide a power series approach to solve fractional partial differential equations.The following is the power series form (PSM):

$$\sum_{n=0}^{\infty} c_n(\varepsilon-\varepsilon_0)^a + c_2(\varepsilon-\varepsilon_0)^{2a} + c_3(\epsilon-\varepsilon_0)^{3a} +$$

The series transitions to a fractional Maclaurin series when 1 0, where 0 m 1 m and n c are constants. A fractional power series if$\sum_{n=0}^{\infty} c_n\,(\varepsilon - \varepsilon_0)^{na}$Whenever is larger than or equal to zero but less than b, where b is a positive number, convergence occurs. If the fractional power series continues,$\sum_{n=0}^{\infty} c_n\,(\varepsilon - \varepsilon_0)^{na}$diverges when d is positive and is greater than d.

4.2 Modified Variation Iteration Method (Mvim)

Guo-Cheng Wu and DumitruBaleanu suggested a novel variant of the variational iteration method employing the Laplace transform and Lagrange multipliers. When different nonlinear fractional derivative problems emerge in mathematical physics and other related fields, the Lagrange multiplier approaches have been frequently applied. He developed the variational iteration method to solve nonlinear equations. Due to the flexibility, dependability, and effectiveness of the VIM method, it has gained popularity among researchers. The technique has been used to initialise boundary value problems , fractional initial problems, and q–difference equations. This algorithm employs a general Lagrange multiplier and typically goes through the following three steps: constructing a correlation functional, identifying the Lagrange multipliers, and deciding on the initial iteration. This issue has been resolved by the modified VIM approach, which also defines the Lagrange multiplier from the Laplace transform and is easily applicable to fractional differential equations with initial value issues. To further understand the MVIM approach, let's take a look at the following nonlinear differential equation**.**

$$\boldsymbol{\frac{d^{mu}}{d\varepsilon m} + R_1(u) + N_1(u) = f(\varepsilon)}$$

In the initial circumstances

$$\boldsymbol{u^{(k)}(0) = u_0^k}$$

For$K = 0,1,2 \ldots .m - 1\ where\ u = u(\varepsilon)R_1$is a regular function because it is linear and N1 is a nonlinear bounded operator.

4.3 Solution of Fractional SIR Model by MVIM Method

In order to solve the fractional Susceptible-Infected Recovered model, the modified VIM approach is now used. By introducing the SIR model, W.O. made a significant contribution to mathematical epidemiology by estimating the proportion of susceptible, ill, and recovered individuals in the community.

Basic notations for the fractional model are as follows: S denotes the number of susceptible individuals at the time, I denotes the number of infectedindividuals at the time, and R is the number of individuals who have recovered at the time.

$$\boldsymbol{S}(\in) + \mathbf{1}(\boldsymbol{\varepsilon}) + \boldsymbol{R}(\boldsymbol{\varepsilon}) = \boldsymbol{N}$$

Where N is the total population.

The following introduction of the fractional order SIR model results from the assumptions:

$$\boldsymbol{D}_{\in}^{\boldsymbol{a}}\boldsymbol{x}(\in) = -\boldsymbol{\beta}\boldsymbol{x}(\boldsymbol{\varepsilon})\boldsymbol{y}(\in)$$

$$\boldsymbol{D}_{\in}^{\boldsymbol{a2}}\boldsymbol{y}(\in) = \boldsymbol{\beta}\boldsymbol{x}(\boldsymbol{\varepsilon})\boldsymbol{y}(\boldsymbol{\varepsilon}) - \boldsymbol{k}\,\boldsymbol{y}(\boldsymbol{\epsilon})$$

$$\boldsymbol{D}_{\in}^{\boldsymbol{a3}}\boldsymbol{z}(\in) = \boldsymbol{k}\boldsymbol{y}(\in)$$

4.4 Application Of The Fractional Reduced Differential Transform Method

For a thorough explanation of the characteristics of various genuine materials, differentiation and integration of arbitrary order are excellent tools. Since the application of fractional models is supported by physical considerations, fractional structural models are more beneficial than earlier integer order models. The best tools for describing the genetic and memory properties of various materials and activities are fractional derivatives.

The paper goal is to give creative theories about the two-dimensional differential transformation approach that can increase its applicability to equations with linear and nonlinear partial derivatives, as well as time and space distinction.Fractional. FRDTM is used to solve a variety of partial differential equations and graphically illustrate how the solutions behave for various fractional orders.Phase plots and error diagrams demonstrate the numerical effectiveness and accuracy, demonstrating the application of FRDTM in a variety of applied science domains.

4.5 One dimensional fractional parabolic equation

First, let's discuss second order partial differential equations.

$$\boldsymbol{g} = \boldsymbol{A}_{\boldsymbol{uxx}} + \boldsymbol{B}\boldsymbol{U}_{\boldsymbol{xc}} + \boldsymbol{C}\boldsymbol{U}_{\in\in} + \boldsymbol{D}\boldsymbol{U}_{\boldsymbol{x}} + \boldsymbol{E}\boldsymbol{U}_{\in} + \boldsymbol{F}\boldsymbol{U}$$

A, B, and so forth are kwon functions. If 4, 2 B AC > and 0 apply, the aforementioned equation will be referred to as parabolic, hyperbolic, and elliptic, respectively.

In this part, using the FRDTM technique, we provide an analytical approximation to a solution to the nonlinear fractional order parabolic equation. Due to its numerous applications in practical sciences that now transcend beyond potential theory, fluid dynamics, the Brownian process, conformal geometry, etc., the parabolic equations with fractional order diffusion is the most popular topic among scholars. Demir and Ozbilge's research on fractional parabolic equations . Consider as though it were the next instance. Similar to a fractional parabolic issue.

5. CONCLUSION

Operators with fractional derivatives are thought to be the best tools for thoroughly describing memory and hereditary characteristics of various materials and complex developments. Therefore, fractional calculus is essential for understanding the dynamics of various complex mathematical models. The study of fractional differ-integral problems encourages the widespread use of these tools in practical fields.

References

1. Gulsu, M., Ozturk, Y., Anapali, A.: Numerical approach for solving fractional Fredholmintegro-differential equation. Int. J. Comput. Math. 90(7), 1413–1434 (2013)
2. Henderson, J., Ouahab, A.: A Filippov's Theorem, Some Existence Results and the Compactness of Solution Sets of Impulsive Fractional Order Differential Inclusions. Mediterr. J. Math. 9(3), 453–485 (2011)
3. Jafari, H., Khalique, C. M., Ramezani, M., Tajadodi, H.: Numerical solution of fractional differential equations by using fractional B-spline. Cent. Eur. J. Phys. 11(10), 1372–1376 (2013)
4. Leung, A. Y. T., Yang, H. X., Zhu, P., Guo, Z. J.: Steady state response of fractionally damped nonlinear viscoelastic arches by residue harmonic homotopy. Comput. Struct. 121, 10–21 (2013)
5. Nerantzaki, M. S., Babouskos, N. G.: Vibrations of inhomogeneous anisotropic viscoelastic bodies described with fractional derivative models. Eng. Anal. Bound. Elem. 36(12), 1894– 1907 (2012)
6. Pedas, A. Tamme, E.: Numerical solution of nonlinear fractional differential equations by spline collocation methods. J. Comput. Appl. Math. 255(1), 216–230 (2014)
7. Pooseh, S., Almeida, R., Torres, D. F. M.: Numerical approximations of fractional derivatives with applications. Asian J. Control 15(3), 698–712 (2013)