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DISCUSSION ABOUT THE APPLICATION AND FUNCTION OF FRACTIONALCALCULUS

YASHVANT KUMAR SWARNAKAR
RESEARCH SCHOLAR SUNRISE UNIVERSITY ALWAR
DR. BIRADAR KASHINATH B.
PROFESSOR SUNRISE UNIVERSITY ALWAR

ABSTRACT

Fractional differential equations (FDEs) have gained significant attention in recent years due to their ability to describe various phenomena in physics, engineering, and other scientific disciplines. The non-local and non-linear nature of FDEs poses significant challenges for their analytical and numerical solutions. This research focuses on studying and developing novel analytical, approximate, and numerical methods to solve different types of fractional differential equations efficiently and accurately. The first phase of the study involves a comprehensive review of existing analytical methods for FDEs, including fractional calculus and integral transforms. This analysis provides a foundation for understanding the fundamental concepts and mathematical tools used in solving FDEs. Building upon this knowledge, new analytical techniques are developed to enhance the existing methods, enabling the solution of a wider range of FDEs and improving the accuracy of the results. In parallel, the research explores approximate methods for solving FDEs. Approximations play a crucial role when exact analytical solutions are difficult or impossible to obtain. Various approximation techniques such as perturbation methods, homotopy analysis methods, and variational iteration methods are investigated and tailored to the specific requirements of FDEs. The aim is to develop efficient and reliable approximate methods that can yield accurate solutions over a wide range of parameter values.

Keywords: - Fractional, Numerical, Equations, Function, Application.

I. INTRODUCTION

Fractional calculus, also known as calculus of fractional order or fractional differential calculus, is an area of mathematics that deals with generalizations of differentiation and integration to non-integer orders. Traditional calculus deals with integer-order derivatives and integrals, but fractional calculus extends these concepts to non-integer orders, allowing for a deeper understanding of phenomena with fractal-like or complex behaviors.

The history of fractional calculus can be traced back to the work of mathematicians like Leonhard Euler, who considered fractional derivatives in the 18th century. However, it wasn't until the 19th century that the theory began to take shape with the contributions of mathematicians like Augustin-Louis Cauchy and Liouville. Since then, fractional calculus has found numerous applications in various scientific and engineering fields, including physics, engineering, biology, finance, and signal processing, among others.

The fractional derivative, denoted by D^(α)f(x), represents the differentiation of a function f(x) to a fractional order α. Similarly, the fractional integral, denoted by I^(α)f(x), represents the integration of a function f(x) to a fractional order α. These operators are defined using various approaches, such as the Riemann-Liouville, Caputo, and Grünwald-Letnikov definitions, each offering different interpretations and properties.

Fractional calculus allows us to describe processes with long memory or non-local effects that cannot be captured by traditional integer-order calculus. It provides a powerful tool to analyze and model complex systems exhibiting fractal patterns or behaviors, self-similarity, and anomalous diffusion.

II. FRACTIONALCALCULUS:HISTORICALOVERVIEW

Fractional Calculus (FC) deals with derivatives and integrals of non-integer orders, wherethe orders may be real or complex.The genesis of FC began with a question raised byFrenchmathematicianL'HôpitaltoLeibniz,aGermanmathematicianofthe17thcentury.Heaskedin aletterdated30September1695thatwhatwouldbetheinterpretationsofthesymbol $\frac{d^{1/2}x}{dx^{1/2}}$?This unprecedented question laid the foundations of FC. During the 19thand 20th centuries, many giants of mathematics such as Laplace, Fourier, Abel, Liouville,Riemann,Letnikov,Mittag-Leffler,Weierstrass,Heaviside,Weyl,Lévy,Feller,Riesz,andsooncontributedprofoundlytothedevelop mentofFC.In19thcentury,S.F.LacroixaFrenchmathematicianmodifiedtheformulaforthe$n-$thorderdif ferentiationofx^m,$m\in$ Nandobtainedtheformulafor$_2$-thorderderivative $\frac{d^{1/2}x}{dx^{1/2}} = \frac{2\sqrt{x}}{\sqrt{\pi}}$. .Apartfromthat,Niels Henrik Abel found the first application of FC by solving tautochrone problem.FordetailedhistoryofFCanditsapplicationswereferthereaderto.

III. APPLICATIONSOFFRACTIONALCALCULUS

FC is one of the most currently active areas of research.In the past few decades, FCattracted many researchers due to its wide applicability especially to the processes in-volving memory effects. To interpret complex physical phenomena occurring in thereal-world, FC-based models give better results as compared to traditional integer-ordermodels.Substantial amount of work has been done theoretically in the field of FC.Moreover, extensive applications of FC have been found in different fields such as fluidmechanics,earthsystemdynamics,imageprocessing,rheology,electronicsandsignalprocessing, bio-engineering,geology,economics,soil hydrology and mechanics, electromagnetism, dynamics and control, radiation physics and many other branches of science and engineering.In,Sebba*et.al.* Used FC to modify the Biot theory, which relates the viscous interactions be-tween fluid and solid framework.In, Carpinteri and Mainardi used space fractionalderivative to describe the mechanics of fractal media and laid down the foundations for newapplicationsofviscoelasticmaterials.In,Assaleh*etal.*demonstratedthemodelingofspeech signals using FC that gives more accurate results with less parameters. Fractionalorderbasedcontrollersareprovenusefulinpathtracingproblemsincontrolofautonomousvehicles.In,Fellah*etal.*suggestedthatfractionalordertimederivativesdescribethebehaviorofsoundwavesinrigidporousmaterials.In,Rekhviashvili*etal.*appliedthe FC based oscillator to characterize strongly damped oscillations.In, Failla andZingales have given an account of the applications of FC in many fields such as materialhereditaries, heat conduction, fractal media, and so on.In, Oliveira *et al.*have dis-cussed the applications of FC in relaxation processes. Recently, Fang *et al.* proposedFC based model to describe the viscoelastic behaviors of solid propellants, and concludesthatthenewmodelismoreaccurateandrequiresfewerparametersascomparedtothetra-ditional model.In, Zhang *et al.*suggested FC based model for electromotive forceand for the charging state of lithium-ion batteries, which improve the model accuracy thattoowithoutincreasingmuchcalculations.

IV. SPECIALFUNCTIONS

SpecialfunctionssuchasGammafunction,Betafunction,Mittag-Lefflerfunction,Wrightfunction [38–41] are the necessary and basic tools of FC as they appear in the solutions offractional differential equations (FDEs), fractional partial differential equations (FPDEs),fractionalintegralequations(FIEs)andsoon.

- **GammaFunction**

Duringthe18thcentury,twofamousmathematiciansBernoulliandGoldbachattemptedtogeneralizethefactorialfunction$m!$,where$m \in \mathbb{N}$;however,EuleraSwissmathematiciansucceeded.Euler'sgammafunctionisdefinedbythefollowingintegral:

$$\Gamma(\varsigma) = \int_0^\infty e^{-x} x^{\varsigma-1} dx, \quad Re(\varsigma) > 0. \qquad (1.3.1)$$

Substituting $e^{-x} = \lim^{m\to\infty} \left(1 - \frac{x}{m}\right)^m$ in eq and performing integration by parts (m-times),weget

$$\Gamma(\varsigma) = \lim_{m\to\infty} \frac{m^\varsigma m!}{\varsigma(\varsigma+1)(\varsigma+2)\cdots(\varsigma+m)}. \qquad (1.3.2)$$

Inviewof(1.3.2),theGammafunctionisdefinedforall$\varsigma \in$Cexceptwhen$\varsigma=\{0,-1,-2,$ — 3,..}.SomeofthepropertiesofGammafunctionareasfollows:

$\Gamma(1)=1, \Gamma(\varsigma+1)=\varsigma\Gamma(\varsigma)$. Inparticular$\Gamma(m+1)=m!, m\in$N.

- **Mittag-LefflerFunction**

Mittag-Leffler(M-L)functionanditsseveralgeneralizationsplaypivotalroleinFDEsas they appear in the solutions of various FDEs.M-L function is a generalization of theexponential function introduced by Swedish mathematician Mittag-Leffler in.M-Lfunctionwithoneparameterμisdefinedas

$$E_\mu(\varsigma) = \sum_{k=0}^{\infty} \frac{\varsigma^k}{\Gamma(\mu k+1)}, \quad Re(\mu) > 0, \quad \varsigma \in \mathbb{C}. \qquad (1.3.3)$$

In1905WimangeneralizedtheoneparameterM-LfunctionandproposedthetwoparameterM-Lfunction

$$E_{\mu,\nu}(\varsigma) = \sum_{k=0}^{\infty} \frac{\varsigma^k}{\Gamma(\mu k+\nu)}, \quad Re(\mu) > 0, \ \varsigma, \ \nu \in \mathbb{C}. \qquad (1.3.4)$$

Observethat$E_\mu(\varsigma)=E_{\mu,1}(\varsigma)$.FormoredetailsaboutM-Lfunctionanditsgeneralizations,werefertothefollowingreferences.

V. CONCLUSION

In conclusion, the study and development of new analytical, approximate, and numerical methods for solving different types of fractional differential equations have proven to be a valuable and exciting area of research. Fractional differential equations are mathematical models that generalize classical differential equations by involving derivatives of non-integer order. These equations have gained significant attention in various scientific and engineering fields due to their ability to describe complex phenomena more accurately.

The development of analytical methods for solving fractional differential equations is a challenging task due to the non-local nature of fractional derivatives. However, researchers have made significant progress in developing specialized techniques such as the Laplace transform, the fractional Fourier transform, and the operational matrix method. These analytical methods allow for the direct solution of certain types of fractional differential equations, providing valuable insights into the behavior of the underlying systems.

Approximate methods, such as the Adomian decomposition method, the variational iteration method, and the homotopy perturbation method, have also been employed to tackle fractional differential equations. These methods offer efficient and computationally feasible approaches to obtain approximate solutions for a wide range of fractional differential equations. By approximating the solution iteratively or through perturbation techniques, these methods provide a useful alternative when analytical solutions are not readily available.

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