



GRAPH THEORY IN COMPUTER SCIENCE

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ABSTRACT

Graph theory is essential for solving many challenging computer science issues. Graphs are one of the most common representations of both organic and man-made systems. Relationships and process dynamics in computer science, physics, biology, and sociology may all be modeled with their help. Graphs are a useful representational tool for many issues with real-world relevance. The academic community is continually interested in the problems that arise in mobile ad hoc networks (MANETs). focuses on the many uses of graph theory in computing, Graphs may be used to illustrate a wide variety of real-world situations. This article demonstrates the relevance of graph theory by examining its many ideas and illustrating them via examples from computer science. These examples serve to propagate the concept of graph theory and highlight the relevance of this area of study to computer science.

KEYWORDS:Ad-hoc networks, Graphs, network, application of graphs, graph algorithms.

INTRODUCTION

Graph theory is a branch of mathematics concerned with the study of graphs, which are structures used to represent relationships between things by means of pairs. In this sense, a graph consists of a collection of vertices (also known as nodes or points) that are linked together by edges (also called links or lines). For clarity, directed graphs have edges that connect their vertices in an asymmetrical fashion, whereas undirected graphs have edges that connect their vertices symmetrically. In the field of discrete mathematics, graphs are among the most studied objects.

Cybernetics is a branch of computer science that makes extensive use of graphs to depict various network topologies, data structures, computational apparatuses, computation flows, etc. For instance, a directed graph may be used to depict a website's link structure, with the vertices standing in for individual web pages and the directed edges for the connections that connect them. The same method may be applied to issues in several other disciplines, including social media, transportation, biology, computer chip design, and tracking the development of neurodegenerative illnesses. Graph algorithms are therefore of significant importance to the field of computer

science. Graph rewriting systems are often used as a formalization and representation for graph transformations. Graph databases are a complement to graph transformation systems, which concentrate on rule-based in-memory modification of graphs, by providing a secure, durable storage and querying mechanism for graph-structured data.

The field of discrete mathematics known as graph theory. Graph theory is the study of graphs, which are mathematical structures used to depict pairwise interactions between objects in math and computer science. Graphs are often used to describe problem-solving methods because they provide an understandable approach before delivering formal description. Two issues are taken into account in the analysis of the graph-theory application.

1- Classical problem

2- Problems from applications

Connectivity, cuts, routes, and flows, coloring issues, and the theoretical facets of graph drawing are all terms used in graph theory to characterize the classical problem. Comparatively, experimental study and the actual implementation of the graph theory methods are given a disproportionate amount of weight when dealing with challenges originating from application. The automated production of drawing graph has vital applications in key computer science technologies including database design, software engineering, circuit designing, network designing, and visual interfaces, making graph drawing an essential issue from an implementation point of view.

The discipline of networking is one of several areas where graph theory may be usefully used. Graph-based representation and network theory are both taken into account for this examination of graph-networking theory's applications. There are several benefits to using a graph-based representation, such as the fact that it offers a new perspective, simplifies difficult problems, and offers a clearer, more precise explanation. While graph analysis and application of network theory through graph representation are provided by network theory. As far as meaning goes, "graph" and "network" are interchangeable. Both terms describe a topology in which nodes (or vertices) are connected by lines (or edges) (i.e., links, lines). Depending on the nature, graphs and networks may provide either high or low levels of organization.

LITERATURE REVIEW

N. Balaji et al (2017) Numerous disciplines rely heavily on mathematics. Graph theory is an important branch of mathematics because it is used to structural models. Structured groupings of things or technology inspire new developments and alterations to the status quo for the purpose of improvement. In 1735, the dilemma of the Koneisberg bridge was the impetus for the development of what would become the science of graph theory. This article provides a survey of graph-theoretic applications in a variety of disciplines, with an emphasis on their usage in computer science. Here, we provide a summary of the research done on the many graph-theory-based works that deal with scheduling ideas and their computer science and application contexts.

AlabbasAlhaj Ali et al (2022) For several business needs, such as fraud detection, scheduling, and recommendation systems, many graph algorithms have been created. Thus, graph-processing

frameworks are developed to ease the deployment of graph-based applications. However, the number of such frameworks has increased dramatically over the last decades, and they all have their own advantages and disadvantages. In order to choose the best approach to a problem, it is crucial to be familiar with the needs and features of each framework. Utilizing a PageRank solution to a real-world business issue based on the Yelp dataset, we compare and contrast the capabilities of two widely used graph-processing frameworks, Neo4j and Apache Spark GraphX.

Akalyadevi K. et al (2022)In this research, we explore the chromatic number of a graph and propose a novel method for identifying the root causes of accident hot spots by use of Pythagorean fuzzy graph vertex coloring.

P. Tharaniya et al (2022)This paper's goal is to compare and contrast the chromatic numbers of different graph types and fractal graphs with high levels of self-similarity. The Fractal Graph has become a ubiquitous symbol of cutting-edge technology and innovation. Many branches of engineering, including computer science, physics, medicine, architecture, etc., have benefited from its expansion and increased use. Color-coding graphs is a cutting-edge technique in Graph Theory that is finding increasing use in several disciplines. Simply assigning a color to each vertex, edge, or area is "coloring the vertices." Vertices and edges that are neighbors do not share the same hue. The chromatic number of a graph is the number of colors required to color its vertices. The chromatic numbers of Cantor Sets, Von Koch Curves, and Hilbert Curves, among others, are examined. The latest research on the topic of Coloring combined with Fractal Graphs has come to a close.

Mercedes Pelegrinet al (2022)Operations Research, and more especially Location Science, has done much research on covering challenges. Assuming that the candidate facility locations, the points to be covered, or both are discrete sets is a common starting point when the location space is a network. This paper investigates the set-covering location issue in which the supply of possible locations and the demand at those locations are both continuous sets on a network. Little research has been done on this version, and the methods that do exist mostly apply to narrow use cases like tree networks and integer covering radii. For networks where no edge is longer than the covering radius, we investigate the issue at large and provide a Mixed Integer Linear Programming (MILP) formulation. Any edge that does not fulfill this criterion may still be partitioned into sub edges of suitable lengths without modifying the underlying issue, thus the model retains its generality. We suggest a preprocessing approach to cut down on the MILP's size, as well as provide stringent big- M constants and sound inequalities. In addition, a second MILP is suggested, this one allowing edge lengths longer than the covering radius. The second suggested MILP is different from previous formulations of the issue in that the number of variables and constraints is independent of the lengths of the network's edges. As the edges of real-world networks are often larger than the covering radius, the second model provides a scalable strategy that is well-suited to these kinds of systems. The advantages and disadvantages of our precise technique are shown by computer tests on both real-world and random networks. We also compare our solutions to a standard, accurate procedure.

APPLICATION IN COMPUTER SCIENCE

Data mining

As far as data mining is concerned, graph theory is most useful when applied to graph mining. Graph mining is a metaphor for the connectedness of data. Graph based data mining may be approached from five different theoretical perspectives. These include classifications of subgraphs, isomorphisms of subgraphs, invariants of graphs, mining metrics, and approaches to solving graph problems.

Operating system

There is a limited number of possible combinations, or "edges," between each pair of "vertices" in a graph. Operating system challenges like work scheduling and resource allocation are ideal candidates for graph-based solutions. In CPU task scheduling difficulties, for instance, the notion of graph coloring may be used; in this setting, jobs are modeled as vertices of the graph, and an edge is drawn between any two jobs that cannot be run at the same time.

Website designing

Web design may be thought of as a graph, with individual web sites standing in for vertices and the hyperlinks connecting them standing in for edges. Web graph is the term for this idea. Which one of them uncovers the most fascinating facts? Graphs also have uses in the online social world. In this form of graph, each vertex stands in for a distinct category of items, and all vertices belonging to the same category are linked to all vertices belonging to other categories. Complete bipartite graph is a term used in graph theory to describe such a structure.

Software engineering

Graph is widely used in the field of software development. For instance, Data Flow diagrams are used throughout the Requirements Specification process, where the vertices indicate the transformations and the edges represent the data flows. Control flow of a program related with McCabe's complexity measure, which uses directed graphs for addressing the sequence of executed instructions and etc. is addressed using graphs during the Testing phase. Network diagrams, which need graph algorithms, are used in other contexts as well, including Software Process Management.

Image processing

What we call "Image Analysis" is the process that's used to glean data from pictures. Digital image processing methods are the most often used for picture analysis. Using a graph theoretic strategy, image processing methods may be enhanced. Graphs may be used for the following purposes in image processing: employing segmentation graph search methods to locate edge boundaries. In order to determine the correct placement of the image, using minimal spanning tree to find mathematical limitations like entropy. The approach uses shortest route computations to find distance transformations of the pixels and determine the distance between the inside pixels.

concise descriptions of graph's many uses, most notably in computing. Theories from graph theory are being implemented in a wide variety of computer-related fields. Here we will go through a few possible uses.

Mobile phone coverage areas and map colors using the Global System for Mobile communications (GSM): Cells in a mobile phone network using the Groups Special Mobile (GSM) standard are hexagonal in shape. There is a communication tower in each cell that links all the mobile phones there. Every cell phone does a neighborhood cell search to join the GSM network. Graph theory dictates that just four colors are available for coloring the cellular zones since GSM only function in four distinct frequency bands. These four hues are utilized to accurately depict the various geographical areas. Therefore, each given GSM mobile phone network may have no more than four distinct frequencies assigned using the vertex coloring approach. The idea is described in the following way by the authors: The four-color theorem asserts that it is always feasible to correctly color the regions of a map using at most four separate colors so that no two neighboring parts are given the same color, whether the map is drawn on the plane or the surface of a sphere. Now, we create a dual graph by inserting a vertex within each map area and linking those vertices together with an edge if their regions share a complete boundary segment. If you color the dual graph correctly, you'll get the correct colors when you color the first map. Graph G is planar, hence coloring its regions is the same as coloring its counterpart graph's vertices, and vice versa. The four-color theorem is used to assign one of four frequencies to each section of the map.

Graph algorithm in computer network security:

Given a basic graph G with n vertices labeled $1, 2, \dots, n$, the vertex cover method seeks a vertex cover of size at most k . Stop if the size of the vertex cover achieved is less than or equal to k at each step. enables researchers to model the spread of stealth worms over big networks and develop effective countermeasures to keep systems safe in real time. Through extensive modeling in a simulated internet-scale virtual network, we found that the network architecture significantly affects the spread of worms. In order to stop the spread of worms in real time, it is crucial to locate their source. The key concept here is to locate the minimal vertex cover in a directed graph where the nodes represent individual routing servers and the edges represent the links between them. Then, a network defensive plan is developed, and the best method for worm propagation is identified. If every vertex in graph G is incident on at least one edge in set g , then g is said to cover G . A covering of a graph G is the collection of edges that surround it. e.g. In a directed graph, a covering is a spanning tree. Hamiltonian circuits are also a kind of covering. Following is an illustration of a computer network with the least vertex cover that would be required.

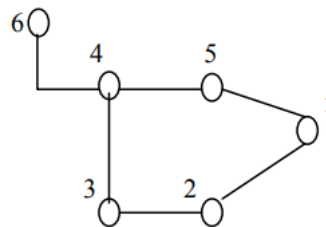


Figure - 1 The vertex Set $g=\{2,4,5\}$ covers all the vertices in G

Graph theory relevant to ad-hoc networks:

The ability to build a communication network without requiring a preexisting infrastructure is a key feature of ad hoc networks, which are decentralized, self-organizing networks. Every computer in an ad hoc network has a radio that can send and receive signals, so they can talk to each other over the airwaves.

There are two major foci in MANET research: The first step is network modeling and simulation, while the second is an understanding of the underlying challenges including connection, scalability, and routing. Because of the mathematical similarity between a network and a graph (there is a bijection between network topology and graph), the ideas from Graph Theory are crucial to the investigation of these basic problems. In addition, the challenges of ad hoc networks have a mathematical foundation. To continue, graphs may be represented algebraically as matrices, allowing the analysis of the network to be mechanized using algorithms.

A network's primary function is to enable data to be sent between any two endpoints. Connectivity to the network is required for this to occur. Since this is the case, connection is one of the MANETs' primary concerns. The connection is mostly determined by the nodes' transmission range and their mobility. Each of the core questions of connection, routing, and topology control may be tackled separately with the use of the ideas of proximity of graphs (UDG, NNG, RNG).

Proximity:The study of proximity graphs is significant for topology management and network connection in mobile ad hoc networks. The proximity between two nodes is a visual representation of their neighbor connection. An edge connects two nodes if the distance between them is small enough. This will have a major impact on the availability of the network. What kind of graph is generated depends on the scale used. Due to the variety of proximity metrics available, there are several proximity graph kinds. The spanning ratio is one such metric. For the purposes of

this definition, Maximum $\left(\frac{\eta(u,v)}{\Omega(u,v)} \right)$ where $\eta(u,v)$ is the distance traveled by the shortest route between vertices u and v , where vertices are connected by edges of length, where vertices are separated by edges of length measured in terms of the Euclidean distance, and $\Omega(u,v)$ represents the shortest possible path using just Euclidean geometry. Several distinct varieties of proximity graphs may be constructed. Some of the more notable examples include the Unit Distance Graph (UDG), Nearest Neighbor Graphs (NNG), Minimum Spanning Trees (MST), Relative Neighborhood Graphs (RNG), Delaunay Triangulation (DT), and Gabriel Graphs (GG).

Unit Disk Graph (UDG):if the Euclidean distance between i and j is less than 1, then the pair of i and j form a link in the set E , and the graph $UDG(V, E)$ contains the set V of vertices and the set E of vertices. Similarly, UDG may be pointed in the direction that the connection points. Typically, UDG is used for simulating ad hoc networks. If the two nodes' Euclidean distances are smaller than r , where r is the radius of the circular transmission range of the antenna of the mobile node, then the resulting UDG is more general. The geodesic of the transmission range of the radio signals covered in this model is almost circular, making it an excellent fit for ad hoc networks. Finding the node transmitters' minimal threshold range for network connectivity is a problem associated with these architectures.

Nearest Neighbor Graph(NNG):A directed graph $G(V, E)$ is a spanning sub-graph of a graph G' with $V=V'$ and there exists a directed edge e between any two nodes i and j if and only if node j is the closest neighbor of node i . (the Euclidean distance is the measure for selecting nearest neighbor). All pair shortest pathways in a network may be found here on this diagram. If the NNG of a particular network can be determined, then packet routing in that network will be streamlined and the network's throughput will rise as a result of less time spent in transit. However, NNG gets cut off since one of its links failed. Therefore, for it to be fault tolerant, a more generic NNG is necessary. Multiple neighbors may be included in NNG as a generalization. The K -NNG, a special case of the NNG, quantifies the number of arcs connecting a given vertex to its K closest neighbors. A given graph may have many NNGs. NNG is a spanning subgraph of Minimum Spanning Tree and may also be an undirected graph (MST). The minimum spanning tree (MST) of a weighted graph G is an acyclic spanning tree of G with the characteristic that the total weight of its edges is minimal. Any given graph G may have numerous MSTs due to the fact that its edges may have the same weights. The Relative Neighborhood Graph (RNG) is a special case of NNG in which every pair of nodes i and j in a graph $G(V, E)$ has an edge between them if and only if every pair of nodes k (in V) has an edge between them, $d(i,j) \leq \text{Maximum}(d(i,k),d(k,j))$.

Restricted Delaunay Graph (RDG (G)):It is commonly known that $DT(G)$ is a spanning sub-graph of the entire graph, although it is not locally constructible and may have very long edges. The RDG idea was developed to counteract these drawbacks. The RDG is defined as the Euclidean spanner of the UDG and comprises all the short edges of the planar graph G . Face routing techniques and the memory-less routing algorithm that combines greedy forwarding with local minimum recovery on the basis of face routing are only two examples of when RDG is put to use. Algorithms for face routing are limited to planar graphs. You just need to come up with an approach to remove certain connections from the supplied (non-planar) network in order to make it planar. Graph theory is crucial for identifying the planarity or non-planarity of a network and for transforming non-planar networks into planar ones.

Graphical representation of algorithm

A flowchart is a visual diagram of a program's logic. Algorithms are formulas that may be followed to get a desired outcome. The steps taken (or assumed) by a program, on the other hand, are peripheral to the algorithm's actual function.

Graph theory in symbol recognition:

An update of these costs must be incorporated into the given algorithm as node costs change due to the connecting two regions R_i and R_j , as discussed in the paper "Symbol Recognition by Error tolerant subgraph matching between region adjacency graphs" (The region adjacency graph is one in which costs are associated with both nodes and arcs, implying that a matching algorithm must be tolerant of errors when matching subgraphs). See below for examples of segmented graphs, region adjacency graphs, and its dual graph.

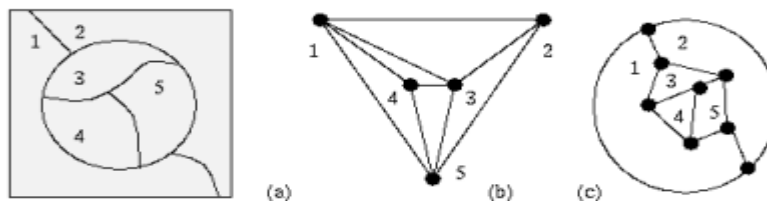


Figure – 2 Segmented graph, Region adjacency graph and its dual graph

The model graph is compared to the subgraph of the distorted picture. This means that the image subgraph and the model graph represent the image region and the model region, respectively. String edit distance aligning polygonally estimated outer bounds of graphs consisting of matched regions and neighbour region candidates is the cost of adding a neighbour to the correspondence. The procedure is adaptable to any representation of a graph based on geographic regions.

Graph based and structural methods for fingerprint classification:

Most often, fingerprint analysis is used in the context of solving crimes. In this system, fingerprints are cataloged and stored in databases. However, if the data is kept in its current format, the database will grow in size, necessitating additional space. The author provides a summary of the state of fingerprint categorization using graph theory in this study. You may classify information using a number of methods, including structural, statistical, and graph-based ones. Only methods based on graphs will be described here. In the past, this was accomplished by manually segmenting fingerprint photos into zones where the ridges all faced the same direction. However, structural information of this kind is useless in recognizing fingerprints based on other classes since all fingerprints share the same basic organizational principles. The graph-based strategy relies on elementary relational diagrams. Fingerprint orientation field segmentation is shown in the following relationship diagram.

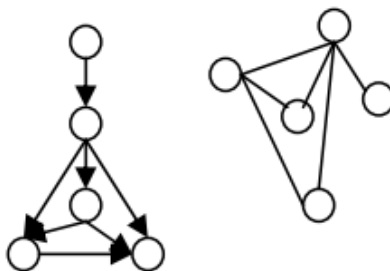


Figure – 3 Relational graphs

The segmentation process may be aided by this methodology. Nodes in a relational network seem more suitable, as they readily map to the regions recovered by the segmentation technique. Each network node corresponds to a segmentation area, and edges connect adjacent nodes based on the adjacency connection between the regions. A technique called error-correcting graph matching is used to calculate relational graphs. In order to classify an input pattern, this technique calculates a measure of the distance between the graph expressing the pattern and a given graph prototype. The term "edit distance" is used to describe this kind of similarity metric. To do this, a deformation model is used to swap in new nodes or remove old ones, or to add or remove edges. It

is argued that the graph-based representation of fingerprints is superior to previous structural techniques for classifying fingerprints.

CONCLUSION

The purpose of this article is to help computer science students learn more about graph theory and its connections to other fields of study, such as operating systems, networks, databases, software engineering, and more. In this study, we explored the many computers science and engineering applications of major graph theory. The primary goal of this study is to introduce researchers to the usefulness of graph theoretical notions in a variety of computational applications so that they may incorporate these ideas into their own investigations. Researchers may learn about graph theory and its uses in the IT industry, and maybe even come up with some new ideas. We have shown how vital graph theory is to solving some of MANET's most pressing problems. Particularly for the purpose of conveying the concept of graph theory, a summary is offered. Therefore, more weight is placed on the graph theory portions of papers than on any others. Scholars may learn about the computer science applications of graph theory and maybe even receive some new ideas for their own study.

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