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Unsteady MHD Blood Flow through Contraction Volumetric Flow Rate

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Abstract : The impact of a static applied magnetic field on pulsatile flow in stenosed arteries is examined in this paper. The Navier Stokes equations model the physical issue. It is investigated how the magnetic field, the Womersley number, and the Darcy number affect the wall shear stress and volume flow rate.

Key-words Unsteady Blood Flow, mild contraction, MHD, Permeability K

Introduction : Both the study of blood flow under healthy physiological conditions and under pathological conditions is significant. Cardiovascular disorders account for the majority of deaths in developed nations, and the majority of these diseases have some connection to irregular blood flow in arteries. These disorders' exact causes are not yet fully understood. The arterial disease atherosclerosis or stenosis affects the arteries by causing a subendothelial build-up of fatty or lipid material rich in cholesterol. For such a condition, high WSS (Wall Shear Stress) plays a significant role in the development of the illness. Many authors have written about the issues of contraction. A mathematical model of arterial blood flow in the presence of moderate stenosis was examined by Sanyal and Maiti in 1998.. They concluded that the pressure gradients increases with the increases in hematocrit value which indicates that there is higher value in systolic and lower value in diastolic pressure. Sanyal and Maiti (1998) studied a mathematical model on arterial blood flow in the presence of mild stenosis and obtained the pressure gradient and wall shear stress by series solution. They concluded that the pressure gradients increases with the increases in hematocrit value which indicates that there is higher value in systolic and lower value in diastolic pressure. Sanyal and Maiti (1998) have worked out pulsatile flow of blood through a stenosed artery. In this paper, it has been attempted to study the magnetic effect on the unsteady blood flow through stenosis when the tube is filled with a porous material of constant permeability K.

Formulation of Problem : Let us consider the pulsatile axially symmetric unsteady flow of blood through stenosed artery. The blood flowing in the tube is assumed to be a suspension of red cells in plasma. The hematocrit dependent blood viscosity is taken as described by the Einstein equation $\mu=\mu_0[1+\beta\, h(r)]$. The diameter of the artery is not less than 1mm so that Fahreus Lindquist effect is not significant. The artery is considered as long cylindrical tube with axis of symmetry coinciding with the z-axis. A static transverse magnetic field of moderate strength has been applied, so that induced E.M.F is assumed negligible result of which a Lorenz an force came into existence opposite to flow direction

and tube is filled with porous material of constant permeability K. Geometry of the stenosis is defined by the function

$$\frac{R(z)}{R_0} = \begin{cases} 1-\dfrac{\delta}{2R_0}\left(1+\cos\dfrac{\pi z}{L}\right) & -L \le z \le L \\ 1 & \textit{otherwise} \end{cases} \qquad (1)$$

2L is the length of stenosis. R(z) is the radius of the tube in the stenotic region; d is the location of the stenosis, R_0 is the radius of unconstructed tube ,δ is maximum thickness of the stenosis

The governing equations of the motion with axisymmetric condition $\frac{\partial}{\partial\theta}(\,)=0$ and impermeable wall in the cylindrical coordinate system (r, θ, z) is given by

$$\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial p}{\partial r} = 0$$

$$\frac{\partial w}{\partial T} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\nu r\frac{\partial w}{\partial r}\right) - \sigma\frac{B_0^2 w}{\rho} - \frac{\mu}{K}w \qquad \ldots(2)$$

The boundary conditions are

w = 0 at r = R(z)

and

$$\frac{dw}{dr} = 0 \text{ at } r = 0 \qquad (3)$$

Introducing following non-dimensional parameter :

$$y = \frac{r}{R_0}\ ;\ t = \frac{T}{t_0}\ ;\ \frac{R_0^2\sigma B_0^2}{\mu_0} = H^2\ ;\ \frac{K}{R_0^{\ 2}} = Da^2\ ;\ \frac{\rho R_0^2}{t_0\ \mu_0}\omega = \alpha^2$$

(4)

Substituting the values of ν, μ, h(r) in the equation (2) and then using (4),we get

$$\frac{\rho R_0^2}{t_0\,\mu_0}\frac{\partial w}{\partial t} = \frac{-R_0^2}{\mu_0}\frac{\partial p}{\partial z} + \frac{1}{y}\frac{\partial}{\partial y}\left[(a-ky^n)y\frac{\partial w}{\partial y}\right] - \frac{R_0^2\,\sigma B_0^2 w}{\mu_0} - \frac{(a-ky^n)}{Da^2}w$$

$\beta h_m = k$

$a = 1 + k$

For the solution of the equation (1.7), let us consider

$$w(y, t) = W(y)\, e^{i\omega t}; \tag{5}$$

$$-\frac{R_0^2}{\mu_0}\frac{\partial p}{\partial z} = ce^{i\omega t} \tag{6}$$

Then we have

$$\frac{1}{y}\frac{d}{dy}\left[(a-ky^n)y\frac{dW}{dy}\right] - (\alpha^2 i + H^2 + \frac{a}{Da^2})W + \frac{ky^n}{Da^2}W = -c \tag{7}$$

The corresponding boundary conditions are transformed to

$$W = 0 \quad \text{at} \quad y = \frac{R}{R_0} \tag{8}$$

and

$$\frac{dW}{dy} = 0 \quad \text{at} \quad y = 0 \tag{9}$$

Method of Solution :

Frobenius method is applied for the solution. For this, it is required that W is bounded at y = 0 and the only admissible solution satisfying the boundary condition (8) is

$$W = D\sum_{m=0}^{\infty} A_m y^m - \frac{c}{4a}\sum_{m=0}^{\infty} \lambda_m y^{m+2} \tag{10}$$

Where D is an arbitrary constant to be determined by the boundary condition (8) and

$$A_{m+1} = \frac{(\alpha^2 i + H^2 + \frac{a}{Da^2})A_{m-1} + k(m+1)(m-n+1)A_{m-n+1} - \frac{k}{Da^2}A_{m-n-1}}{a(m+1)^2} \tag{11}$$

and

$$\lambda_{m+1} = \frac{(\alpha^2 i + H^2 + \frac{a}{Da^2})\lambda_{m-1} + k(m+3)(m-n+3)\lambda_{m-n+1} - \frac{k}{Da^2}\lambda_{m-n-1}}{a(m+3)^2}$$

(12)

with $A_0 = \lambda_0 = 1$ and $A_{-m} = \lambda_{-m} = 0$

Here the constant D involved in the solution (10) is obtained with the help of (8) as

$$W = 0 \text{ at } y = \frac{R(z)}{R_0}$$

$$\therefore \qquad D = \frac{c}{4a}\frac{\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}}{\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}} \tag{13}$$

Hence

$$W = \frac{c}{4a}\frac{\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}A_m y^m - \sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\sum_{m=0}^{\infty}\lambda_m y^{m+2}}{\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}}$$

(14)

Since $w(y, t) = W(y)\, e^{i\omega t}$, therefore

$$w(y,t) = \frac{c}{4a}\frac{e^{i\omega t}\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}A_m y^m - \sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\sum_{m=0}^{\infty}\lambda_m y^{m+2}}{\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}}$$

(15)

In particular, in the absence of the hematocrit, the average velocity w_0 is given by w_0

$$= \frac{c_0 e^{i\omega t}}{\left(\alpha^2 i + \frac{1}{Da^2}\right)}\left[1 - \frac{I_0\left(\sqrt{\alpha^2 i + \frac{1}{Da^2}}\, y\right)}{I_0\left(\sqrt{\alpha^2 i + \frac{1}{Da^2}}\right)}\right]$$

(16)

The dimensionless form of w with respect to w_0 is now obtained from equations (15) and (16) as

$$\frac{w}{w_0}=\frac{(\alpha^2 i+\frac{1}{Da^2})c}{4ac_0}\frac{\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}A_m y^m-\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\sum_{m=0}^{\infty}\lambda_m y^{m+2}}{\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\left(1-\frac{I_0(\sqrt{\alpha^2 i+\frac{1}{Da^2}}y)}{I_0(\sqrt{\alpha^2 i+\frac{1}{Da^2}})}\right)}$$

(17)

Volumetric Flow Rate : The volumetric flow rate Q of the fluid in the stenotic region is given by

$$Q = 2\pi\ R_0 \int_0^{R/R_0} wydy$$

(18) Using the equations (6) and (15) , we have

$$Q = 2\pi\ R_0 \int_0^{R/R_0}\frac{\left[\frac{ce^{i\omega t}}{4a}\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}A_m y^m-\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\sum_{m=0}^{\infty}\lambda_m y^{m+2}\right]ydy}{\sum A_m\left(\frac{R}{R_0}\right)^{m}}$$

$$\frac{\frac{\pi R_0^3}{2a\mu_0}\left(\frac{\partial p}{\partial z}\right)\sum_{m=0}^{\infty}\lambda_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}\frac{A_m}{m+2}\left(\frac{R}{R_0}\right)^{m+2}-\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}\sum_{m=0}^{\infty}\frac{\lambda_m}{m+4}\left(\frac{R}{R_0}\right)^{m+4}}{\sum_{m=0}^{\infty}A_m\left(\frac{R}{R_0}\right)^{m}}$$

(19)

Let Q_0 denotes the flow rate of plasma fluid in unconstricted tube when M=0 and H=0 , Then

$$Q_0 = -\frac{\pi R_0^3}{8\mu_0}\left(\frac{\partial p}{\partial z}\right)_0 \qquad (20)$$

$\left(\frac{\partial p}{\partial z}\right)_0$ being the pressure gradient of the fluid in unconstricted uniform tube.

Thus non-dimenional flow rate $\overline{Q} = \frac{Q}{Q_0}$ is given as:

$$\overline{Q} = \frac{4}{a} \frac{\frac{\partial p}{\partial z}}{\left(\frac{\partial p}{\partial z}\right)_0} \frac{\sum_{m=0}^{\infty} \lambda_m \left(\frac{R}{R_0}\right)^{m+2} \sum_{m=0}^{\infty} \frac{A_m}{m+2}\left(\frac{R}{R_0}\right)^{m+2} - \sum_{m=0}^{\infty} A_m \left(\frac{R}{R_0}\right)^{m} \sum_{m=0}^{\infty} \frac{\lambda_m}{m+4}\left(\frac{R}{R_0}\right)^{m+4}}{\sum_{m=0}^{\infty} A_m \left(\frac{R}{R_0}\right)^{m}}$$

(21)

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