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EXAMINES NUMERICAL APPROXIMATION OF NEUTRAL DIFFERENTIAL EQUATIONS AND COQ

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ABSTRACT

This paper examines numerical approximation of neutral differential equations with infinite delay, asymptotic stability of infinite delay differential equations, half-discretization and first order numerical hyperbolic solution of partial differential equations. Selected composition and intersection are examples of certain theoretical operations on partial functions and the class of algebras which are isomorphic for a collection of functions equipped with these are examined. Typical issues addressed include whether the class in any first-order logic is axiomatizable or, in fact, finite axiomatizable, in which computational complexity its first-order theories are compared to, and whether or not it is decisive whether a finite algebra is a class one. The initial change to the fundamental image demands that the isomorphisms transform any existing entities into unions and/or infima, and analyze the resulting class. We demonstrate that supreme and infima criteria are equal to composition, intersection and anti-domain combined.

Keywords: - Number, Function, Partial, Real, COQ

I. INTRODUCTION

Rational numbers have been utilized for centuries to figure territories, charges, and so on. Calculus has been created in the eighteenth century and now all understudies are accustomed to knowing and processing with 3, 1 7, √ 13, exp(1), and π. Calculus and computations are even observed as an approach to choose the accepted "best" understudies. Real analysis is undoubtedly a decent experiment for understudies: knowing definitions and theorems, they are evaluated for thinking and accurately applying theorems. With regards to proof collaborators, the circumstance is the inverse: they are intended for thinking, yet are to be evaluated for definitions and theorems about real analysis. The intrigue of that work is that it gathers proof composed by power clients of those formal frameworks.

Partial functions are universal in mathematical practice. For instance, the division function/over the mind-boggling numbers (or some other field) is partial, since z1/z2 is possibly characterized if z2 6= 0. Another noticeable model is the square root function over the real numbers, which is just characterized for non-negative contentions. Partial functions show up in exceptionally fundamental arithmetic, for instance the deduction function over the characteristic numbers, just as in further developed science, for instance the Lebesgue indispensable function R over real functions, which maps Lebesgue basic functions to their Lebesgue integrals however is unclear on other real functions. As can be seen from these models, partial functions can be either unary or of higher arity; for rearranging the piece, we will focus on unary partial functions for the remainder of this presentation.

II. COQ: STANDARD LIBRARY AND C-CORN/MATHCLASSES

The formal language of Coq†† is based on the Calculus of Inductive Constructions which combines both a higher-order logic and a richly-typed functional programming language Programs can be extracted from proofs to external programming languages like OCaml, Haskell, or Scheme. As a proof development system, Coq provides interactive proof methods, decision and semi-decision algorithms, and a tactic language for letting the user define new proof methods. The Coq library is structured into two parts: the initial library, which contains elementary logical notions and data-types, and the standard library, a general-purpose library containing various developments and axiomatizations about sets, lists, sorting, arithmetic, real numbers, etc. The standard library reals are axiomatic and described in Section 3.1.2. Recent developments in Nijmegen led to a different library called C-CoRN and its successor MathClasses The main idea is to provide Bishop-like constructive mathematics. Another important library comes from the Mathematical Components project. It is a comprehensive formalization when it comes to number theory and algebraic structures, but real analysis is in its infancy, so we leave this library out of this survey.

III. REAL ANALYSIS FOR COQ

Coq Standard Library

As our library, the next two libraries use the Coq5 formal language. Coq is based on the Calculus of Inductive Constructions which combines both a higher-order logic and a richly-typed functional programming language.

This logic lacks several axioms that are available to many other formal systems. These include extensionality, Hilbert's operator ε or its variant the ι operator, and excluded middle.

Moreover, even if a predicate satisfies the excluded-middle property, Coq does not allow its truth value to be tested inside the body of a function; it can be decided only inside proofs.

Coq's Standard Axioms for Real Numbers

Traditionally, real numbers are represented by Cauchy Sequences or Dedekind Cuts. These representations are mathematically rigorous and expressible in Coq, and implemented in libraries like the Coq Repository at Nijmegen (CoRN). However, these representations are difficult both to express and to compute with.

Coq's standard library takes a very different approach to the real numbers: An axiomatic approach.

Module OurR.

Parameter R : Set

Delimit Scope R_scope with R.

Local Open Scope R_scope.

We'll start by declaring two real numbers - the important ones.

Parameter R0 : R.

Parameter R1 : R.

Along with methods for obtaining (some of) the others

Parameter Rplus : R → R → R.

Parameter Rmult : R → R → R.

Parameter Ropp : R → R.

Parameter Rinv : R → R.

Infix "+" := Rplus : R_scope.

Infix "*" := Rmult : R_scope.

Notation "- x" := (Ropp x) : R_scope.

Notation "/ x":= (Rinv x):R_scope.

Other basic operations are given in terms of our declared ones

Definition Rminus (r1 r2: R): R := r1 + - r2.

```
Definition Rdiv (r1 r2: R): R := r1 * / r2.

Infix "-”:=Rminus :R_scope.

Infix "/" := Rdiv : R_scope
```

We'd like to be able to convert natural numbers to Rs, thereby allowing ourselves to write numbers like 0, 1, 2, 3..

```
Fixpoint INR (n :nat) : R :=

  match n with

  | O ⇒ R0

  | 1 ⇒ R1

  | S n' ⇒ R1 + INR n'

  End
```

The standard library defines a coercion from Z to R which is slightly more useful but also more difficult to parse.

Coercion tells Coq to try applying a given function whenever types mismatch.

For instance, Rplus 4 5 will currently give a type error

```
Fail Check (4 + 5).

Coercion INR: nat>-> R.

Check 4 + 5.

Compute (4 + 5)
```

IV. LIMITED PRINCIPLE OF OMNISCIENCE

The conjunction of completeness and total order T causes any formula that satisfies the excluded-middle principle to become decidable. This strong property is of little interest for our development though. For doing real analysis, one can derive a more useful property from the axioms defining real numbers in Coq: the limited principle of omniscience (LPO). Let P be a decidable predicate on natural numbers. The LPO states that one can decide whether the property never holds. Moreover, if P(n) happens to hold for some number n, the LPO produces such a number. The original idea of the proof comes from [19]; the CoqTail project later improved it by removing the need for the not all ex not consequence of the excluded-middle axiom. We have improved it further by getting rid of the sizable amount of analysis it needed (geometric series, logarithm, and so on). Let us sketch our Coq proof. Since P is decidable, we can build a function f(n) that returns 1/(n+ 1) if P(n) holds and 0 otherwise. Let us consider the subset of real numbers $\{f(n) \mid n \in N\}$.

It is nonempty and bounded by 1, thus its supremum is given by completeness. This supremum can be tested against 0 by total order T. If it is zero, we deduce $\forall n, \neg P(n)$. Otherwise we compute its discrete inverse with archimed, which is a value n such that P(n) holds.

Compactness

Another important tool is the property of compactness, which has numerous applications in traditional mathematics. For instance, a function continuous on a compact set is uniformly continuous. Unfortunately, the compactness property is inherently classical, up to the point that constructive mathematics tend to redefine continuity so that it actually means uniform continuity in order to avoid compactness. Our goal is to stay as close as possible to traditional analysis, so dropping compactness is not under consideration. One of the definitions of a compact set is a set such that, from any cover with open sets, one can extract a finite subcover. Yet in most of the proofs we are interested in, we do not need the full strength of this property. Indeed, the extracted sets are useless; only their minimum diameter matters. Moreover, the finiteness property is only useful so that this minimum is nonzero.

V. CONCLUSION

First, we reiterate what was said in the introduction to this thesis: that partial functions have, ingeneral, more favourable logical and computational properties than binary relations. The results in thisthesis only reinforce this viewpoint. Consider those operations with a first-order definition—by whichwe mean definable in the manner required by the fundamental theorem, Theorem 3.1.6. It had alreadybeenestablishedthatwhenconsideringthesetypesofoperations,generallytherepresentationclassesarefinitelyaxiomatisableandhaveequationaltheoriesoflowcomplexity,thefiniterepresentationpropertyis satisfied, and representability of finite algebras is simple to decide. And it had been found that theseremarksextendtomultiplacefunctionsaswell.Thisisallincontrasttohowrelationsbehave.

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