HEAT AND MASS TRANSFER ON MHD FLOW OVER A VERTICAL STRETCHING SURFACE WITH CHEMICAL REACTION, HEAT SOURCE AND THERMAL STRATIFICATION EFFECTS

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ABSTRACT

An analysis has been carried out to obtain the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. An approximate numerical solution for the flow problem has been obtained by solving the governing equations using numerical technique. A magnetic field is applied transversely to the direction of the flow. Adopting the similarity transformation, governing nonlinear partial differential equations of the problem are transformed to nonlinear ordinary differential equations. Then the numerical solution of the problem is derived using Quasilinearization of Newton's method, for different values of the dimensionless parameters. The results obtained show that the flow field is influenced appreciably by the presence of thermal stratification, chemical reaction and magnetic field.

Keywords: Chemical reaction, Magnetic field, Heat source, Thermal stratification and Quasilinearization of Newton's method.

1. Introduction

In many mixed flows of practical importance in natural as well as in many engineering devices, the environment is thermally stratified. The discharge of hot fluid into enclosed region often results in a stable thermal stratification with lighter fluid overlying denser fluid. The thermal stratification effects of heat transfer over a stretching surface is of interest in polymer extrusion processes where the object, after passing through a die, enters the fluid for cooling below a certain temperature. The rate at which such objects are cooled has an important bearing on the properties of the final product. In the process of cooling the fluids, the momentum boundary layer for linear stretching of sheet was first studied by crane [1].

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with chemical and hydrometallurgical industries. In order to study the thermal stratification effects over the above-mentioned problem, an attempt has been made have to analyze the nonlinear hydro magnetic flow with heat and mass transfer over a vertical stretching surface with chemical reaction and thermal stratification effects.

In the past decades, the penetration theory of Hughie 1935 had been widely applied to unsteady state diffusion problems with and without chemical reaction. As far as we can ascertain, all the solution with chemical reaction were obtained for the case of a semi-infinite body of liquids, although physical absorption into a finite film was considered. Among some of the interesting problems which were studied are the analyses of laminar forced convection mass transfer with homogeneous chemical reaction, [2]. The effect of different values of Prandtl number of the fluid along the surface was analyzed by Gebhart [3]. A study on heat and mass transfer over a stretching surface with suction or blowing was carried out by Gupta and Gupta [4]. The same type of problem with inclusion of constant surface velocity and power-law temperature variation were studied by Soundalgekar and Ramamurthy [5]. Grubka and Bobba [6] studied the power-law temperature variations in the case of a stretching continuous surface. Chen and Char [7] investigated the effect of power-law temperature and power-law surface heat flux in the heat transfer characteristics of a continuous linear stretching surface. Atul Kumar Singh [8] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. The paper deals with the study of free convection and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical plate in the presence of large suction and under the influence of uniform magnetic field considering heat source and thermal diffusion. The problem of a stretching surface with constant surface temperature was analyzed by Noor Afzal [9]. Processes involving the mass transfer effect have a long been recognized as important principally in chemical processing equipment. Recently, Acharya et al. [10] have studied heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing.

Here we analyze the non linear hydromagnetic flow and chemical reaction, heat and mass transfer over a vertical stretching surface with thermal stratification and heat source hence we considered the problem of this kind.

2. Mathematical Formulation.

Two-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, and electrically conducting and Boussinesq's fluid flowing over a vertical stretching surface in the presence of a uniform magnetic field has been considered with heat, mass transfer, chemical reaction and thermal stratification effects. According to the coordinate system, the x-axis is chosen parallel to the vertical surface and the y-axis is taken normal to it. The fluid properties are assumed to be constant in a limited temperature range. The value of C_∞ is set as zero in the problem as the concentration of species far from the wall, C_∞, is infinitesimally small [11] and hence the Soret and Dufour effects are negligible. The chemical reactions are taking place in the flow and the physical properties ρ, μ, D and the rate of chemical reaction, k_1 are constant throughout the fluid. It is assumed that the induced magnetic field, the external electric field and the electric field due to polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion neglecting viscous and Joules dissipation under Boussinesq's approximation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \left(\frac{\sigma B_0^2}{\rho}\right)u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + Q(T_\infty - T) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1 C \tag{4}$$

The boundary conditions are

$U=U(x) = ax, \quad v = 0,$

$T=T_w(x), \quad C=C_w(x) \quad \text{at } y=0$ (5)

$u=0, T = T_\infty(x) = (1-n)\, T_o + n\, T_w(x), \quad C=C_\infty \quad \text{as} \quad y\to\infty$ (6)

Where 'a' is a dimensional constant and n is a constant which is the thermal stratification parameter and is such that 0 < n < 1. The n defined as thermal stratification parameter is equal to $m_1 / (1+m_1)$ of Nakayama and Koyama [12] where m_1 is a constant. T_o is constant reference temperature say, T_∞ (0). The suffixes w and ∞ denote surface and ambient conditions.

As in Acharya et al. [10] the following change of variables are introduced:

$$\psi = \left(\nu x U(x)\right)^{\frac{1}{2}} f(\eta)$$
$$\eta = \left(\frac{U(x)}{\nu x}\right)^{\frac{1}{2}} y \qquad (7)$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \qquad (8)$$

It can be easily verified that the continuity Eq. (1) is identically satisfied. Similarity solution exist if we assume that U(x) =ax and introduce the non-dimensional form of temperature and concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, Gr_x = \frac{g\beta\nu\left(T_w - T_\infty\right)}{U^3}, Gc_x = \frac{g\beta^*\nu\left(C_w - C_\infty\right)}{U^3}$$

$$P_r = \frac{\mu c_p}{K}, \quad S_c = \frac{\nu}{D}, \quad M^2 = \frac{\sigma B_0^2}{\rho a}. \quad \gamma = \nu k_1 / U^2, \; \mathrm{Re}_x = Ux/\nu \qquad (9)$$

In this work, temperature variation of the surface is taken into account and is also given by the power-law temperature, $T_w - T_\infty = Nx^n$ where N and n are constants. Also concentration variation is given by $C_w - C_\infty = N_1 x^{n^1}$ where N_1 and n^1 are constants. The nonlinear equations and boundary conditions are obtained as

$$f''' + Gc_x \operatorname{Re}_x \phi + Gr_x \operatorname{Re}_x \theta - (f')^2 - (M^2/\operatorname{Re}_x) f' + ff'' = 0 \qquad (10)$$

$$\theta'' - P_r f'(\theta + n/(1-n)) + p_r f\theta' - SP_r\theta = 0 \qquad (11)$$

$$\phi'' - S_c(\theta\gamma \operatorname{Re}_x + f'\phi) + S_c f\phi' = 0 \qquad (12)$$

The boundary conditions are given by

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ &f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \theta(\infty) = 0 \end{aligned} \qquad (13)$$

3. Method of solution

Eqs. (10) – (12) with boundary conditions (13) is solved using Quasilinearization of Newton's method.

4. Results and Discussions:

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The dimensionless velocity profiles for different values of magnetic field with constant chemical reaction parameter and thermal stratification parameter are presented in Fig. 1. It is observed that the velocity of them fluid decreases with the increase of magnetic parameter. Fig. 2 represents the dimensionless temperature profiles for different values of magnetic field with constant Chemical reaction parameter and thermal stratification parameter. It is clear that the temperature of the fluid increases with the increase of magnetic parameter.

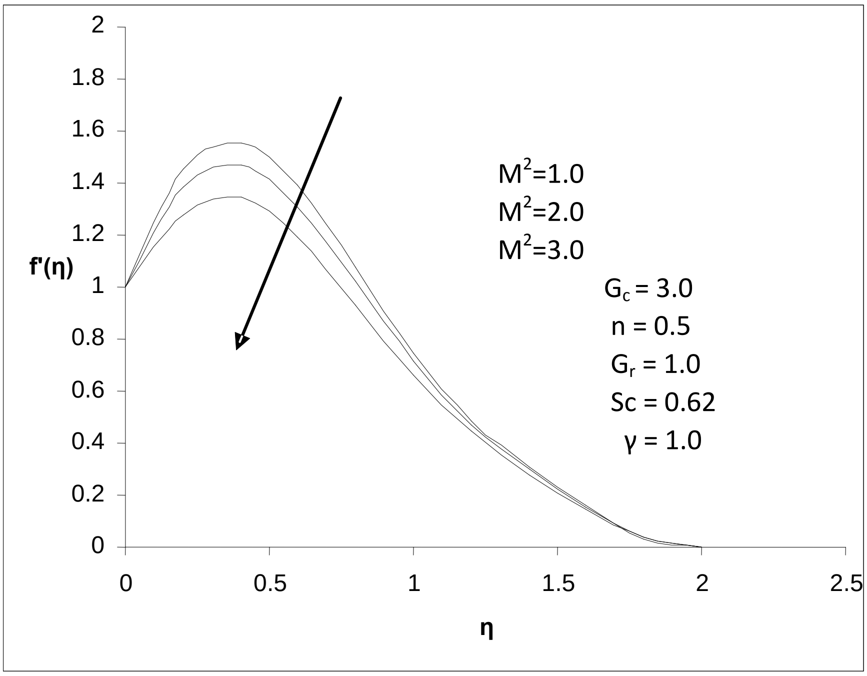

Fig (1). Velocity profiles for different magnetic parameter.

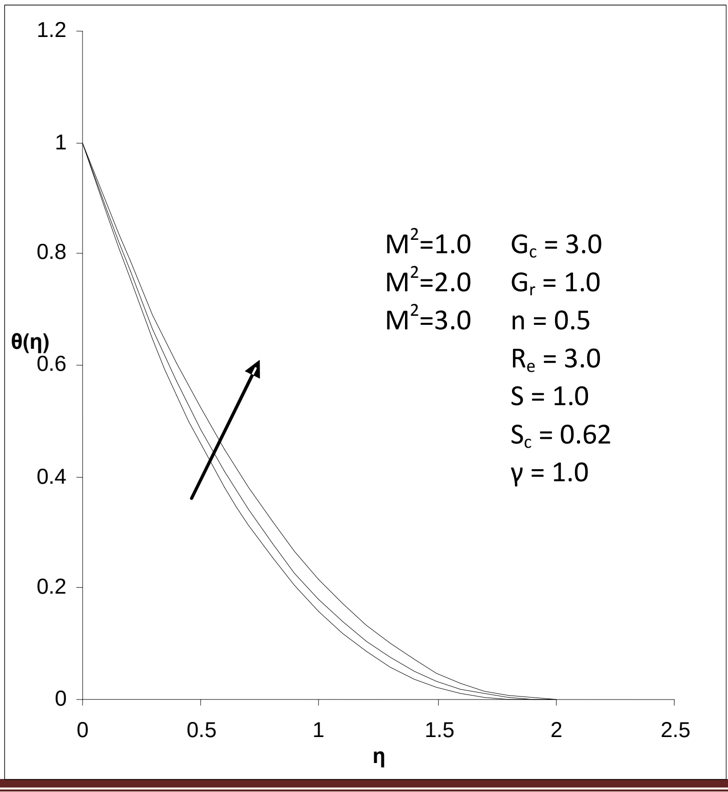

Fig (2) Magnetic effect over the temperature profiles.

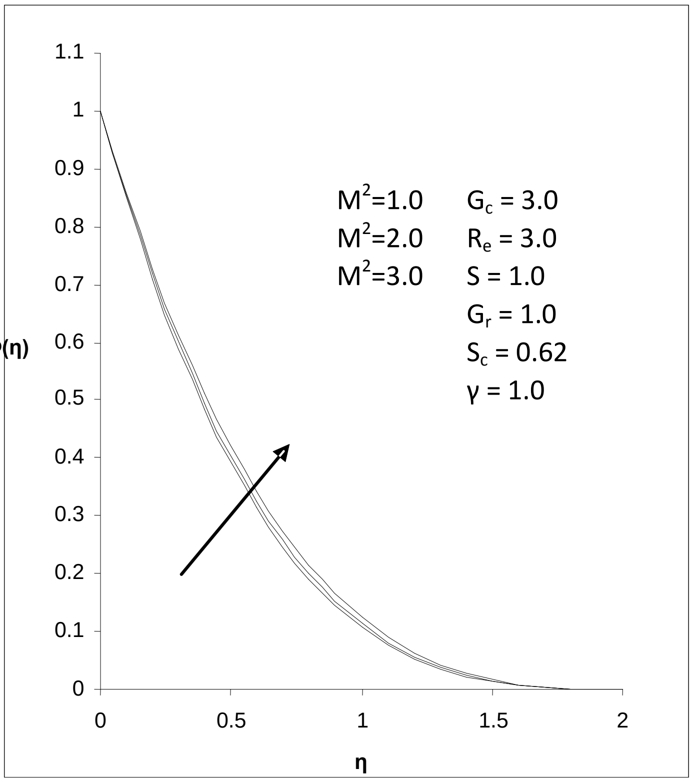

Fig (3) Magnetic effect over the concentration profiles.

The dimensionless concentration profiles for different values of magnetic field with constant chemical reaction parameter and thermal stratification parameter are demonstrated in Fig. (3). It is seen that the concentration of the fluid increases with the increase of magnetic parameter. Fig.(4) depicts the dimensionless velocity profiles for different values of thermal stratification parameter with constant chemical reaction parameter and the uniform magnetic field. It is observed that the velocity of the fluid decreases with the increase of thermal stratification parameter. The dimensionless temperature profiles for different values of thermal stratification parameter with constant chemical reaction parameter and the uniform magnetic field are shown in Fig. (5). It is clear that the temperature of the fluid decreases with the increase of thermal stratification parameter

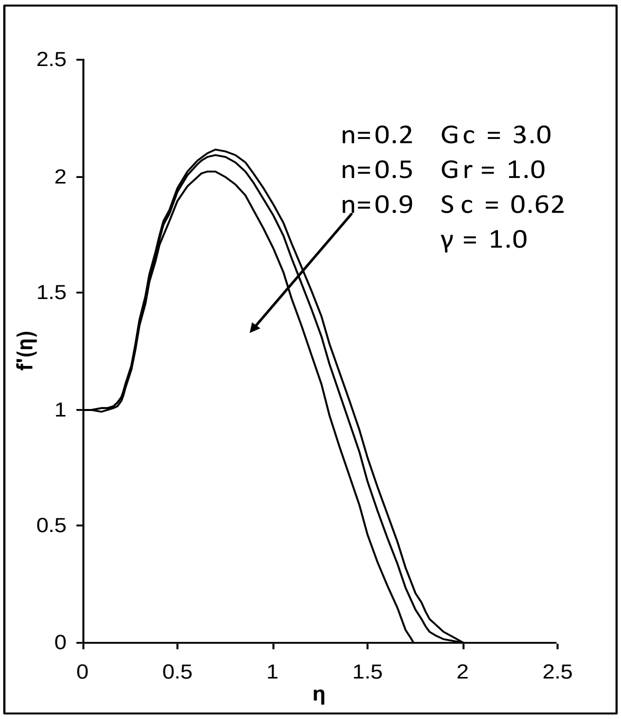

Fig (4) Thermal stratification effects over the velocity profiles.

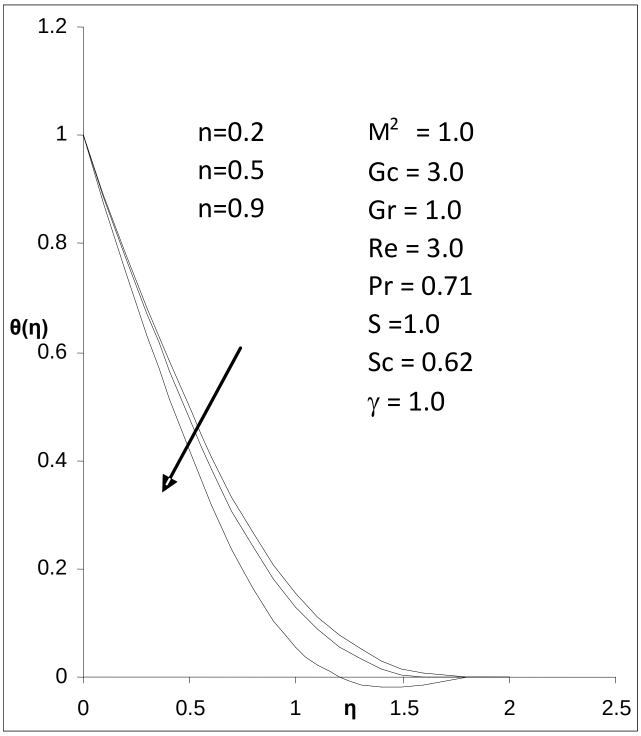

Fig (5) Thermal stratification effects over the temperature profiles.

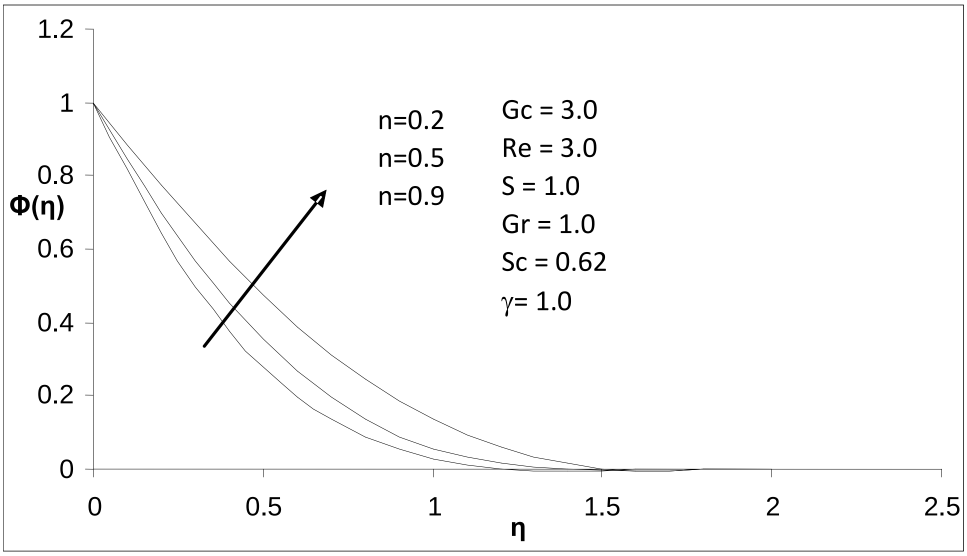

Fig (6) Thermal stratification over the concentration profiles.

Fig. (6) demonstrates the dimensionless concentration profiles for different values of thermal stratification parameter with constant chemical reaction parameter and the uniform magnetic field. It is seen that the concentration increases with the increase of thermal stratification parameter. The dimensionless velocity profiles for different values of chemical reaction parameter c with uniform magnetic field and constant thermal stratification parameter are depicted in Fig.(7). It is observed that the velocity of the fluid decreases with the increase of Chemical reaction parameter.

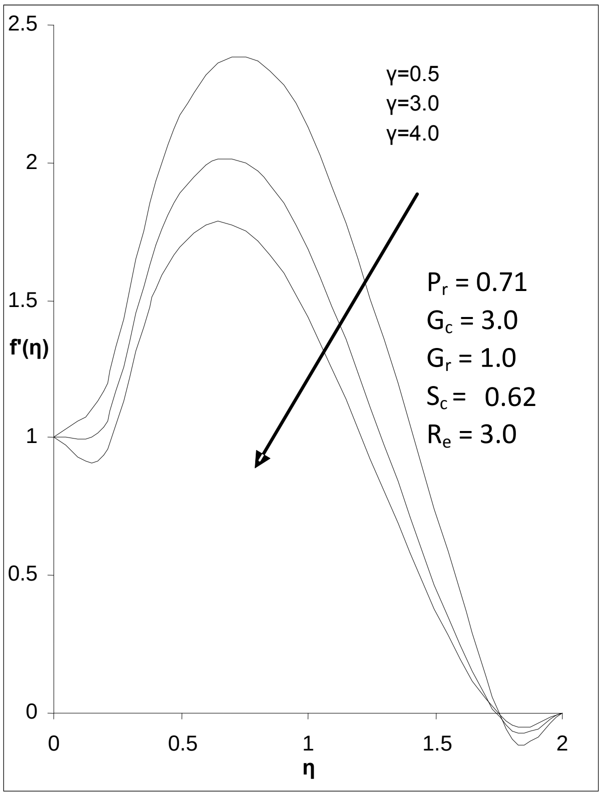

Fig (7) Chemical reaction over the velocity profiles.

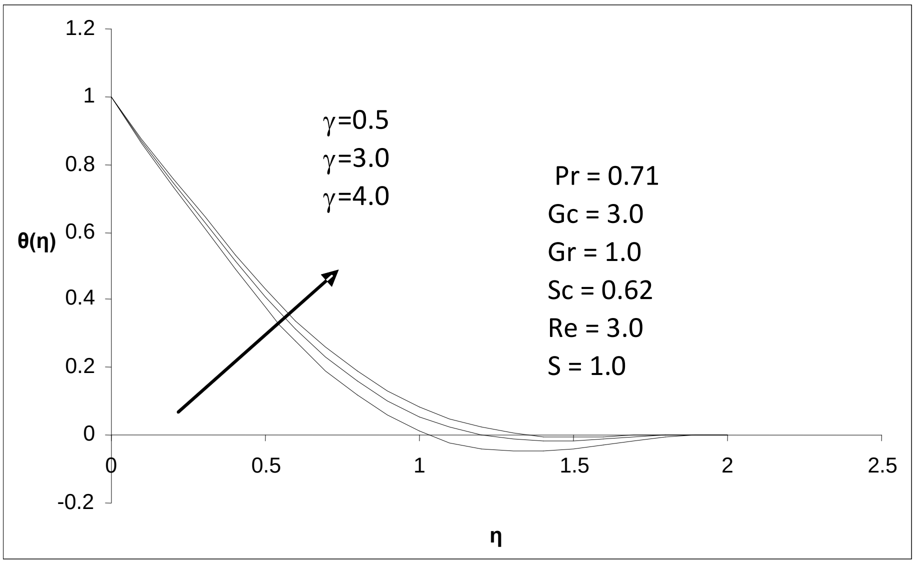

Fig (8) Chemical reaction over the temperature profiles.

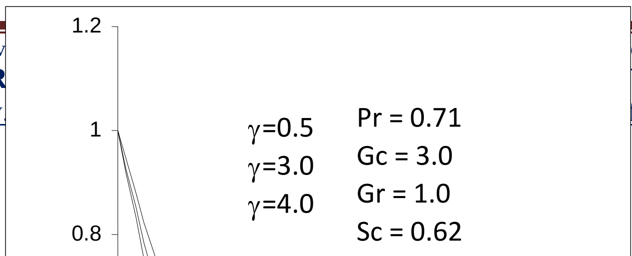

Fig (9) Chemical reaction over the concentration profiles.

The dimensionless temperature profiles for different values of chemical reaction parameter with uniform magnetic field and constant thermal stratification parameter are displayed in Fig.(8). It is seen that the temperature of the fluid increases with the increase of chemical reaction parameter. The concentration of the fluid decreases with the increase of chemical reaction parameter and this is noted through Fig (9).

5. Conclusions

(1) Due to the uniform magnetic field and thermal stratification parameter, the velocity and concentration of the fluid decrease and the temperature of the fluid increases with the increase of chemical reaction parameter.

(2). In the case of constant chemical reaction and thermal stratification parameter, the velocity of the fluid decreases and the temperature and concentration of the fluid increase with the increase of magnetic parameter.

(3). Due to uniform magnetic field with constant chemical reaction parameter, the velocity and the temperature of the fluid decrease and the concentration of the fluid increases with the increase of thermal stratification parameter.

(4). A comparison of velocity profiles shows that the velocity increases near the plate and thereafter decreases.

(5). It is to note that an increase in magnetic field leads to a rise in temperature at slow rate in comparison to the velocity profiles.

(6). The skin friction, rate of mass transfer decreases and rate of heat transfer increases with increasing magnetic parameter.

(7). With increasing chemical reaction parameter skin friction, rate of mass transfer decreases while rate of heat transfer increases.

(8). The skin friction, rate of heat and mass transfer decreases with increasing thermal stratification parameter.

It is hoped that the present investigation of the study of physics of flow over a vertical surface can be utilized as the basis for many scientific and engineering applications and for studying more complex vertical problems. The findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

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