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SOME RESULTS ON BIPOLAR FUZZY GRAPHS

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ABSTRACT

In this paper, strongly irregular bipolar fuzzy graphs and strongly total irregular bipolar fuzzy graphs are introduced. Some results on strongly irregular bipolar fuzzy graphs and strongly total irregular bipolar fuzzy graphs are established.

Keywords: Degree of bipolar fuzzy graph, regular bipolar fuzzy graph, irregular bipolar fuzzy graph, highly irregular bipolar fuzzy graph, strongly irregular bipolar fuzzy graph and strongly total irregular bipolar fuzzy graph.

1. Introduction: In 1965,L.A.Zadeh[1] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram[2] introduced the concept of bipolar fuzzy graphs and defined different operations on it. A.Nagoorgani and K.Radha[3,4] introduced the concept of regular fuzzy graphs in 2008 and discussed about the degree of a vertex in some fuzzy graphs. K.Radha and N.Kumaravel[5] introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs. S.Arumugam and S.Velammal[6] discussed edge domination in fuzzy graphs. A.Nagoorgani and M.Baskar Ahamed[7] discussed order and size in fuzzy graph. A.Nagoorgani and J.Malarvizhi [8] discussed properties of μ complement of a fuzzy graph. In this paper we introduce strongly irregular bipolar fuzzy graph and strongly total irregular bipolar fuzzy graph. We provide some results on strongly irregular bipolar fuzzy graphs and strongly total irregular bipolar fuzzy graphs.

2.PRELIMINARIES.

In this section, some basic definitions and preliminary ideas are given which is useful for proving theorems.

2.1. Bipolar fuzzy graph [1]: By a bipolar fuzzy graph we mean a pair $G=(A,B)$ where $A=(\mu_A{}^P,\mu_A{}^N)$ is a bipolar fuzzy set in V and $B=(\mu_B{}^P,\mu_B{}^N)$ is a bipolar fuzzy relation on E, such that

$\mu_B{}^P(xy)\le\min(\mu_A{}^P(x),\mu_A{}^P(y))$ and $\mu_B{}^N(xy)\ge\max(\mu_A{}^N(x),\mu_A{}^N(y))$

2.2. Degree of a vertex [4]: Let $G=(A,B)$ be a bipolar fuzzy graph on $G^*=(V,E)$. The degree of a vertex $(\mu_A{}^P(x),\mu_A{}^N(x))$ is $d_G(\mu_A{}^P(x),\mu_A{}^P(x))$ =$\sum_{x\neq y}(\mu_B{}^P(xy),\mu_B{}^N(xy))$ = $\sum_{xy\in E}(\mu_B{}^P(xy),\mu_B{}^N(xy))$.

2.3. Total degree of a vertex [8]: Let $G=(A,B)$ be a bipolar fuzzy graph on $G^*=(V,E)$. The total degree of a vertex $(\mu_A{}^P(x),\mu_A{}^N(x))$ is $t\,d_G(\mu_A{}^P(x),\mu_A{}^N(x))$ =$\sum_{\substack{x\neq y\\ xy\in E}}(\mu_B{}^P(xy),\mu_B{}^N(xy))+(\mu_A{}^P(x),\mu_A{}^N(x))$.

2.4. Complement of a bipolar fuzzy graph [15]: The complement of a bipolar fuzzy graph $G=(A,B)$ is a bipolar fuzzy graph $G^c=(A^c,B^c)$. Where $(\mu_A{}^P(x),\mu_A{}^N(x)) = (\mu_{A^c}{}^P(x),\mu_{A^c}{}^N(x))$

$\left(\mu_{B^c}{}^P(xy),\mu_{B^c}{}^N(xy)\right)=\left(\mu_A{}^P(x),\mu_A{}^N(x)\right)\cap\left(\mu_A{}^P(y),\mu_A{}^N(y)\right)-\left(\mu_B{}^P(xy),\mu_B{}^N(xy)\right)$

For all $\left(\mu_A{}^P(x),\mu_A{}^N(x)\right),\left(\mu_A{}^P(y),\mu_A{}^N(y)\right)$ $in\ V$.

2.5. Irregular bipolar fuzzy graph [2]: Let $G=(A,B)$ be a bipolar fuzzy graph. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

2.6. Complete bipolar fuzzy graph [2]: A bipolar fuzzy graph $G=(A,B)$ is a complete bipolar fuzzy graph

$\mu_B{}^P(xy)=\min(\mu_A{}^P(x),\mu_A{}^P(y))$ and $\mu_B{}^N(xy)=\max(\mu_A{}^N(x),\mu_A{}^N(y))$.

2.7. Neighbourly irregular bipolar fuzzy graph [2]: Let $G=(A,B)$ be a connected bipolar fuzzy graph. G is said to be a neighbourly irregular bipolar fuzzy graph, if every two adjacent vertices of G have distinct degrees

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2.8. Total irregular bipolar fuzzy graph [2]: Let $G=(A,B)$ be a bipolar fuzzy graph. Then G is total irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

2.9. Neighbourly total irregular bipolar fuzzy graph [2]: If every two adjacent vertices of a bipolar fuzzy graph $G = (A, B)$ have distinct total degree, then G is said to be a neighbourly total irregular bipolar fuzzy graph.

2.10. Highly irregular bipolar fuzzy graph [2]: Let $G = (A, B)$ be a connected bipolar fuzzy graph. G is said to be a highly irregular bipolar fuzzy graph, if every vertex of G is adjacent to vertices with distinct degrees.

2.11. Highly total irregular bipolar fuzzy graph [2]: Let $G = (A, B)$ be a connected bipolar fuzzy graph. G is said to be a highly total irregular bipolar fuzzy graph, if every vertex of G is adjacent to vertices with distinct total degrees.

3. Properties of strongly irregular bipolar fuzzy graphs.

3.1. Strongly irregular bipolar fuzzy graph [15]: Let $G = (A, B)$ be a connected bipolar fuzzy graph. G is said to be a strongly irregular bipolar fuzzy graph, if every pair of vertices in G have distinct degrees.

3.2. Example. Define $G = (A, B)$ by $\left({\mu_A}^P(u), {\mu_A}^N(u)\right) = (0.8, -0.6)$, $\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = (0.5, -0.4)$, $\left({\mu_A}^P(w), {\mu_A}^N(w)\right) = (0.7, -0.9)$, $\left({\mu_A}^P(x), {\mu_A}^N(x)\right) = (0.6, -0.8)$ and $\left({\mu_B}^P(uv), {\mu_B}^N(uv)\right) = (0.3, -0.4)$, $\left({\mu_B}^P(ux), {\mu_B}^N(ux)\right) = (0.6, -0.5)$, $\left({\mu_B}^P(wv), {\mu_B}^N(wv)\right) = (0.4, -0.3)$, $\left({\mu_B}^P(wx), {\mu_B}^N(wx)\right) = (0.6, -0.8)$ and $d\left({\mu_A}^P(u), {\mu_A}^N(u)\right) = (0.9, -0.9)$, $d\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = (0.7, -0.7)$, $d\left({\mu_A}^P(w), {\mu_A}^N(w)\right) = (1.0, -1.1)$, $d\left({\mu_A}^P(x), {\mu_A}^N(x)\right) = (1.2, -1.3)$.

3.3. Proposition. A neighbourly irregular bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

3.4. Example. Define $G = (A, B)$ by $\left({\mu_A}^P(u), {\mu_A}^N(u)\right) = (0.6, -0.6)$, $\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = (0.7, -0.5)$, $\left({\mu_A}^P(w), {\mu_A}^N(w)\right) = (0.8, -0.7)$, $\left({\mu_A}^P(x), {\mu_A}^N(x)\right) = (0.5, -0.8)$ and $\left({\mu_B}^P(uv), {\mu_B}^N(uv)\right) = (0.6, -0.5)$, $\left({\mu_B}^P(ux), {\mu_B}^N(ux)\right) = (0.2, -0.3)$, $\left({\mu_B}^P(wv), {\mu_B}^N(wv)\right) = (0.1, -0.4)$, $\left({\mu_B}^P(wx), {\mu_B}^N(wx)\right) = (0.5, -0.6)$, $d\left({\mu_A}^P(u), {\mu_A}^N(u)\right) = (0.8, -0.8)$, $d\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = (0.7, -0.9)$, $d\left({\mu_A}^P(w), {\mu_A}^N(w)\right) = (0.6, -1.0)$, $d\left({\mu_A}^P(x), {\mu_A}^N(x)\right) = (0.7, -0.9)$.

Here $d\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = d\left({\mu_A}^P(x), {\mu_A}^N(x)\right)$

So G is not strongly irregular bipolar fuzzy graph.

3.5. Proposition. A highly irregular bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

3.6. Example. Define $G = (A, B)$ by $\left({\mu_A}^P(u), {\mu_A}^N(u)\right) = (0.4, -0.8)$, $\left({\mu_A}^P(v), {\mu_A}^N(v)\right) = (0.3, -0.6)$, $\left({\mu_A}^P(w), {\mu_A}^N(w)\right) = (0.5, -0.7)$, $\left({\mu_A}^P(x), {\mu_A}^N(x)\right) = (0.7, -0.9)$ and $\left({\mu_B}^P(uv), {\mu_B}^N(uv)\right) = (0.3, -0.6)$, $\left({\mu_B}^P(ux), {\mu_B}^N(ux)\right) = (0.4, -0.5)$, $\left({\mu_B}^P(wv), {\mu_B}^N(wv)\right) = (0.2, -0.4)$, $\left({\mu_B}^P(wx), {\mu_B}^N(wx)\right) = (0.3, -0.6)$,

$d(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.7, -1.1)$, $d(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.5, -1.0)$, $d(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.5, -1.0)$, $d(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.7, -1.1)$.

Here $d(\mu_A{}^P(u), \mu_A{}^N(u)) = d(\mu_A{}^P(x), \mu_A{}^N(x))$

and $d(\mu_A{}^P(v), \mu_A{}^N(v)) = d(\mu_A{}^P(w), \mu_A{}^N(w))$.

3.7. Proposition. A total irregular bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

3.8. Example. Define $G = (A, B)$ by $(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.3, -0.6)$, $(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.4, -0.7)$, $(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.5, -0.8)$, $(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.6, -0.9)$ and $(\mu_B{}^P(uv), \mu_B{}^N(uv)) = (0.2, -0.5)$, $(\mu_B{}^P(ux), \mu_B{}^N(ux)) = (0.2, -0.5)$, $(\mu_B{}^P(wv), \mu_B{}^N(wv)) = (0.4, -0.6)$, $(\mu_B{}^P(wx), \mu_B{}^N(wx)) = (0.4, -0.6)$,
$d(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.4, -1.0)$, $d(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.6, -1.1)$, $d(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.8, -1.2)$, $d(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.6, -1.1)$,
$t\,d(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.7, -1.6)$, $t\,d(\mu_A{}^P(v), \mu_A{}^N(v)) = (1.0, -1.8)$, $t\,d(\mu_A{}^P(w), \mu_A{}^N(w)) = (1.3, -2.0)$, $t\,d(\mu_A{}^P(x), \mu_A{}^N(x)) = (1.2, -2.0)$.

3.9. Theorem. If $G = (A, B)$ is a strongly irregular bipolar fuzzy graph then it is both highly irregular bipolar fuzzy and neighbourly irregular bipolar fuzzy graph.

Proof.

If G is strongly irregular bipolar fuzzy graph, then every pair of vertices in G have distinct degrees. Obviously every two adjacent vertices have distinct degrees and every vertex of G adjacent vertices with distinct degrees. Hence G is neighbourly irregular and highly irregular bipolar fuzzy graph.

3.10. Proposition. A highly irregular bipolar fuzzy and neighbourly irregular bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

3.11. Example. Define $G = (A, B)$ by $(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.4, -0.6)$, $(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.3, -0.9)$, $(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.5, -0.8)$, $(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.7, -0.7)$, $(\mu_A{}^P(y), \mu_A{}^N(y)) = (0.7, -0.6)$ and $(\mu_B{}^P(uv), \mu_B{}^N(uv)) = (0.3, -0.5)$, $(\mu_B{}^P(ux), \mu_B{}^N(ux)) = (0.1, -0.6)$, $(\mu_B{}^P(wv), \mu_B{}^N(wv)) = (0.1, -0.6)$, $(\mu_B{}^P(vy), \mu_B{}^N(vy)) = (0.2, -0.4)$, $d(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.4, -1.1)$, $d(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.6, -1.5)$, $d(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.1, -0.6)$, $d(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.1, -0.6)$, $d(\mu_A{}^P(y), \mu_A{}^N(y)) = (0.2, -0.4)$.

3.12. Proposition. A complete bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

3.13. Example. Define $G = (A, B)$ by $(\mu_A{}^P(u), \mu_A{}^N(u)) = (0.8, -0.6)$, $(\mu_A{}^P(v), \mu_A{}^N(v)) = (0.7, -0.7)$, $(\mu_A{}^P(w), \mu_A{}^N(w)) = (0.7, -0.8)$, $(\mu_A{}^P(x), \mu_A{}^N(x)) = (0.9, -0.6)$, $(\mu_A{}^P(y), \mu_A{}^N(y)) = (0.5, -0.2)$ and $(\mu_B{}^P(uv), \mu_B{}^N(uv)) = (0.7, -0.6)$, $(\mu_B{}^P(uy), \mu_B{}^N(uy)) = (0.5, -0.2)$, $(\mu_B{}^P(wv), \mu_B{}^N(wv)) = (0.7, -0.7)$, $(\mu_B{}^P(wx), \mu_B{}^N(wx)) = (0.7, -0.6)$, $(\mu_B{}^P(xy), \mu_B{}^N(xy)) = (0.5, -0.2)$, $d(\mu_A{}^P(u), \mu_A{}^N(u)) = (1.2, -0.8)$, $d(\mu_A{}^P(v), \mu_A{}^N(v)) = (1.4, -1.3)$, $d(\mu_A{}^P(w), \mu_A{}^N(w)) = (1.4, -1.3)$, $d(\mu_A{}^P(x), \mu_A{}^N(x)) = (1.2, -0.8)$, $d(\mu_A{}^P(y), \mu_A{}^N(y)) = (1.0, -0.4)$.

4. PROPERTIES OF STRONGLY TOTAL IRREGULAR BIPOLAR FUZZY GRAPH

4.1. Definition. Let $G = (A, B)$ be a connected bipolar fuzzy graph. G is said to be a strongly total irregular bipolar fuzzy graph, if every pair of vertex in G have distinct total degrees.

4.2. Example. Define $G = (A, B)$ by $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.4, -0.8)$, $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.5, -0.3)$, $\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.6, -0.9)$, $\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.3, -0.5)$, $\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (0.7, -0.6)$ and $\left(\mu_B{}^P(uv), \mu_B{}^N(uv)\right) = (0.4, -0.2)$, $\left(\mu_B{}^P(uy), \mu_B{}^N(uy)\right) = (0.3, -0.4)$, $\left(\mu_B{}^P(wv), \mu_B{}^N(wv)\right) = (0.1, -0.6)$, $\left(\mu_B{}^P(wx), \mu_B{}^N(wx)\right) = (0.2, -0.3)$, $\left(\mu_B{}^P(xy), \mu_B{}^N(xy)\right) = (0.3, -0.4)$, $d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.7, -0.6)$, $d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.5, -0.8)$, $d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.3, -0.9)$, $d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.5, -0.7)$, $d\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (0.6, -0.8)$ and $t\,d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (1.1, -1.4)$, $t\,d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (1.0, -1.1)$, $t\,d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.9, -1.8)$, $t\,d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.8, -1.2)$, $t\,d\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (1.3, -1.4)$

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4.3. Proposition. A strongly irregular bipolar fuzzy graph need not be a strongly total irregular bipolar fuzzy graph.

4.4. Example. Define $G = (A, B)$ by $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.7, -0.6)$, $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.5, -0.5)$, $\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.8, -0.9)$, $\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.5, -0.4)$ and $\left(\mu_B{}^P(uv), \mu_B{}^N(uv)\right) = (0.2, -0.3)$, $\left(\mu_B{}^P(vw), \mu_B{}^N(vw)\right) = (0.5, -0.5)$, $\left(\mu_B{}^P(ux), \mu_B{}^N(ux)\right) = (0.3, -0.4)$, $\left(\mu_B{}^P(wx), \mu_B{}^N(wx)\right) = (0.3, -0.5)$, $d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.5, -0.7)$, $d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.7, -0.8)$, $d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.8, -1.0)$, $d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.6, -0.9)$, and $t\,d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (1.2, -1.3)$, $t\,d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (1.2, -1.3)$, $t\,d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (1.6, -1.9)$, $t\,d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (1.1, -1.3)$.

Here $t\,d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = t\,d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right)$

Therefore, The graph G is strongly irregular bipolar fuzzy graph but not a strongly total irregular bipolar fuzzy graph.

4.5. Proposition. A strongly total irregular bipolar fuzzy graph need not be a strongly irregular bipolar fuzzy graph.

4.6. Example. Define $G = (A, B)$ by $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.4, -0.7)$, $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.5, -0.5)$, $\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.6, -0.6)$, $\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.1, -0.8)$, $\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (0.3, -0.9)$and $\left(\mu_B{}^P(uv), \mu_B{}^N(uv)\right) = (0.2, -0.3)$, $\left(\mu_B{}^P(uy), \mu_B{}^N(uy)\right) = (0.3, -0.6)$, $\left(\mu_B{}^P(wv), \mu_B{}^N(wv)\right) = (0.4, -0.4)$, $\left(\mu_B{}^P(wx), \mu_B{}^N(wx)\right) = (0.1, -0.5)$, $\left(\mu_B{}^P(yx), \mu_B{}^N(yx)\right) = (0.1, -0.5)$, $d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.5, -0.9)$, $d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.6, -0.7)$, $d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.5, -0.9)$, $d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.2, -1.0)$, $d\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (0.4, -1.1)$and $t\,d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.9, -1.6)$, $t\,d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (1.1, -1.2)$, $t\,d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (1.1, -1.5)$, $t\,d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.3, -1.8)$, $t\,d\left(\mu_A{}^P(y), \mu_A{}^N(y)\right) = (0.7, -2.0)$

Here $d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right)$

Therefore, The graph G is strongly total irregular bipolar fuzzy graph but not a strongly irregular bipolar fuzzy graph.

4.7. Proposition. If $G = (A, B)$ is a strongly irregular bipolar fuzzy graph. Then G^cneed not be a strongly irregular bipolar fuzzy graph.

4.8. Example. Define $G = (A, B)$ by $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.3, -0.9)$, $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.6, -0.7)$, $\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.7, -0.5)$, $\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.5, -0.8)$and
$\left(\mu_B{}^P(uv), \mu_B{}^N(uv)\right) = (0.2, -0.5)$, $\left(\mu_B{}^P(ux), \mu_B{}^N(ux)\right) = (0.2, -0.6)$, $\left(\mu_B{}^P(wv), \mu_B{}^N(wv)\right) = (0.4, -0.4)$, $\left(\mu_B{}^P(wx), \mu_B{}^N(wx)\right) = (0.3, -0.4)$,
$d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.4, -1.1)$, $d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.6, -0.9)$, $d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.7, -0.8)$, $d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.5, -1.0)$ and $\left(\mu_{B^c}{}^P(uv), \mu_{B^c}{}^N(uv)\right) = (0.1, -0.2)$,
$\left(\mu_{B^c}{}^P(vw), \mu_{B^c}{}^N(vw)\right) = (0.2, -0.1)$, $\left(\mu_{B^c}{}^P(ux), \mu_{B^c}{}^N(ux)\right) = (0.1, -0.2)$, $\left(\mu_{B^c}{}^P(wx), \mu_{B^c}{}^N(wx)\right) = (0.2, -0.1)$,
$d^c\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.2, -0.4)$, $d^c\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.3, -0.3)$, $d^c\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.4, -0.2)$, $d^c\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.3, -0.3)$
Here $d^c\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right)$
Therefore, The graph G^c is not strongly irregular bipolar fuzzy graph.

4.9. Proposition. A complete bipolar fuzzy graph need not be a strongly total irregular bipolar fuzzy graph.

4.10. Example. Define $G = (A, B)$ by $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (0.5, -0.4)$, $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (0.6, -0.5)$, $\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (0.7, -0.7)$, $\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (0.8, -0.8)$and
$\left(\mu_B{}^P(uv), \mu_B{}^N(uv)\right) = (0.5, -0.4)$, $\left(\mu_B{}^P(ux), \mu_B{}^N(ux)\right) = (0.5, -0.4)$, $\left(\mu_B{}^P(wv), \mu_B{}^N(wv)\right) = (0.6, -0.5)$, $\left(\mu_B{}^P(wx), \mu_B{}^N(wx)\right) = (0.7, -0.7)$,
$d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (1.0, -0.8)$, $d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (1.1, -0.9)$, $d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (1.3, -1.2)$, $d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (1.2, -1.1)$, $t\,d\left(\mu_A{}^P(u), \mu_A{}^N(u)\right) = (1.5, -1.2)$,
$t\,d\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) = (1.7, -1.4)$, $t\,d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = (2.0, -1.9)$, $t\,d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right) = (2.0, -1.9)$
Here $t\,d\left(\mu_A{}^P(w), \mu_A{}^N(w)\right) = t\,d\left(\mu_A{}^P(x), \mu_A{}^N(x)\right)$
Therefore, The graph G is not strongly total irregular bipolar fuzzy graph.

4.11. Theorem. Let $G = (A, B)$ be a bipolar fuzzy graph and $(\mu_A{}^p, \mu_A{}^N)$ be a constant function. Then G is strongly irregular bipolar fuzzy graph if and only if G is strongly total irregular bipolar fuzzy graph.
Proof.

Let $G = (A, B)$ be a bipolar fuzzy graph and $(\mu_A{}^p, \mu_A{}^N)$ be a constant function.
Assume that $G = (A, B)$ is a strongly irregular bipolar fuzzy graph. That is every pair of of vertices in G has distinct degrees.
Claim:

G is strongly total irregular bipolar fuzzy graph.

Since G is strongly irregular bipolar fuzzy graph, the vertices in G have distinct degrees.

Let $(\mu_A{}^p(u_1), \mu_A{}^N(u_1)), (\mu_A{}^p(u_2), \mu_A{}^N(u_2)), , \ldots \ldots . (\mu_A{}^p(u_n), \mu_A{}^N(u_n))$ be the vertices of G.

Let $d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = K_i$ and $d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) = K_j$ where $K_i \neq K_j$

Also $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = C$ for all $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \in V(G)$ where C is constant, $C \in [-1,1]$

$\therefore t\, d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) + (\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = K_i + C$

And $t\, d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) = d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) + \left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) = K_j + C$

To prove:

$t\, d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq t\, d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$

Since $k_i \neq K_j$

$K_i + C \neq K_j + C$

$t\, d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq t\, d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$ for all $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)), \left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) \in V(G)$

Thus G is total strongly irregular bipolar fuzzy graph.

Conversely assume that G is strongly total irregular bipolar fuzzy graph.

Claim: G is strongly irregular bipolar fuzzy graph.

Let $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)), \left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$ be any two vertices in G, and $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = C$ for all i where C is constant and $C \in [-1,1]$.

Since G is strongly total irregular,

$t\, d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq t\, d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$.

Consider the vertices $(\mu_A{}^P(u_i), \mu_A{}^N(u_i))$ *and* $\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$ have degrees K_i and K_j respectively.

That is $d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = K_i$ and $d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) = K_j$

And $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) = c$ for all i

To prove

$d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$

Since $t\, d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq t\, d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$

$d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) + (\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) + \left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$

$K_i + C \neq K_j + C$

$K_i \neq K_j$

$d(\mu_A{}^P(u_i), \mu_A{}^N(u_i)) \neq d\left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right)$ for all $(\mu_A{}^P(u_i), \mu_A{}^N(u_i)), \left(\mu_A{}^P(u_j), \mu_A{}^N(u_j)\right) \in V(G)$

Therefore the degree of any two vertices of the graph G are distinct. Hence G is strongly irregular bipolar fuzzy graph.

5. CONCLUSION: In this paper, we have found some relationship between strongly irregular bipolar fuzzy graphs and strongly total irregular bipolar fuzzy graphs and studied some results between neighbourly irregular and neighbourly total irregular bipolar fuzzy graphs. The results discussed may be useful for further study on strongly vertex irregular bipolar fuzzy graphs.

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