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TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM WITH TWO THRESHOLDS, DIFFERENT EPOCHS FOR EXITS AND CORRELATED INTER-DECISIONS

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ABSTRACT

In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment involving two thresholds for a single grade manpower system with attrition generated by its policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times are exchangeable and constantly correlated exponential random variables and inter- exit times form an ordinary renewal process. The analytical results are numerically illustrated with relevant findings by assuming specific distributions.

Keywords

Single grade manpower system; decision and exit epochs; correlated inter-decision times; univariate policy of recruitment with two thresholds; ordinary renewal process; variance of the time to recruitment.

1. Introduction

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2, 19] the authors have discussed manpower planning models using

Markovian and renewal theoretic approach. In [6] the author has considered a single grade manpower system and obtained system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequence by employing a univariate policy of recruitment. In [8, 10, 12] the authors have studied the problem of time to recruitment in the single grade manpower system with attrition for different types of thresholds. In [9] the author has obtained the optimum cost of recruitment for a single grade manpower system using several univariate and bivariate policies of recruitment. The authors in [5, 11, 15, 16, 17, 18] have studied this problem corresponding to correlated inter-decision times under different policies of recruitment. In [7] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in [3] and the variance of time to recruitment is obtained when the inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. In [4] the authors have studied their work in [3] using a different probabilistic analysis. Recently, in [20] the authors have studied the work in [3] by considering optional and mandatory thresholds which is a variation from the work of [7] in the context of considering non-instantaneous exits at decision epochs. In the present paper, for a single grade manpower system, a mathematical model is constructed in which attrition due to policy decisions takes place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the system has different epochs for policy decisions and exits and the inter-exit times form an ordinary renewal process. The present paper extends the research work in [20] for exchangeable and constantly correlated exponential inter-decision times.

2. Model Description

Consider an organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th} exit point and S_k be the total loss of manpower up to the first k exit points. It is assumed that X_i's are independent and identically distributed random variables with probability density function m(.), distribution function M(.) and mean $\frac{1}{\alpha}$(α>0). Let U_k be the continuous random variable representing the time between the (k-1)th and kth policy decisions. It is assumed that U_k's are exchangeable and constantly correlated exponential random variables with probability density function $f(.)$, distribution function F(.) and mean u. Let R be the correlation between U_i and U_j, $i \neq j$ and v = u(1-R). Let W_i be the continuous random variable representing the time between the (i-1)th and ith exit times. It is assumed that W_i's are independent and identically distributed random variables with probability density function g(.), probability distribution function G(.). Let $N_e(t)$ be the number of exit points lying in (0,t]. Let Y be the optional threshold level and Z the mandatory threshold level (Y< Z) for the cumulative depletion of manpower in the organization with probability density function h(.) and distribution function H(.). Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that $q \neq 0$. Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function l(.), mean E(T) and variance V(T). Let $A^*(.)$ be the Laplace - Stieltjes transform of A(.). The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of manpower in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower exceeds the optional threshold.

3. Main Result

In this section, analytical expressions for the performance measures namely mean and variance of the time to recruitment are obtained for different threshold distributions.

P(T > t) = P$\{$ Total loss of manpower at the exit points in (0, t] does not exceed Y or the total loss of manpower at the exit points in (0, t] exceeds Y but lies below Z and the organization is not making recruitment $\}$

i.e. $$\mathrm{P(T>t)} = \sum_{k=0}^{\infty} P[N_e(t)=k]\,P(S_k \le Y) + p\sum_{k=0}^{\infty} P[N_e(t)=k]\,P(S_k > Y)\,P(S_k \le Z) \quad (1)$$

Note that $L(t) = 1 - P(T > t)$; $\mathrm{E(T^r)} = (-1)^r [\frac{d^r}{ds^r} L^*(s)]_{s=0}$, r = 1, 2, … .

From Renewal theory [13], we have $P[N_e(t)=k] = G_k(t) - G_{k+1}(t)$ and $\mathrm{G}_0(t) = 1$ (2)

It can be shown that the distribution function G(.) of the inter-exit times W satisfy the relation $G(x) = q\sum_{n=1}^{\infty}(1-q)^{n-1}F_n(x)$. (3)

Therefore $G^*(s) = q\sum_{n=1}^{\infty}(1-q)^{n-1}F_n^{\,*}(s)$

and

Using [14], we get

$$E(W) = \frac{v}{(1-R)q}, \quad E(W^2) = \frac{2(1+u^2R^2\bar{q})}{q^2}, \text{ where } \bar{q} = 1-q \quad (4)$$

We now determine the variance of the time to recruitment for different forms of the distribution of the thresholds by assuming $M(x) = 1 - e^{-\alpha x}$, $\mathrm{F}(x) = 1 - e^{-\lambda x}$, $G(x)$.

Case (i): $H(y) = 1 - e^{-\theta_1 y}$ *and* $\mathrm{H(z)} = 1 - \mathrm{e}^{-\theta_2 z}$

In this case, it can be shown that

$$P(S_k \le Y) = \int_0^{\infty} P[S_k \le Y / S_k \le x]\, m_{S_k}(x)dx = \int_0^{\infty} e^{-\theta_1 x} m_{S_k}(x)dx = a^k$$

$P(S_k \le Z) = b^k$, where $a = E[e^{-\theta_1 X}], b = E[e^{-\theta_2 X}]$ with $\bar{d} = 1 - d$ (5)

From (1), (2) & (3) and on simplification, we get

$$L(t) = \bar{a}\sum_{k=1}^{\infty} G_k(t)\, a^{k-1} + p\left\{\bar{b}\sum_{k=1}^{\infty} G_k(t)\, b^{k-1} - \overline{ab}\sum_{k=1}^{\infty} G_k(t)\,(ab)^{k-1}\right\}$$

$$L^*(s) = \frac{\bar{a}\, G^*(s)}{1 - a\, G^*(s)} + p\left\{\frac{\bar{b}\, G^*(s)}{1 - b\, G^*(s)} - \frac{\overline{ab}\, G^*(s)}{1 - ab\, G^*(s)}\right\}$$

$$E(T) = E(W)\left\{\frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}}\right\} \tag{6}$$

and

$$E(T^2) = \frac{2\bar{a}(1+u^2R^2\bar{q})(1-R)^2 + 2a\,v^2}{(1-R)^2 q^2 (\bar{a})^2} + p\left\{\frac{2\bar{b}(1+u^2R^2\bar{q})(1-R)^2 + 2b\,v^2}{(1-R)^2 q^2 (\bar{b})^2} - \frac{2\overline{ab}(1+u^2R^2\bar{q})(1-R)^2 + 2ab\,v^2}{(1-R)^2 q^2 (\overline{ab})^2}\right\} \tag{7}$$

where $\bar{a}$, $\bar{b}$, $\overline{ab}$ are given by (5).

We know that

$$V(T) = E(T^2) - [E(T)]^2 \tag{8}$$

Equation (6) together with (7) and (8) give the variance of the time to recruitment for case(i).

Case (ii): $H(y) = [1 - e^{-\theta_1 y}]^2$ and $H(z) = [1 - e^{-\theta_2 z}]^2$ which are the extended exponential distributions with scale parameter θ and shape parameter 2.

In this case, it can be shown that

$$P(S_k \le Y) = \int_0^{\infty} P[S_k \le Y / S_k \le x]\, m_{S_k}(x)dx = \int_0^{\infty} [2e^{-\theta_1 x} - e^{-2\theta_1 x}]\, m_{S_k}(x)dx = 2a^k - b^k$$

$P(S_k \le Z) = 2a_1^{\,k} - b_1^{\,k}$, where $a = a(\theta_1) = E(e^{-\theta_1 X}), b = b(2\theta_1) = E(e^{-2\theta_1 X})$,

$a_1 = a(\theta_2), \mathrm{b}_1 = \mathrm{b}(2\theta_2)$ with $\bar{d} = 1 - d$. (9)

$$E(T) = E(W)\left\{\frac{2}{\bar{a}} - \frac{1}{\bar{b}} + \frac{2p}{\bar{a}_1} - \frac{p}{\bar{b}_1} - \frac{4p}{\overline{aa_1}} + \frac{2p}{\overline{ab_1}} + \frac{2p}{\overline{a_1 b}} - \frac{p}{\overline{bb_1}}\right\} \tag{10}$$

and

$$E(T^2)=\frac{2\bar{a}E(W^2)+4a[E(W)]^2}{(\bar{a})^2}-\frac{\bar{b}E(W^2)+2b[E(W)]^2}{(\bar{b})^2}+p\left\{\frac{2\bar{a_1}E(W^2)+4a_1[E(W)]^2}{(\bar{a_1})^2}-\frac{\bar{b_1}E(W^2)+2b_1[E(W)]^2}{(\bar{b_1})^2}\right.$$

$$\left.-\frac{4\overline{aa_1}E(W^2)+8aa_1[E(W)]^2}{(\overline{aa_1})^2}+\frac{2\overline{ab_1}E(W^2)+4ab_1[E(W)]^2}{(\overline{ab_1})^2}+\frac{2\overline{a_1b}E(W^2)+4a_1b[E(W)]^2}{(\overline{a_1b})^2}-\frac{\overline{bb_1}E(W^2)+2bb_1[E(W)]^2}{(\overline{bb_1})^2}\right\} \quad (11)$$

where $\bar{a}$, $\bar{b}$, $\bar{a_1}$, $\bar{b_1}$, $\overline{aa_1}$, $\overline{ab_1}$, $\overline{a_1b}$, $\overline{bb_1}$ and $E(W)$, $E(W^2)$ are given by (9) and (4) respectively. Equation (10) together with (11) and (8) give the variance of the time to recruitment for case (ii).

Case (iii): $H(y)=p_1e^{-(\theta_3+\mu_1)y}+q_1e^{-\theta_4 y}$, where $p_1=\frac{\theta_3-\theta_4}{\mu_1+\theta_3-\theta_4}, q_1=1-p_1$ and

$H(z)=p_2e^{-(\theta_5+\mu_2)z}+q_2e^{-\theta_6 z}$, where $p_2=\frac{\theta_5-\theta_6}{\mu_2+\theta_5-\theta_6}, q_2=1-p_2$ are the distribution functions with SCBZ property.

In this case, it can be shown that

$$P(S_k\le Y)=\int_0^\infty P[S_k\le Y/S_k\le x]\,m_{S_k}(x)dx=\int_0^\infty[p_1e^{-(\theta_1+\mu_1)x}+q_1e^{-\theta_2 x}]\,m_{S_k}(x)dx=p_1a^k+q_1b^k$$

$P(S_k\le Z)=p_2a_1{}^k+q_2b_1{}^k$, where $a=a(\theta_3+\mu_1)=E(e^{-(\theta_3+\mu_1)X})$, $b=b(\theta_4)=E(e^{-\theta_4 X})$,

$a_1=a(\theta_5+\mu_2)$, $b_1=b(\theta_6)$ with $\bar{d}=1-d$. (12)

$$E(T)=E(W)\left\{\frac{p_1}{\bar{a}}+\frac{q_1}{\bar{b}}+\frac{pp_2}{\bar{a_1}}+\frac{pq_2}{\bar{b_1}}-\frac{pp_1p_2}{\overline{aa_1}}-\frac{pp_1q_2}{\overline{ab_1}}-\frac{pp_2q_1}{\overline{a_1b}}-\frac{pq_1q_2}{\overline{bb_1}}\right\} \quad (13)$$

and

$$E(T^2)=\frac{p_1\bar{a}E(W^2)+2ap_1[E(W)]^2}{(\bar{a})^2}+\frac{q_1\bar{b}E(W^2)+2bq_1[E(W)]^2}{(\bar{b})^2}+p\left\{\frac{p_2\bar{a_1}E(W^2)+2a_1p_2[E(W)]^2}{(\bar{a_1})^2}+\frac{q_2\bar{b_1}E(W^2)+2b_1q_2[E(W)]^2}{(\bar{b_1})^2}\right.$$

$$-\frac{p_1p_2\overline{aa_1}E(W^2)+2aa_1\,p_1p_2[E(W)]^2}{(\overline{aa_1})^2}-\frac{p_1q_2\overline{ab_1}E(W^2)+2ab_1\,p_1q_2[E(W)]^2}{(\overline{ab_1})^2}-\frac{p_2q_1\overline{a_1b}E(W^2)+2a_1bp_2q_1[E(W)]^2}{(\overline{a_1b})^2}-$$

$$\left.\frac{q_1q_2\overline{bb_1}E(W^2)+2bb_1\,q_1q_2[E(W)]^2}{(\overline{bb_1})^2}\right\} \quad (14)$$

where $\bar{a}$, $\bar{b}$, $\overline{a_1}$, $\bar{b}_1$, $\overline{aa_1}$, $\overline{a_1 b}$, $\overline{ab_1}$, $\overline{bb_1}$ and $E(W), E(W^2)$ are given by (12) and (4) respectively.

Equation (13) together with (14) and (8) give the variance of the time to recruitment for case (iii).

Note:

(i)When p=0,our results for cases (i),(ii)& (iii) agree with the results in [3] for the manpower system having only one threshold which is the mandatory threshold.
(ii)When q=1,our results for cases (i),(ii)&(iii) agree with the results in [7] for the manpower system with optional and mandatory thresholds having instantaneous certain exits in the decision epochs.

4. Numerical Illustration

The mean and variance of time to recruitment for the cases(i),(ii) and (iii) are numerically illustrated by varying the three nodal parameters R, α and p one at a time and keeping the other parameters fixed. The effect of the nodal parameters on E(T) and V(T) is shown in the following table. In the computations, it is assumed that $\theta_1 = 0.01, \theta_2 = 0.0067, q = 0.5, \text{v} = 2,$ $\theta_3 = 0.008, \theta_4 = 0.012, \theta_5 = 0.004,\ \theta_6 = 0.009,\ \mu_1 = 0.012,\ \mu_2 = 0.009.$

Table: Effect of nodal parameters on E(T) and V(T)

R	α	p	Case(i)		Case(ii)		Case(iii)	
			E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.5	0.2	0.4	225.9527	0.4288 x10^5	322.5269	0.5403 x10^5	225.1492	0.3052x10^5
0.5	0.3	0.4	334.5572	0.9606 x10^5	479.2673	1.2096 x10^5	333.3098	0.6856x10^5
0.5	0.4	0.4	443.1592	1.7040 x10^5	636.0048	2.1448 x10^5	441.4678	1.2177x10^5
0.5	0.5	0.4	551.7602	2.6589 x10^5	792.7412	3.3460 x10^5	549.6248	1.9016x10^5
0.5	0.1	0.1	95.3347	0.7822 x10^4	137.4440	0.9931x10^4	95.2446	6.2210x10^3
0.5	0.1	0.2	102.6693	0.8936 x10^4	146.8880	1.1364x10^4	102.4892	6.8037x10^3
0.5	0.1	0.3	110.0040	0.9943x10^4	156.3319	1.2618x10^4	109.7338	7.2815x10^3
0.5	0.1	0.4	117.3386	1.0842 x10^4	165.7759	1.3694x10^4	116.9784	7.6543x10^3
0.6	0.2	0.5	300.5511	7.2125 x10^4	426.4483	9.0321x10^4	299.2956	4.9698x10^4
0.7	0.2	0.5	400.7347	1.2900 x10^5	568.5978	1.6167x10^5	399.0607	8.9125x10^4
0.8	0.2	0.5	601.1021	2.9115 x10^5	852.8967	3.6504 x10^5	598.5911	2.0143x10^5
0.9	0.2	0.5	1.2022x10^3	1.1720x10^6	1.7058x10^3	1.4707x10^6	1.1972x10^3	8.1313x10^5

5. Findings

From the above table, the following observations are presented which agree with reality.

1. When α increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases. In fact, decrease in α increases the loss of manpower on the average which in turn prepone the time to recruitment.
2. When R increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.
3. As p increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

6. Conclusion

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) provision of optional and mandatory thresholds. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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