International Research Journal of Marketing and Economics
Vol. 2, Issue 11, Nov 2015 IF- 2.988 ISSN: (2349-0314)

Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

BETA CALCULATION AND ROBUST REGRESSION METHODS: AN EXAMPLE FROM THE ISTANBUL STOCK EXCHANGE[1]

Prof. Dr. Ali Alp[2], Dr. Hakan Bilir[3]
[2]Department of Business Administration/TOBB University of Economics and Technology, Turkey
[3]Competition Authority, Turkey

ABSTRACT

Capital Asset Pricing Model (CAPM) is the most widely used method for asset valuation and investment selection. As a systematic risk estimator, beta is the most important element of the CAPM, and many investors trust it for selecting stocks or portfolios. The Ordinary Least Square (OLS) is the most preferred regression method for beta calculation, indicating a relation between stock and market index. Although the OLS is adequate in the case of normal distribution, tail or other distribution cannot be handled successfully by the model. To eliminate standard parametric model inefficiency, robust regression techniques have been developed. In this paper, we propose the robust regression method Least Median Squares (LMS) to estimate beta risk. We compare the behavior of the OLS and LMS estimation methods using monthly returns (adjusted price for US Dollar) for firms listed on the Istanbul Stock Exchange. Our results are also consistent with many authors' works.

KEY WORDS: Beta Coefficient, CAPM, LMS, OLS, Robust Statistics
JEL Classification: G32

1. Introduction

The decision process of investment opportunities requires measuring the expected return and risk. Generally, risk can be defined as the variation from the expected return. It is universally accepted that a high return is achieved by taking high risk. Capital Asset Pricing Model (CAPM) is related to the linear relationship between an asset's systematic risk (beta) and the return expected. The model is extensively accepted because of its simplicity. However, there

[1] We are very grateful to Dr. Murat Çetinkaya and M. Cengiz Aydın for their valuable contributions.

are many criticisms about the application of CAPM. Therefore, it is very important to not only use the model, but also to do so correctly. As a systematic risk estimator, beta is the most important element of CAPM and many investors trust it for selecting stocks or portfolios. The Ordinary Least Square (OLS) regression method is widely used for beta calculation. Although the model fits well under linear estimation and normal distribution assumption, sometimes the expected returns include extreme data, named outlier, and the OLS method is not capable of detecting such extreme data. However, a small fraction of beta values may lead to unexpected financial losses. Thus, to eliminate the unintentional influence of outlier data, alternative robust statistics techniques are advised in many works. In this study, we propose Least Median Squares (LMS) as a robust beta estimator. We compare the behavior of the OLS and LMS beta estimation methods using monthly returns (adjusted price for US dollar) for firms listed on the Istanbul Stock Exchange (Borsa Istanbul) from the BIST 100 database. We analyze 293 firms that have been listed for 12 years of data during the period 2000 to 2012 plus first six months of 2013.

In this paper, we aim to show that if the data set includes an outlier, different regression methods will have highly different results, and this may result in biased financial decisions. The remainder of the paper proceeds as follows. In section 2, we briefly introduce the CAPM and beta coefficient. In section 3, we establish and compare the model of OLS and LMS methods to estimate beta coefficient. In section 4, we apply OLS and LMS methods in empirical studies to check whether the data set includes outliers and whether beta coefficient will have different values. Finally, section 5 concludes the paper.

2. Capital Asset Pricing Model (CAPM) and Beta Coefficient

In the financial world, the value of any asset, especially stocks or portfolio as, is estimated mostly by CAPM, explaining the risk-return relation of any asset. CAPM asserts that nondiversifiable risk (or systematic risk) is only one valid factor determining expected returns. This risk is measured by the covariance between the return on this asset and a market portfolio including all available assets in the market. Beta is the name of the factor measuring systematic risk (Ajlouni et al., 2013: 432).

The Mean-Variance model of Harry Markowitz (1952) is accepted as the beginning of CAPM. According to this model, investors are risk-averse, and at least two conditions must be satisfied for efficient portfolios: "*(1) minimize the variance of portfolio return, given the expected return, and (2) maximize the expected return, given the variance.*" Later, two other key assumptions were added by William Sharpe (1964) and John Lintner (1965), complete agreement, and borrowing and lending at a risk-free rate, respectively. After the introduction of CAPM by Sharpe and Lintner, it has been further developed by many others. Not only individual stock but also portfolios' value and betas have long been estimated by analysts for using CAPM (Miao, 2013: 6-7). CAPM is a simple model but includes a strong assumption. It implies that the expected return of stock depends on a single factor (index). According to the model, the beta is a relative risk measure of securities as a part of a well-diversified portfolio (Zaimoviç, 2013: 31). The equation of the model is as follows:

$$E(R_i) = R_f + [E(R_M) - R_f] * \beta_i \tag{1}$$

where:

$E(R_i)$: is the expected return of an asset

R_f : is the risk-free rate of interest

β_i : is the sensitivity of the expected asset returns to market returns

$E(R_M)$: is the expected market returns

$E(Ri) - R_f$: is the risk premium

$E(R_M) - R_f$: is the market premium

As stated above, individual stock risk and its contribution to a well-diversified portfolio can be measured by beta. We know from the finance and statistics textbooks that risk is related to the existence of the probability of expected return. Volatility of expected return creates this probability and standard deviation is the most commonly used method for risk measurement. Thus, it is accepted that the more one stock's return fluctuates than that of the others, the riskier it is (Allen et al., 2009: 2).

Under the CAPM model, there are two types of risks: systematic and unsystematic. While the former is related to the marketwide movement and affects all firms and investments, the latter is firm-specific and affects only one firm or stock. Thus, while there is no way to eliminate systematic risk with portfolio diversification, it is possible to remove unsystematic risk with proper diversification. In CAPM, there is no need to take and reward a risk being eliminated by diversification. The systematic risk (or nondiversifiable risk) is the only risk rewarding (Simonoff, 2011: 1). β_i is conceived as a measure of systematic risk and can be calculated as:

$$\beta_{iM} = Cov(R_i, R_M) / \sigma^2 (R_M) \quad (2)$$

where:

R_i is the return of asset i

R_M is the return of the market

σ^2 is the variance of the returns of the market

$Cov(R_i, R_M)$ is the covariance between asset i and market returns

We know from the assumptions of the model that investors are rational and risk-averse. Accordingly, a rational investor accepts higher risk only in return of a higher expected return. This means that higher beta values imply higher risk and expected returns. If a stock's beta is greater than one, it implies that this stock is more volatile and riskier than the market index. However, if it is smaller than one, then this stock is less volatile and riskier than the market index. If a stock's beta is equal to zero, an investor can get only a risk-free rate of return and if equal to one, an investor can get only market risk premium (Zaimoviç, 2013: 32).

3. Beta Calculation and Robust Regression Techniques

It can be accepted that methodologically, the CAPM is a general equilibrium model. The aim of the model is to measure the future expected returns and beta values. However, the data used in the model belongs to the past. Therefore, the CAPM is an ex-post analysis of ex-ante expectations. Under the CAPM, historical data is assumed as proxies for future expectations. The most widely used method for CAPM is the so-called Market Model (MM) and OLS estimators (Milionis and Patsouri, 2011: 6–7). While the systematic risk is denoted by beta, unsystematic risk is displayed by the error term of the OLS application of CAPM (Allen et al., 2009: 2). In this way, beta is estimated as the slope coefficient in the regression:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon \quad (3)$$

Where ε is the stochastic disturbance (error term) and αi is a constant. As seen above, the risk-free rate does not exist in the model. Realistic changes of the risk-free rate value create only a small amount of difference in the estimated beta values (Milionis and Patsouri 2011, 6–7). "OLS (or LS) fits the line by finding the intercept and slope that minimize the sum of squared residuals (SSR). Consider the data-generating process $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, where ε_i is independently and identically distributed $N(0,\sigma)$. Given a realization of n observations in X,Y pairs, called the sample, the goal is to estimate the parameters, β_0 and β_1. This is usually done by fitting a line to the observations in the sample and using its intercept, b_0, and slope, b_1, as the parameter estimates. More formally, the optimization problem looks like this" (Barreto, 2001: 2):

$$\min_{b_0, b_1} SSR = \sum_{i=1}^{n} \left(Y_i - (b_0 + b_1 X_i)\right)^2 \tag{4}$$

OLS is accepted as the best linear unbiased estimate (BLUE) of linear model coefficients when the errors are Gaussian (Martin and Simin, 1999: 2). Thus, generally, researchers use the OLS method as "idealized assumptions about the error term, ε, to be independent and identically distributed with a normal distribution, N(0, σ2), for the purpose of statistical inferences." However, because of giving equal weights to each observation in achieving the parameter estimates, the model is vulnerable to extreme data (Cheng et al., 2005: 384). Although the OLS is adequate in the case of normal distribution, tail or other distribution cannot be handled successfully by the model. Existence of outlier or extreme data can create an efficiency problem for the OLS regression model (Jiang, 2011: 10). While one of the assumptions of the CAPM is that investors have quadratic utility functions or stock returns are normally distributed (with thick tail (leptokurtosis) and skewness, there are many empirical works stating that asset returns are not distributed normally (Hodgson, 2000: 1). Mandelbrot (1963), Fama (1965), Kon (1984), Roll (1988), Connolly (1989), and Richardson and Smith (1994) separately showed non-normal distribution of asset returns in their works (Martin and Simin, 1999: 2). Moreover, Shalit and Yitzhaki (2002) and Martin and Simin (2003) showed the outlier's problem if the OLS regression method is used for beta estimation (Tofallis 2004, 4). In addition, outliers appear much more than expected (Jiang, 2011: 3).

To eliminate standard parametric model inefficiency, robust regression techniques have been developed. Nowadays, there are many robust statistical methods for measuring β, instead of OLS. These methods are not affected by outliers and hence provide more reliable results (Genton, 2007: 2). In this paper, we prefer to use one of the robust regression techniques, the LMS, for calculating beta coefficient. Formally, the LMS fit is determined by solving the following optimization problem:

$$\min_{b_0, b_1} \underset{i}{med}\, SR_i = Median\left\{\left(Y_1 - (b_0 + b_1 X_1)\right)^2, \left(Y_2 - (b_0 + b_1 X_2)\right)^2, \ldots \left(Y_n - (b_0 + b_1 X_n)\right)^2\right\} \tag{5}$$

As seen above, the OLS method minimizes the sample mean, which is vulnerable to outliers . However, in place of the mean, LMS uses the median, which is less sensitive to extreme data. This makes LMS a robust estimator. “While we are analyzing alternative objectives in fitting a line, it is important to keep in mind that the data generation process is not an issue here. The model remains $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, where ε_i is independently and identically distributed $N(0,\sigma)$. The question is, which algorithm should we apply to the realized X,Y data points in order to best estimate the parameters?” (Barreto, 2001: 7).

One of the factors causing the superiority of the robust techniques comes from the breakdown point (Cheng et al., 2005: 384). “The smallest percentage of bad data that can cause the fitted line to explode is defined as the breakdown point.” While robust techniques have a higher breakdown point, for OLS it is zero.This is because only one extreme data can fail the least squares line (Barreto, 2001: 3). While OLS has a breakdown point of 0%, that of the LMS estimator is 50%. This 50% breakdown point can be accepted as being as good as possible. However, in practice, it is advisable for researchers to estimate both LMS and OLS regression methods together. If the results are close to each other, the OLS result can be trusted. However, if significant differences exist, the LMS method can be used to detect outlier data (Rousseeuw, 1984: 872–874).

In the next section, we will compare the performance of the OLS and LMS estimators for calculating beta coefficient.

4. Empirical Analysis

4.1. Data

We compare the behavior of the OLS and LMS (robust method) beta estimates using monthly returns (adjusted price for US dollar) for firms listed on the Istanbul Stock Exchange (Borsa Istanbul) from the BIST 100 database (https://datastore.borsaistanbul.com). We include 293 firms that have been listed for 12 years of data during the period 2000 to 2012 and the first six months of 2013.

To calculate for stocks monthly return; we use the formula depicted below:

$$R_i = \frac{\mathrm{R_{it}} - R_{it-1}}{\mathrm{R_{it-1}}}$$

R_i : monthly return of security i

R_{it} : closing price of security i in t month

R_{it-1}: closing price of security i in t - 1 month

To calculate the Index (BIST 100) monthly return; we use the following formula:

$$R_{bist\,100} = \frac{\mathrm{Bist100_t} - \mathrm{Bist100_{t-1}}}{\mathrm{Bist100_{t-1}}}$$

$R_{bist100}$: average return for market

$Bist100_t$: market return in t month

$Bist100_{t-1}$: market return in t-1 month

4.2. Empirical Results

Stata 10.1 package is used to estimate the model. According to our analysis, although most of the betas (224 of 293 securities) for the robust estimator are fairly close to the OLS beta, considerable amount of betas (69 of 293 betas) are highly different (more than 10%) remarks. In other words, 23,5% of the firms have differences larger than 0.1 and 6% have differences larger than 0.2. These differences are likely to be financially significant to many investors. Even some of the security betas have more than 20% differences. There are 69 firms that have at least 10% different beta results, as listed in Table 1.

Table 1. Proportional Difference Between OLS and LMS Beta

	FIRMS	OLS BETA	LMS BETA	=>10	=>20	INDUSTRY
1	ADANA A	0,9542	0,8446	0,114780167		
2	AFM	0,5465	0,4722	0,136015799		
3	ALYAG	0,7295	0,8319	-0,140370117		
4	ANELT	0,7619	0,8491	-0,114450715		
5	ANSA	0,6558	0,7893		-0,203568161	Investment B.
6	ARFYO	1,0317	0,8902	0,137152273		
7	ASLAN	0,8694	0,6581		0,243041178	Cement
8	ASUZU	1,0065	1,1130	-0,105812221		

9	ASYAB	1,1346	1,2536	-0,104882778		
10	ATAGY	0,6631	0,5876	0,113859146		
11	AYCES	0,8627	0,7075	0,179900313		
12	BAKAB	0,1391	0,0578		0,584471603	Packing
13	BJKAS*	1,0252	0,6676		0,348809988	Sports
14	BOYNR	1,2669	1,0788	0,14847265		
15	BRMEN	0,5937	0,6655	-0,1209365		
16	BOROVA	0,9831	0,7905	0,195910894		
17	BRYAT	0,8968	0,7933	0,115410348		
18	BSOKE	-0,0321	0,0534		2,663551402	Cement
19	CCOLA	0,6095	0,6792	-0,11435603		
20	CMENT	0,8132	0,5732		0,295130349	Cement
21	DAGHL	0,9927	0,8818	0,111715523		
22	DENIZ	0,8298	0,6343		0,23559894	Banking
23	DEVA	0,7772	0,9171	-0,180005147		
24	DGZTE	1,2576	1,0947	0,129532443		
25	DNZYO	1,1243	0,7300		0,350707107	Investment B.
26	DOBUR	1,2040	1,0190	0,153654485		
27	DOHOL	1,2264	1,1036	0,100130463		
28	DYOBY	1,0965	0,9794	0,106794346		
29	EDIP	0,8048	0,6349		0,21110835	Real Estate
30	EGPRO	0,5490	0,4250		0,225865209	Construction
31	EMKEL	0,9938	0,8332	0,161601932		
32	ERSU	0,7803	0,6707	0,140458798		
33	ESEMS	0,9047	0,7678	0,15132088		
34	FENER	0,6466	0,4366		0,32477575	Sports
35	FFKRL	0,9757	0,8756	0,10259301		
36	FINBN	0,9486	0,7631	0,195551339		
37	FONFK	0,9318	0,7724	0,171066753		
38	GDKGS	0,5305	0,4743	0,105937795		
39	GDKYO	0,5915	0,5176	0,124936602		
40	GEREL	0,9478	0,7071		0,253956531	Electrical
41	GOLTAS	0,7809	0,6702	0,141759508		
42	GRNYO	1,0125	0,8356	0,174716049		
43	GSRAY	0,5326	0,4255		0,201088997	Sports
44	GUBRF	0,9025	1,0363	-0,148254848		
45	GUSGR	0,8223	0,9071	-0,10312538		
46	HZNDR	0,7784	0,6893	0,11446557		
47	ICGYH	1,1058	0,7677		0,305751492	Investment B.
48	IHLAS	1,1259	0,9455	0,160227374		
49	KRDMB	1,2273	1,0805	0,119612157		
50	KRSTL	1,0166	0,8105		0,202734606	Beverage
51	MEMSA	1,0701	0,9179	0,142229698		
52	MRSHL	0,8527	0,7529	0,117039991		
53	MZHLD	1,0150	0,8272	0,185024631		
54	NUHCM	0,6479	0,5645	0,128723568		
55	OZGYO	0,7761	0,6449	0,16905038		

56	PENGD	0,7236	0,6059	0,162658928		
57	PIMAS	1,0127	0,8997	0,111582897		
58	PRKME	0,9790	1,0998	-0,123391216		
59	RYSAS	1,1204	0,7844		0,299892895	Logistics
60	SASA	0,9426	0,8282	0,121366433		
61	TCRYO	0,7431	0,6636	0,106984255		
62	TRNSK	0,8369	0,7103	0,151272553		
63	TSPOR	0,5126	0,5929	-0,156652361		
64	TTKOM	0,5891	0,6621	-0,123917841		
65	VESBE	1,2534	1,0904	0,130046274		
66	YESIL	0,1574	0,2307		-0,465692503	Footwear
67	YGYO	1,0512	0,8972	0,146499239		
68	YKBYO	0,9702	0,8689	0,104411462		
69	YKSGR	1,0627	0,9132	0,140679402		

Note: For all the stocks, we have regression results p<0.01 (for beta coefficient and market returns).

Another problematic issue also comes from the beta value itself. We know from the CAPM analogy that a beta of one indicates that the security's price will move with the market. A beta less than one means that the security will be less volatile than the market. A beta greater than 1 indicates that the security's price will be more volatile than the market. However, as seen from Table 2, two different methods might result in two distinct outcomes for the same stock in terms of beta movement.

Table 2. Comparison of Beta Values for Market Index Relation

	FIRMS	OLS BETA	LMS BETA	% DIFFERENCE
1	ADANA C.	1,0147	0,9617	0,052192
2	ARFYO	1,0317	0,8902	0,137152
3	BJKAS	1,0252	0,6676	0,34881
4	CELHA	1,0473	0,9679	0,075814
5	DGGYO	1,0075	0,9518	0,055285
6	DNZYO	1,1243	0,7300	0,350707
7	DYOBY	1,0965	0,9794	0,106794
8	ECZYT	0,9439	1,0153	-0,07564
9	GRNYO	1,0125	0,8356	0,174716
10	GUBRF	0,9025	1,0363	-0,14825
11	ICGYH	1,1058	0,7677	0,305751
12	IHLAS	1,1259	0,9455	0,160227
13	ISGYO	1,0018	0,9949	0,006888
14	KRSTL	1,0166	0,8105	0,202735
15	MEMSA	1,0701	0,9179	0,14223
16	MNDRS	0,9930	1,0064	-0,01349

17	MZHLD	1,0150	0,8272	0,185025
18	NTHOL	1,0390	0,9971	0,040327
19	NTTUR	1,0225	0,9805	0,041076
20	PIMAS	1,0127	0,8997	0,111583
21	PRKME	0,9790	1,0998	-0,12339
22	RYSAS	1,1204	0,7844	0,299893
23	SONME	0,9298	1,0167	-0,09346
24	USAK	1,0710	0,9896	0,076004
25	VAKFN	0,9847	1,0141	-0,02986
26	VESTL	1,0523	0,9779	0,070702
27	YATAS	1,0115	0,9775	0,033613
28	YAZIC	1,0033	0,9276	0,075451
29	YGYO	1,0512	0,8972	0,146499
30	YKSGR	1,0627	0,9132	0,140679

For example, while with the OLS regression, BJKAS beta is 1,0252, with the LMS regression, it is calculated as 0,6676. There is a 34% difference between these two results. Under the CAPM, when beta is greater than one, it implies that the security is more volatile than the market index and has a chance of winning more than the market if the market index will increase. However, while one model indicates that the BJKAS beta value is greater than one, another indicates that it is lesser than one. It is also very difficult to decide whether the security is more volatile than the market index or not.. Table 2 shows that 30 firms of 293 have different beta scores when OLS regression results are greater than one and LMS results are lesser than one; and when OLS regression results are lesser than one and LMS results are greater than one.

5. Conclusion

In our analysis, we determine two main problems when we use OLS and LMS methods for the same data set. We compare the behavior of the OLS and LMS method beta estimates using monthly returns (adjusted price for US dollar) for firms listed on the Istanbul Stock Exchange (Borsa Istanbul) from the BIST 100 database. We include 293 firms that have been listed for 150 months. Firstly, there are huge difference between OLS and LMS beta scores 69 of 293 firms. Secondly, in our analysis, OLS and LMS methods give us different beta scores for 30 firms in terms of volatility measurement. In other word, while one method indicates that these securities are less volatile from the market but the other method indicates they are more volatile from the market., These results create great confusion for the many investor in portfolio selection process. This can be happened if there is an outlier in data.

Even only one outlier can cause these problems. Thus, We propose to use robust statistics model for calculating beta score.

REFERENCES

Journal Papers

1. Ajlouni, Moh'd M, Dima W.H. Alrabadi Tariq K. Alnader, Forecasting the Ability of Dynamic versus static CAPM: Evidence from Amman Stock Exchange, *Jordan Journal of Business Administration*, *Volume 9, No. 2.* 2013
2. Zaimović, Azra, Testing The CAPM In Bosnia And Herzegovina With Continuously Compounded Returns, *South East European Journal of Economics and Business. Volume 8 (1)*, 31-39.
3. Cheng,Tsung-Chi, Hung-Neng Lai and Chien-Ju Lu, Industrial Effects and the CAPM: From the Views of Robustness and Longitudinal Data Analysis, *Journal of Data Science 3(2005),* 381-401, 2005.
4. Martin, R. Douglas and Tim Simin, Outlier-Resistant Estimates of Beta, *Financial Analysts Journal*, *Vol. 59, No. 5*, 56-69, September/October, 2003.
5. Rousseeuw, Peter J., Least Median of Squares Regression, *Journal of the American Statistical Association, December, Volume 79, Number 388*, Theory and Methods Section,1984.

Thesis

1. Miao, Dingquan, *Empirical Researches of the Capital Asset Pricing Model and the Fama-French Three-factor Model on the U.S. Stock Market*, Bachelor Thesis in Economics, Division of Economics, The School of Business, Society and Engineering (EST) Mälardalens University Västerås, 2013

Others

1. Allen, David E., Abhay Kumar Singh and Robert Powell, Asset Pricing, the Fama-French Factor Model and the Implications of Quantile Regression Analysis, School of Accounting, Finance and Economics & FEMARC Working Paper Series Edith Cowan University, Working Paper *0911*, October, 2009.
https://www.ecu.edu.au/__data/assets/pdf_file/0013/40432/wp0911da.pdf
2. Simonoff, Jeffrey S., CAPM: Do you want fries with that?, 2011.
http://people.stern.nyu.edu/jsimonof/classes/2301/pdf/mcdonald.pdf
3. Milionis, Alexandros E. and Dimitra K. Patsouri, A Conditional CAPM; Implications For The Estimation Of Systematic Risk, Working Paper, 131, Bank of Greece, Athens, 2011.

http://www.emeraldinsight.com/doi/abs/10.1108/15265941111158488

4. Barreto, Humberto, An Introduction to Least Median of Squares, 2001 Barretoh@wabash.edu, http://www.wabash.edu/econexcel

5. Martin, R. Douglas and Tim Simin, Estimates of Small-Stock Betas are Often Very Distorted by Outliers, Technical Report No. 351, Department of Statistics, University of Washington, May, 1999. https://www.stat.washington.edu/research/reports/1999/tr351.pdf

6. Jiang, Chuanliang, On the Robustness of Beta Risk In The Cross-Sectional Regressions Of Stock Return, 2011. https://www2.bc.edu/chuanliang-jiang/Working_Paper.pdf

7. Hodgson, Douglas J, Oliver Linton, Keith Vorkink, Testing The Capital Asset Pricing Model Efficiently Under Elliptical Symmetry: A Semiparametric Approach, Discussion Paper, July, 2000. https://ideas.repec.org/p/fmg/fmgdps/dp382.html

8. Tofallis, Chris, Investment Volatility: A Critique of Standard Beta Estimation and A Simple Way Forward, University of Hertfordshire, United Kingdom, 2014. http://arxiv.org/ftp/arxiv/papers/1109/1109.4422.pdf

9. Genton, Marc G. and Elvezio Ronchetti, Robust Prediction of Beta, 2007. http://stsda.kaust.edu.sa/Documents/2008.GR.GilliBook.pdf