

A Common Fixed Point Theorem On Lohani And Bhadshah Using A- Compatible And S-Compatible Mappings

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ABSTRACT:

The aim of this paper is to present a common fixed point theorem in a metric space which extends the result of P. C Lohani V H Bhadshah using the weaker conditions and generalize common fixed point theorem proved by Bijendra Singh by introducing the two types of weak reciprocally continuous mappings. The conditions of continuity, compatibility and completeness of a metric space are replaced by two different types of weak reciprocally continuous mappings along with some weaker conditions such as Weakly compatible and the Associated sequence.

Keywords: Fixed point, Self maps, reciprocally Continuous, Compatible maps, Weakly Compatible mappings, Associated sequence, A-Compatible mapping, S-compatible mapping, A-weak reciprocally Continuous mappings, S- reciprocally Continuous mappings.

1. INTRODUCTION

G.Jnngck[1] gave a common fixed point theorem for commuting mapping maps, which generalizes the Banach's fixed point theorem and he also introduced the concept of compatible maps which is weaker than weakly commuting maps. S-Sessa[5] defined weak commutativity and proved common fixed point theorem for weakly commuting maps. G.Jungk[1] initiated the concept of compatible maps which is weaker than weakly commuting maps of type(v). pathak extended the concept of compatibility to two definitions namely v- compatible and s- compatible later Jungcj abd rloades [4]defined weaker class of maps known as weakly compatible maps. Pant [2] introduced a new notion of continuity namely reciprocal continuity for a pair of self maps and proved some common fixed point theorem. Further Pant [2] etal introduced concept of weak reciprocally continuity.

The purpose of this paper is to prove a common fixed point theorem for four self maps using weakly compatibility map and Complete Metric Space are replaced by two different types of weak reciprocally continuous mappings along with some weaker conditions.

DEFINITIONS AND PRELIMINARIES

1.1.1 Definition : If S & T are mappings from a metric space (X, d) into itself are called weakly commuting mappings on X . If $d(STx, TSx) \leq d(Sx, Tx)$ for all x in X .

1.1.2 Compatible Mappings : Two self maps A & S of a metric space (X, d) are said to be compatible mappings [1] if $\lim_{n \rightarrow \infty} d(ASx_n, SAsx_n) = 0$, whenever $\langle x_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

1.1.3 Weakly Compatible Mappings: Two self maps A and S of a metric space (X, d) are said to be weakly compatible mappings [4] if they commute at their coincidence point i.e if $Au = Su$ for some $u \in X$ then $ASu = SAu$.

It is clear that every compatible pair is weakly compatible but its converse need not be true. P.C.Lohani & V.H.Badshah [6] proved the following theorem.

1.1.4 Reciprocally continuous mappings : Two self maps A & S of a metric space (X, d) are said to be reciprocally continuous [2] if $\lim_{n \rightarrow \infty} ASx_n = At$ & $\lim_{n \rightarrow \infty} SAsx_n = St$ whenever $\langle x_n \rangle$ is a sequence such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

1.1.5 Weak Reciprocally continuous mappings : Two self maps A & S of a metric space (X, d) are said to be weak reciprocally continuous [11] iff $\lim_{n \rightarrow \infty} ASx_n = At$ or $\lim_{n \rightarrow \infty} SAsx_n = St$ whenever $\langle x_n \rangle$ is a sequence such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

Now we define the weak reciprocally continuous mappings by introducing into two analogous definitions.

1.1.6 A-Weak Reciprocally continuous mappings : Two self maps A and S of a metric space (X, d) are said to be A-weak reciprocally continuous iff $\lim_{n \rightarrow \infty} ASx_n = At$ whenever $\langle x_n \rangle$ is a sequence such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

1.1.7 S-Weak Reciprocally continuous mappings : Two self maps A and S of a metric space (X, d) are said to be S-weak reciprocally continuous iff $\lim_{n \rightarrow \infty} SAsx_n = St$ whenever $\langle x_n \rangle$ is a sequence such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

1.1.8 A-Compatible mappings : Two self maps A and S of a metric space (X, d) are A-compatible iff $\lim_{n \rightarrow \infty} d(ASx_n, SAsx_n) = 0$ Whenever $\langle x_n \rangle$ is a sequence $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t$ for some $t \in X$.

1.1.9 S-Compatible mappings: Two self maps A and S of a metric space (X, d) are S-compatible iff

$\lim_{n \rightarrow \infty} d(AAx_n, S Ax_n) = 0$ Wherever $\langle x_n \rangle$ is a sequence $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ax_n = t$ for some $t \in X$.

It is clear that every compatible pair is weakly compatible but its converse need to be true. P.C Lohani and V.H Badshah [6] proved the following theorem.

2. Theorem(A): Let A, B, S & T be self mappings from a completed metric space (X, d) into itself satisfying the following conditions

$$S(X) \subset B(X) \text{ \& } T(X) \subset A(X) \dots\dots\dots(2.1.1)$$

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1+d(Ax, Sx)]}{[1+d(Ax, By)]} + \beta d(Ax, By) \dots\dots\dots(2.1.2)$$

For all x, y in X where $\alpha, \beta \geq 0, \alpha + \beta < 1$

One of A, B, S & T is continuous(2.1.3)

Pair S, A & T, B are compatible on X (2.1.4)

Further if X is a complete metric space.....(2.1.5)

Then A, B, S & T have a unique common fixed point in X .

Now we use definition of associated sequence [10] that plays an important role improving our Theorem.

2.1 Associated Sequence:

Suppose A, B, S & T are self maps of a metric space (X, d) satisfying the conditions (2.1.1)

Then for arbitrary $x_0 \in X$ such that $Sx_0 = Bx_1$ and for some point x_1 There exists a point x_2 in X such that $Sx_1 = Ax_2$ and for this point x_2 in X such that $Tx_2 = Bx_3$ and so on.

Proceeding in the similar manner, we can define sequence $\langle y_n \rangle$ in X

such that $y_{2n} = Sx_{2n} = Bx_{2n+1}$ and $y_{2n+1} = Tx_{2n+1} = Ax_{2n+2}$ for $n \geq 0$

We shall call this sequence as an "Associated sequence of x_0 " relative to the four self maps A, B, S and T.

2.2 Lemma: Let A, B, S and T be self mappings from a complete metric space (X, d) into itself satisfying the conditions $S(x) \subset B(x)$ and $T(x) \subset A(x)$ and

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1+d(Ax, Sx)]}{[1+d(Ax, By)]} + \beta d(Ax, By) \dots\dots\dots(2.1.2)$$

For all x, y in X where $\alpha, \beta \geq 0, \alpha + \beta < 1$

$$\begin{aligned} [d(y_{2n+1}, y_{2n})] &= \alpha d(Sx_{2n}, Tx_{2n+1}) \\ &\leq \alpha \frac{d(Bx_{2n+1}, Tx_{2n+1})[1 + d(Ax_{2n}, Sx_{2n})]}{[1 + d(y_{2n-1}, y_{2n})]} + \beta d(y_{2n-1}, y_{2n}) \\ &\leq \alpha d(y_{2n}, y_{2n+1}) + \beta d(y_{2n-1}, y_{2n}) \\ &= (1 - \alpha) d(y_{2n}, y_{2n+1}) \leq \beta d(y_{2n-1}, y_{2n}) \end{aligned}$$

$$d(y_{2n}, y_{2n+1}) \leq \frac{\beta}{(1-\alpha)} d(y_{2n}, y_{2n+1})$$

$$d(y_{2n}, y_{2n+1}) \leq h d(y_{2n-1}, y_{2n}) \text{ where } h = \frac{\beta}{1-\alpha}$$

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &\leq h d(y_{n-1}, y_n) \\ &\leq h^2 d(y_{n-2}, y_{n-1}) \dots \dots h^n d(y_0, y_1) \end{aligned}$$

For every integer $p > 0$ we get

$$d(y_n, y_{n+p}) \leq d(y_n, y_{n+1}) + \dots \dots + d(y_{n+p-1}, y_{n+p})$$

$$\leq d(y_0, y_1) + h^{n+1} d(y_0, y_1) \dots \dots h^{n+p-1} d(y_0, y_1)$$

$$\leq h^n (1 + h + \dots + h^{p-1}) d(y_0, y_1)$$

Since $h < 1$, $h^n \rightarrow 0$ as $n \rightarrow \infty$, so that $d(y_n, y_{n+p}) \rightarrow 0$. This shows that the sequence $\{y_n\}$ is a Cauchy sequence in X and since X is a complete metric space, it converges to a limit, say $z \in X$.

The converse of the Lemma is not true, that is A, B, S and T are self maps of a metric space (X, d) satisfying (2.1.1) and (2.1.3), even if for any $x_0 \in X$ the associated converges, the metric space (X, d) need not be complete. The following example establishes this.

2.3 Example : Let $X = (-1, 1)$ with $d(x, y) = |x - y|$

$$\begin{aligned} Sx = Tx = & \begin{cases} \frac{1}{5} & \text{if } -1 < x < \frac{1}{6} \\ \frac{1}{6} & \text{if } \frac{1}{6} < x < 1 \end{cases} \\ Ax = & \begin{cases} \frac{1}{5} & \text{if } -1 < x < \frac{1}{6} \\ \frac{6x+5}{36} & \text{if } \frac{1}{6} \leq x < 1 \end{cases} \\ Bx = & \begin{cases} \frac{1}{5} & \text{if } -1 < x < \frac{1}{6} \\ \frac{1}{3} - x & \text{if } \frac{1}{6} \leq x < 1 \end{cases} \end{aligned}$$

Then $S(X)=T(X)=\{\frac{1}{5}, \frac{1}{6}\}$ while $A(x)=\{\frac{1}{5} \cup [\frac{1}{6}, \frac{11}{36}]\}$, $B(x)=\{\frac{1}{5} \cup [\frac{1}{6}, \frac{-2}{3}]\}$,
 So that $S(X) \subset B(X)$ and $T(X) \subset A(X)$ proving the condition (2.1.1). Clearly (X, d) is not a complete metric space. It is easy to prove that the associated sequence $Sx_0, Tx_1, Sx_2, Tx_3, \dots, Sx_{2n}, Tx_{2n+1}, \dots$ converges to $\frac{1}{5}$ if $-1 < x < \frac{1}{6}$ or $\frac{1}{6} \leq x < 1$, the associated sequence is converges to $\frac{1}{6}$. Now we prove our theorem.

3.MAIN THEOREM:

3.1 Theorem (B): Let A, B, S and T be self maps of a metric space (X, d) satisfying the condition

$S(x) \subset B(x)$ and $T(x) \subset A(x)$ and

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1+d(Ax, Sx)]}{[1+d(Ax, By)]} + \beta d(Ax, By) \quad \forall \alpha + \beta < 1, \quad \alpha, \beta \geq 0$$

- i) The pair $(A's)$ is A- Weak reciprocally continue & A-Compatible (or)
- ii) The pair $(A's)$ is S- Weak reciprocally continue & S-Compatible
- iii) The pair (B, T) is weakly compatible
- iv) For any $x_0 \in X$ the associated sequence reduce to four self maps A, B, S and T such that the sequence $Sx_0, Tx_1, \dots, Sx_{2n}, Tx_{2n+1}, \dots$ belongs to $z \in x$ as $n \rightarrow \infty$

The A, B, S and T have a unique common fixed point Z in X .

Proof : Using the condition (V)

We have $Sx_{2n} \rightarrow Z, Tx_{2n+1} \rightarrow Z, Bx_{2n+1} \rightarrow Z, Ax_{2n+1} \rightarrow Z$ as $n \rightarrow \infty \dots \dots (3.1.1)$

Case(i)

Since S is weak reciprocally condition then $\lim_{n \rightarrow \infty} SAx_{2n} \rightarrow SZ$

Since the pair $(A's)$ is S-Compatible theorem

$$\lim_{n \rightarrow \infty} d(SAx_{2n}, AAx_{2n}) = 0$$

given that $\lim_{n \rightarrow \infty} SAx_{2n} = \lim_{n \rightarrow \infty} AAx_{2n} = Sz$ -----(3.1.2)

Put $x=Ax_{2n}, y=x_{2n+1}$ in condition (ii) we have

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1 + d(Ax, Sx)]}{[1 + d(Ax, By)]} + \beta d(Ax, By)$$

letting $n \rightarrow \infty$ on both sides and using the conditions (3.1.1), (3.1.2) then we get

$$d(SAx_{2n}, Tx_{2n+1}) \leq \alpha \frac{d(Bx_{2n+1}, Tx_{2n+1})[1 + d(AAx_{2n}, SAx_{2n})]}{1 + d(AAx_{2n}, Bx_{2n+1})} + \beta d(AAx_{2n}, Bx_{2n+1})$$

$$d(SAx_{2n}, Tx_{2n+1}) \leq \alpha \frac{d(z, z)[1 + d(Sz, Sz)]}{1 + d(sz, z)} + \beta d(Sz, z)$$

$$d(Sz, z) \leq \beta d(Sz, z)$$

$$(1 - \beta)d(sz, z) \leq 0, \text{ since } \alpha + \beta < 1, \alpha, \beta \geq 0 \text{ we have}$$

$$\Rightarrow d(Sz, z) = 0 \text{ giving that } Sz = z$$

Put $x = \alpha y = x_{2n+1}$ in condition (ii) we have

$$d(AZ, Tx_{2n+1}) \leq \alpha \frac{d(Bx_{2n+1}, Tx_{2n+1})[1 + d(AZ, SZ)]}{1 + d(AZ, Bx_{2n+1})} + \beta d(AZ, Bx_{2n+1})$$

letting $n \rightarrow \infty$ on both sides and using the conditions $Sv = z$ (3.1.1) then we get

$$d(AZ, Z) \leq \alpha \frac{d(z, z)[1 + d(Az, Sz)]}{1 + d(Az, z)} + \beta d(Az, z)$$

$$d(AZ, Z) \leq \beta d(Az, z)$$

$$d(Az, z)(1 - \beta) \leq 0, \text{ since } \alpha + \beta < 1, \alpha, \beta \geq 0 \text{ we have}$$

$$d(Az, z) = 0, \text{ giving that } Az = z.$$

Since $S(x) \subseteq B(x)$ implies there exists $u \in X$ such that $z = Sz = Bu$.

To prove $Tu = z$, put $x = z, y = u$ in condition (ii) we have

$$d(Sz, Tu) \leq \alpha \frac{d(Bu, Tu)[1 + d(Az, Sz)]}{1 + d(Az, Bu)} + \beta d(Az, Bu)$$

$$d(z, Tu) \leq \alpha \frac{d(z, Tu)[1 + d(z, z)]}{1 + d(z, z)} + \beta d(z, z)$$

$$d(z, Tu) \leq \alpha d(z, Tu)$$

$d(z, Tu)(1 - \alpha) \leq 0$, since $\alpha + \beta < 1$, $\alpha, \beta \geq 0$ we have

$$d(z, Tu) = 0, \text{ giving that } Tu = z.$$

Hence we have $Az = Sz = Tu = z$.

Since (B, T) is weakly compatible $BTu = TBu$

$$\Rightarrow Bz = Tz$$

Now we prove $Tz = z$

put $x = x_{2n}, y = z$ in condition (ii) we have

$$d(Sx, Ty) \leq \frac{[d(By, Ty)[1 + d(Ax, Sx)]}{[1 + d(Ax, By)]} + \beta d(Ax, By)$$

letting $n \rightarrow \infty$ on both sides and using the conditions $Tz = Bz$, (3.1.1) then we get

$$d(Sx_{2n}, Ty) \leq \alpha \frac{d(Bz, Tz)[1 + d(Ax_{2n}, Sx_{2n})]}{1 + d(Ax_{2n}, Bz)} + \beta d(Ax_{2n}, Bz)$$

$$d(z, Tz) \leq \alpha \frac{d(z, Tz)[1 + d(z, z)]}{1 + d(z, z)} + \beta d(z, z)$$

$$d(z, Tz) \leq \alpha d(z, Tz)$$

$d(z, Tz)(1 - \alpha) \leq 0$, since $\alpha + \beta < 1$, $\alpha, \beta \geq 0$ we have

$$d(z, Tz) = 0, \text{ giving that } Tz = z.$$

Hence $Bz = Tz = z$

Case(ii)

Since A is weakly reciprocally continuous then $\lim_{n \rightarrow \infty} ASx_{2n} \rightarrow Az$

Since each the pair (A, S) is A -Compatible then $\lim_{n \rightarrow \infty} d(ASx_{2n}, SSx_{2n}) = 0$

giving that $\lim_{n \rightarrow \infty} SAx_{2n} = \lim_{n \rightarrow \infty} AAx_{2n} = Az$ -----(3.1.3)

Put $x=Ax_{2n}, y=Ax_{2n+1}$ in condition(ii) we have

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1 + d(Ax, Sx)]}{[1 + d(Ax, By)]} + \beta d(Ax, By)$$

$$d(SSx_{2n}, Tx_{2n+1}) \leq \alpha \frac{d(Bx_{2n+1}, Tx_{2n+1})[1 + d(ASx_{2n}, SSx_{2n})]}{1 + d(ASx_{2n}, Bx_{2n+1})} + \beta d(ASx_{2n}, Bx_{2n+1})$$

letting $n \rightarrow \infty$ on both sides and using the conditions(3.1.1),(3.1.3) then we get

$$d(Az, z) \leq \alpha \frac{d(z, z)[1 + d(Az, Az)]}{1 + d(Az, z)} + \beta d(Az, z)$$

$$d(Az, z) \leq \beta d(Az, z)$$

$$(1 - \beta)d(Az, z) \leq 0 \text{ Since } \alpha + \beta > 1, \alpha, \beta \geq 0 .$$

$$d(Az, z) = 0$$

$$Az = z$$

since $S(x) \subseteq B(x)$ implies there exists $v \in X$ such that $z = Sz = Bv$.

To prove $Tv = z$, put $x = Ax_{2n}, y = v$ in condition (ii) we have

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1 + d(Ax, Sx)]}{[1 + d(Ax, By)]} + \beta d(Ax, By)$$

$$d(Sx_{2n}, Tv) \leq \frac{[d(Bv, Tv)][1 + d(Ax_{2n}, Sx_{2n})]}{[1 + d(Ax_{2n}, Bv)]} + \beta d(Ax_{2n}, Bv)$$

letting $n \rightarrow \infty$ on both sides and using the conditions $Av = Tv = z$ (3.1.1) then we get

$$d(z, Tv) \leq \alpha \frac{d(z, Tv)[1 + d(z, z)]}{1 + d(z, z)} + \beta d(z, z)$$

$$d(z, Tv) \leq \alpha d(z, Tv)$$

$$d(z, Tv)(1 - \alpha) \leq 0 \text{ since } \alpha + \beta < 1, \alpha, \beta \geq 0 \text{ we have}$$

$$d(z, Tv) = 0, \text{ giving that } Tv = z.$$

The pair (B, T) is weakly compatible) $\Rightarrow BTv=TBv \Rightarrow Bz=Tz$

Now we prove that $Tz=z$

Put $x=x_{2n}$, $y=z$ in condition(ii) we have

$$d(Sx, Ty) \leq \frac{[d(By, Ty)][1 + d(Ax, Sx)]}{[1 + d(Ax, By)]} + \beta d(Ax, By)$$

$$d(Sx_{2n}, Tz) \leq \frac{[d(Bz, Tz)][1 + d(Ax_{2n}, Sx_{2n})]}{[1 + d(Ax_{2n}, Bz)]} + \beta d(Ax_{2n}, Bz)$$

letting $n \rightarrow \infty$ on both sides and using the conditions $Av=Tz=z$ (3.1.1) then we get

$$d(z, Tv) \leq \alpha \frac{d(Tz, Tz)[1 + d(z, z)]}{1 + d(z, z)} + \beta d(z, Tz)$$

$$d(z, Tz) \leq \beta d(z, Tz)$$

$$d(z, Tz)(1 - \beta) \leq 0 \text{ since } \alpha + \beta < 1, \alpha, \beta \geq 0 \text{ we have}$$

$$d(z, Tz) = 0, \text{ giving that } Tz=z.$$

$$\text{Hence } Bz=Tz=z$$

Since $T(x) \subset A(x) \Rightarrow w \in X$ such that $Tz=z=Aw$ since the pair (A, S) is A compatible then $\lim_{n \rightarrow \infty} d(ASx_n, SSx_n) = 0$ implies $d(ASw, SSw) = 0$ implies $ASw = SSw$

$$\text{implies } Az=Sz=z$$

Since $Az=Bz=Sz=Tz=z$, we get z is a common fixed point of A, B, S , and T . The uniqueness of the fixed point can be easily proved.

3.2 .Remark:

From the example given above, clearly the pairs (S, A) and (T, B) are weakly compatible as they commute at coincident points $\frac{1}{8}$ and $\frac{1}{10}$. But the pairs (S, A) and (T, B) are not compatible and not reciprocally continuous.

For this ,take a sequence $x_n = (\frac{1}{10} + \frac{1}{10^n})$ for $n \geq 1$, then $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ax_n = \frac{1}{10}$ and $\lim_{n \rightarrow \infty} SAs_{2n} = \frac{1}{8}$ also $\lim_{n \rightarrow \infty} ASx_{2n} = \frac{1}{10}$ also .So that $\lim_{n \rightarrow \infty} d(ASx_{2n}, SSx_{2n}) \neq 0$. Also note that none of the mappings are continuous and the rational inequality holds for the values of since $\alpha + \beta < 1$, $\alpha, \beta \geq 0$. Clearly $\frac{1}{6}$ is the unique common fixed point of A,B,S and T.

3.3 Remark:

Theorem (B) is a generalization of Theorem (A) by virtue of the weaker conditions such as S-weak reciprocally continuous and S compatible in the pair (A,S) and (B,T) is weakly compatible ,which are weaker conditions than compatibility of the pairs(A,S) and (B,T) assumed in the theorem(A); The continuity of any one of the mappings is being dropped and the convergence of associated sequence relative to four maps A,B,S and T in place of the complete metric space assumed in the theorem (A).

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