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OPTIMAL REPLENISHMENT POLICY FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH IMPRECISE DEMAND RATE

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ABSTRACT

This paper investigates an inventory model for non-instantaneous deteriorating item when demand rate is imprecise.The objective function in fuzzy sense is defuzzified using Modified Graded Mean Integration Representation Method.The convexity of defuzzified objective function is established using numerical example. The effect of imprecisenessis studied rigorously and some managerial implications are presented.

Keywords: Inventory, non-instantaneous deterioration, imprecise demand.

1. Introduction

The traditional inventory models for deteriorating items are developed with common assumption that deterioration of items in inventory starts from the instant of their arrival. However, many items maintain freshness or original condition for a certain period of time. In other words, deterioration occurs after particular period of time. Wu et al. (2006) promoted this phenomenon and studied inventory model for non-instantaneous deteriorating items with partial backlogging where demand is assumed to be stock-dependent. Motivated by own work, Wu et al. (2009) formulated and solved an inventory system with non-instantaneous deteriorating items and price-sensitive demand. Chang et al. (2010) amended Wu et al.'s (2006) model by allowing (1) a profit-maximization model, (2) a maximum inventory ceiling to reflect the facts that most retail outlets have limited shelf space, (3) an ending-inventory to be nonzero when shortages are not desirable.Uthayakumar and Geetha (2009) and Chang and Lin (2010) investigated partial backlogging inventory model for non-instantaneous deteriorating items considering stock-dependent consumption rate under inflation and time discounting over a finite planning horizon. Wu et al. (2009) established pricing and replenishment policy for non-instantaneous deteriorating items with price sensitive demand. Yang et al. (2009) and Valliathal and Uthayakumar (2011) extended the work of Wu et al. (2009) by incorporating partial backlogging.Shah et al. (2013) presented generalized inventory model for non-instantaneous deteriorating items by considering time sensitive holding cost and deterioration rate and established optimal marketing and replenishment policy for the proposed model.

The demand rate in aforesaid studies are assumed to be precisely known.However, due to the presence of various uncertainties, it is difficult for retail planners and merchandisers to have an accurate gauge of demand.This work is aim to propose an EOQ model for non-instantaneous deteriorating items in the fuzzy sense, wherein demand rateis characterized as Triangular Fuzzy Number (TFN). Making use of *Modified Graded Mean Integration Representation Method*, an objective function is defuzzified.Finally, numerical examples are presented to illustrate the proposed model and the effect of impreciseness on optimal solution is studied.

2. Notation and assumptions

The following notations (similar to Wu et al.; 2009 and Soni and Patel; 2013) and assumptions are used to develop the model. Some additional notations will be introduced later when they are needed.

2.1 Notation

K	:	The ordering cost per order.
t_d	:	The length of deterioration free time.
D	:	Demand rate units per unit time which is imprecise in nature and characterized by triangular fuzzy number $D_1 = \left(D-\delta_1, D, D+\delta_2\right)$, where $\delta_1, \delta_2 \geq 0$.
c_d	:	The deterioration cost per unit.
h	:	Unit holding cost per unit time.

Q	:	The order quantity.
T	:	Length of replenishment cycle ($t_d \le T$).
θ	:	The deterioration rate of the on-hand inventory over [t_d, T].
$I_1(t)$	:	The inventory level at time t ($0 \le t \le t_d$) in which the product has no deterioration.
$I_2(t)$	:	The inventory level at time t ($t_d \le t \le T$) in which the product has deterioration.
$TC(T)$	:	The total cost per unit time of inventory system.

3.2 Assumptions

(These assumptions are mainly adopted from Wu et al.; 2009)

(1) The inventory system involves single non-instantaneous deteriorating item.

(2) The on-hand inventory deteriorate with constant rate θ, where $0 < \theta < 1$.

(3) There is no replacement or repair of deteriorated units during the period underconsideration.

(4) Shortages are not allowed to avoid the lost sales.

(5) Replenishment rate is infinite and lead time is zero.

(6) The system operates for an infinite planning horizon.

3. Model Formulation

3.1 Crisp inventory model

The inventory system evolves as follows: Q units of items arrive at the inventory system at the beginning of each cycle. The inventory level is declining only due to demand rate over time interval [0, t_d]. The inventory level is reducing to zero owing to demand and deterioration during the time interval [t_d, T].The process is repeated as mentioned above. The pattern of inventory level is depicted in Figure 1.

Based on above description and assumptions, the differential equations describing the variation of inventory level$I$ (t), at any instant of time $t \in [0, T]$ are as follows:

$$\frac{dI_1(t)}{dt} = -D, \quad 0 \le t \le t_d \qquad (1)$$

$$\frac{dI_2(t)}{dt} = -\theta I_2(t) - D, \quad t_d \le t \le T \qquad (2)$$

with terminal condition $I_1(0) = Q$ and $I_2(T) = 0$.

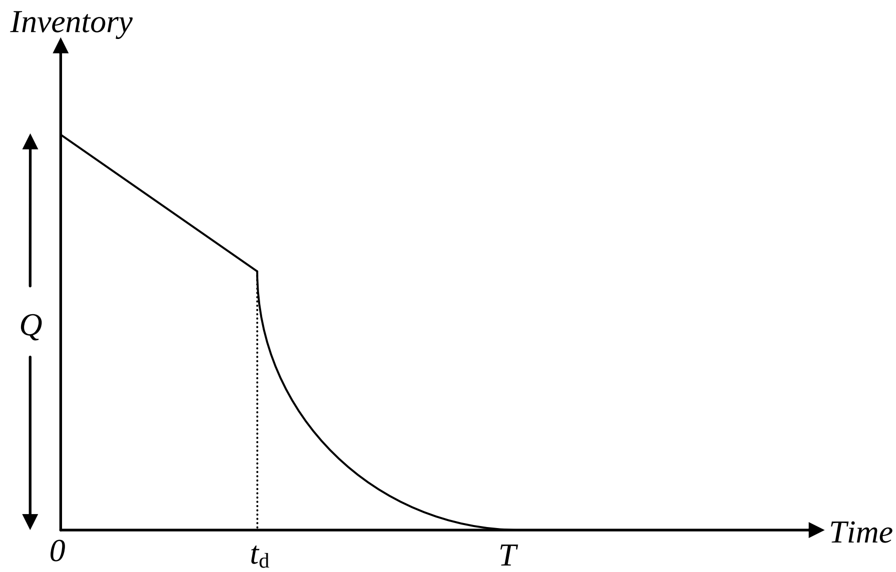

Figure 1: The graphical representation for the inventory system

The solution to (1)–(2) is,

$$I_1(t) = Q - Dt, \quad 0 \le t \le t_d \qquad (3)$$

$$I_2(t) = \frac{d\left(e^{\theta(T-t)} - 1\right)}{\theta}, \quad t_d \le t \le T \qquad (4)$$

Since $I_1(t) = I_2(t)$ at $t = t_d$, it follows from (4) and (5) that,

$$Q - dt_d = \frac{d\left(e^{\theta(T-t)} - 1\right)}{\theta}$$

which yields the order quantity$Q$, as

$$Q = \frac{d}{\theta}\left[\theta t_d + \left(e^{\theta(T-t)} - 1\right)\right] \qquad (5)$$

The elements comprising the cost function per cycleare listed below:

1. The ordering cost is K
2. The holding cost is

$$h\left[\int_0^{t_d} I_1(t)dt+\int_{t_d}^{T} I_2(t)dt\right]=hD\left[\frac{t_d^2}{2}+\frac{\left(e^{\theta(T-t_d)}-1\right)t_d}{\theta}+\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta^2}\right]$$

3. The deterioration cost is

$$c_d\left(I_2(t_d)-D(T-t_d)\right)=c_dD\left[\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta}\right]$$

Assembling above cost components, the total costper unit of time (denoted by TC (T)) is given by

$TC(T)=\{\text{ordering cost} + \text{holding cost} + \text{deterioration cost}\}$

$$=\frac{D}{T}\left\{h\left[\frac{t_d^2}{2}+\frac{\left(e^{\theta(T-t_d)}-1\right)t_d}{\theta}\right]+\left(h+\theta c_d\right)\left[\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta^2}\right]\right\}+\frac{K}{T} \tag{6}$$

4.2 Fuzzy inventory model

In this study, we have characterized the demand rate D, as a triangular fuzzy number to tackle the reality in more effective way.Thus, the objective function defined in (10) can be constructed under fuzzy framework as follows:

$$FTC(T)=\left(FTC_1(T),FTC_2(T),FTC_3(T)\right) \tag{7}$$

where,

$$FTC_1(T)=\frac{(D-\delta_1)}{T}\left\{h\left[\frac{t_d^2}{2}+\frac{\left(e^{\theta(T-t_d)}-1\right)t_d}{\theta}\right]+\left(h+\theta c_d\right)\left[\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta^2}\right]\right\}+\frac{K}{T} \tag{8}$$

$$FTC_2(T)=\frac{D}{T}\left\{h\left[\frac{t_d^2}{2}+\frac{\left(e^{\theta(T-t_d)}-1\right)t_d}{\theta}\right]+\left(h+\theta c_d\right)\left[\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta^2}\right]\right\}+\frac{K}{T} \tag{9}$$

$$FTC_3(T)=\frac{(D+\delta_2)}{T}\left\{h\left[\frac{t_d^2}{2}+\frac{\left(e^{\theta(T-t_d)}-1\right)t_d}{\theta}\right]+\left(h+\theta c_d\right)\left[\frac{\left(e^{\theta(T-t_d)}-\theta(T-t_d)-1\right)}{\theta^2}\right]\right\}+\frac{K}{T} \tag{10}$$

In order to obtain equivalent deterministic form of (7), we employ Graded Mean Integration Representation method proposed by Chen and Hsieh (1999) which is based on the integral value of graded mean α-level of generalized fuzzy number.If $A=\left(a-\delta_1, a, a+\delta_2\right)$ be a TFN, then the modified graded mean integration representation ofTFN A is

$$P(A)=\frac{\left(a-\delta_1\right)+4a+\left(a+\delta_2\right)}{6} \tag{11}$$

Through Eqs. (8) – (10) and using formula (11) we have

$$P\left(FTC(T)\right)=\frac{FTC_1(T)+4FTC_2(T)+FTC_3(T)}{6} \tag{12}$$

Thus, the problem is reduced to

$$\begin{aligned} &\text{Minimize } P\left(FTC(T)\right) \\ &\text{Subject to } T \geq t_d \end{aligned} \tag{13}$$

Our objective is to determine the optimalcycle time which minimizes the total cost per unit time in fuzzy sense. For this, we set first order derivative of $P\left(FTC(T)\right)$ with respect to T to be zero, i.e. $dP\left(FTC(T)\right)\big/dT=0$. (14)

The non-linear nature of objective function in (12) indicates that it is notpossible to obtain closed form solution. However, using Maple 14, the local solutionis obtained in the next section.

4. Numerical example

Example 1: An inventory system with the following data is considered.

$D=7000$ units/year, h = \$ 2/unit/year, K = \$ 150 / order, c_d = \$ 20 / unit, t_d = 15/365 year, θ = 0.08, δ_1 = 50, δ_2 = 150. Solving Eq. (14) by Maple 14 gives T^* = 0.1121 year, Q^* = 786.27 units and $P\left(FTC\left(T^*\right)\right)=\$2379.22$.Figure 2 denotes convexity of total cost per unit time for$T$.

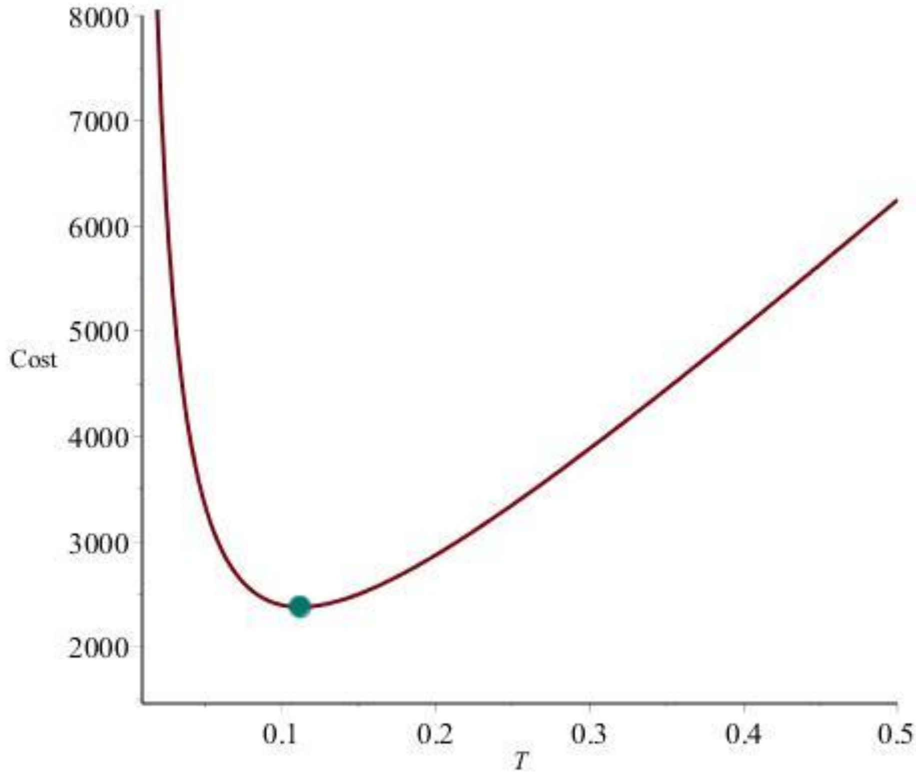

Figure 2: Convexity of $P\left(FTC\left(T\right)\right)$ with respect to T

Example 2: In this example we shall assess the impact of the extent of impreciseness in demand rate over the decision variable. For this, let us first consider the crisp inventory model with same set of input data as in Example 1 except $\delta_1 = 0,\ \delta_2 = 0$.

The crisp model gives the optimal result as $T^* =$ 0.1122 year, $Q^* =$ 786.81 units and $TC\left(T^*\right) = \$2376.75$.

Computational results are summarized in Table 1 for various set of values of fuzzy parameters $\delta_1 = \{50, 150, 300, 550\}$ and $\delta_2 = \{100, 150, 350, 500\}$.

Table 1: Optimal solution for different values of δ_1 and δ_2

δ_1	δ_2	T^*	Q^*	$P\left(FTC\left(T^*\right)\right)$
50	100	0.1122	786.71	2377.99
	150	0.1121	786.27	2379.23
	350	0.1119	784.52	2384.17
	500	0.1117	783.21	2387.86
150	100	0.1123	787.59	2375.51
	150	0.1122	787.15	2376.75
	350	0.1120	785.39	2381.70
	500	0.1118	784.08	2385.40
300	100	0.1125	788.92	2371.79
	150	0.1124	788.48	2373.03
	350	0.1122	786.71	2377.99
	500	0.1120	785.39	2381.70
550	100	0.1128	791.15	2365.57
	150	0.1127	790.70	2366.81
	350	0.1125	788.92	2371.79
	500	0.1123	787.59	2375.51

From the Table 1 it can be observed that for fixed value of δ_1, the optimal length of replenishment cycle (T*) andthe optimal order quantity (Q*) decrease whereas the optimal cost in fuzzy sense increases with an increase in the value of δ_2. On the other hand, for fixed value of δ_2, the optimal length of replenishment cycle (T*) and the optimal order quantity (Q*) increase whereas the optimal cost in fuzzy sense decreases with an increase in the value of δ_1. It may also interesting to note that when demand rate $\left(D\right)$ be symmetrical TFN then the optimal results are equivalent to crisp case (see the results when δ_1 = δ_2 = 150).

5. Conclusion

In this study, an inventory model for non-instantaneous deteriorating item is investigated to determine replenishment policy when demand rate is imprecise. The objective function in fuzzy sense is defuzzified using Modified GradedMean Integration Representation Method.

Furthermore, we illustrated the behavior of our model with respect to key parameters in numerical examples.

The work presented herein could have several possible extensions. For example, this model can be extended to accommodate shortages, variable deterioration rate, stochastic demand, and so forth. One could also incorporate the different preservation investment in the model formulation.

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