A LOSS-AVERSE NEWSVENDOR GAME WITH INVENTORY INACCURACY

Kuo-Hsien Wang,
Department of Business Administration, Takming University of Science and Technology, No.56, Sec.1, Huanshan Rd., Neihu District,Taipei City, Taiwan 11451, R.O.C

Yuan-Chih Huang,
Department of Business Administration, Takming University of Science and Technology, Taipei, Taiwan.

Che-Tsung Tung,
Department of International Trade, Takming University of Science and Technology, Taipei, Taiwan.

ABSTRACT

Considering the impacts of loss aversion and inventory inaccuracy upon a newsvendor game, this paper addresses a loss-averse retailer and a risk-neutral manufacturer decentralized supply chain wherein the manufacturer offers an item to the retailer in a stochastic demand market. The objective is to jointly determine the item's order quantity and wholesale price that optimize both members' profits, during which we learn that high level of the loss aversion damages more profit toward the manufacturer, whereas high rate of the inventory inaccuracy, including temporary and permanent inventory shrinkages, drastically impairs both members' profits. Meanwhile, the game would no longer be profitable if one of the two shrinkages is relatively high. Many managerial insights are observed from the examples hereafter.

***Keywords*:** Loss aversion; Newsvendor; Inventory inaccuracy; Supply chain

1. Introduction

It is well known that the accuracy of an inventory information deeply influences a supply chain profit efficiency, as stated by Hollinger and Adam (2010) who pointed out that inventory shrinkages accounted for 1.44% of total annual sales in USA, and retailers lost more than $33 billion in 2009 as a result of inventory errors. Before that, Atali et al. (2006) categorized inventory errors as temporary shrinkage, permanent shrinkage and transaction error. The temporary shrinkage could refer to product misplacement that affects available inventory but would return for salvage at the end of the selling season; the permanent shrinkage could refer to product theft that affects available inventory and would not be returned; and the transaction error refers to scanning problem that only affects inventory records but would not affect the physical inventory. Xu et al. (2012) thus examined the three inventory errors in the framework of a supply chain, and then investigated how RFID technology reducing inventory errors would economically benefit the supply chain. However, due to advanced modern scanning technology, it is believed that the transaction error could be reduced as much as a decision maker expects. Still, the other two shrinkages are unavoidable. Therefore, this paper zeros in on the two shrinkages in which the following questions will be explored. (1) How will the two shrinkages affect the chain profit? (2) From profit standpoint, which one of the two shrinkages is more intolerant? (3) Whose profit is more damaged by the two shrinkages?

In general, most existing newsvendor problems are risk-neutral assumed. For example, Tsay (1999) studied a quantity flexibility contract in a newsvendor supply chain, Yao et al (2005) dealt with demand uncertainty and return policies for style-good retailing competition, Bose and Anand (2007) contributed a practical finding to return policies with exogenous pricing, Yao et al. (2008) analyzed the impact of price-sensitivity factors on a supply chain, and Chen (2011) tackled a wholesale-price-discount contract in the context of a newsvendor setting, all of which assumed a risk-neutral supply chain.

Other than the risk-neutral newsvendor, a loss-averse one is now widely adopted when describing newsvendor behaviors because it is closer to human nature. The loss aversion, originated from Kahneman and Tversky (1979), indicates that people are more averse to losses than attracted by same-sized gains. Subsequently, Schweitzer and Cachon (2000) contended that an optimal order quantity of a loss-averse newsvendor is less than that of a risk-neutral newsvendor as shortage cost is negligible. Wang and Webster (2007) then extended the loss-averse newsvendor to a supply chain setting where a risk-neutral manufacturer offers an item to a loss-averse retailer. Wang and Webster (2009) further found that, if shortage cost is relatively high, a loss-averse newsvendor may play more order than a risk-neutral newsvendor does. Wang (2010) modified a single loss-averse retailer to multiple loss-averse retailers who then compete for inventory from a risk-neutral manufacturer. Recently, Liu et al. (2013) studied a newsvendor game in which two substitutable items are sold by two different loss-averse retailers who face stochastic customer demand and deterministic product substitution.

Unlike the mentioned articles, this paper considers a loss-averse retailer, a risk-neutral manufacturer decentralized supply chain, simultaneously taking into account the effects of the loss aversion and the contemporary and permanent inventory shrinkages. What interests us will be as follows. (1) How will the loss aversion affect both members' profits? (2) What are the combined impacts on profit performances when incorporating the loss aversion with the two shrinkages?

The remainder of this paper is organized as follows. Assumptions and notation are provided in Section 2 where related newsvendor models will be proposed and analyzed. Numerical examples are conducted in Section 3, along with managerial insights. A summary of the paper and potential directions for further explorations are presented in Section 4 to end the study.

2. The models

The scenario investigated in the study is as follows. A risk-neutral manufacturer supplies an item to a loss-averse retailer in a stochastic demand market with a demand variable x defined on [0, M], of which $f(\cdot)$ is the possibility density function and $F(\cdot)$ is the cumulative distribution function. Assume that Q is the order quantity, r is the unit retail price, w is the unit wholesale price, c is the unit production cost, and s is the unit salvage value for leftover inventory. It is reasonable to assume that $r > w > c > s$. Let $\alpha \in [0,1)$ be a ratio representing the temporary inventory shrinkage that would return for salvage, $\beta \in [0,1)$ be a ratio representing the permanent inventory shrinkage that would not be returned, and $\alpha + \beta \in [0,1)$. Also, define δ =1- α - β; thus,(1- β) Q is the inventory in store, including the misplaced inventory αQ and the available inventory δQ on shelf for sale.

According to Schweitzer and Cachon (2000), we assume the following piecewise-linear loss aversion utility function $U(W)$ for the retailer.

$$U(W) = \begin{cases} W - W_0 & W \geq W_0 \\ \lambda(W - W_{0)} & W \leq W_0 \end{cases} \tag{1}$$

where W_0 is the retailer's reference wealth level at the beginning of the selling period, W is the retailer's final wealth level after the selling period, and λ >1 is the retailer's loss aversion level. Without loss of generality, W_0 =0 is normalized.

Accordingly, the loss-averse retailer's expected utility function $E[U(\pi_r(Q))]$ is calculated by

$$\begin{aligned} E[U(\pi_r(Q))] &= \int_0^{\delta Q} (rx + s((1-\beta)Q - x) - wQ) f(x)dx \\ &+ \int_{\delta Q}^{M} (r\delta Q + s\alpha Q - wQ) f(x)dx \\ &+ (\lambda - 1)\int_0^{AQ} (rx + s((1-\beta)Q - x) - wQ) f(x)dx \\ &= -wQ + \int_0^{\delta Q} ((r-s)x + s(1-\beta)Q) f(x)dx \\ &+ \int_{\delta Q}^{M} (r\delta + s\alpha) Q f(x)dx \\ &+ (\lambda - 1)\int_0^{AQ} ((r-s)x - (w - s(1-\beta))Q) f(x)dx \end{aligned} \tag{2}$$

where $A = \frac{w - s(1-\beta)}{r - s}$ and AQ is explained as the retailer's break-even quantity. And the risk-neutral manufacturer's expected profit is given by

$$E[\pi_m(w)] = (w - c)Q \tag{3}$$

From Eq. (1), to ensure a well-defined $E[U(\pi_r(Q))]$, $A < \delta$ is needed, and this leads to $r\delta + s\alpha > w$ which not only implies that only a positive unit profit can prompt the retailer to join the game, it also gives a tolerable maximal temporary shrinkage $\overline{\alpha} = \frac{r-w}{r-s}$ and a tolerable maximal permanent shrinkage $\overline{\beta} = \frac{r-w}{r}$; clearly, $\overline{\alpha} > \overline{\beta}$. Therefore, the following remarks are observed.

Remark 1 The retailer would join the game only if his profit per unit is positive.

Remark 2 Compared to the temporary shrinkage, the permanent shrinkage is more intolerant.

To optimize $E[U(\pi_r(Q))]$, we take the first- and second-order derivatives with respect to Q as follows.

$$\frac{dE[U(\pi_r(Q))]}{dQ} = r\delta + s\alpha - w - (r-s)\delta F(\delta Q) - (\lambda - 1)(w - s(1-\beta))F(AQ) \tag{4}$$

$$\frac{d^2 E[U(\pi_r(Q))]}{dQ^2} = -(r-s)\delta^2 f(\delta Q) - (\lambda - 1)\frac{(w - s(1-\beta))^2}{r-s} f(AQ) < 0 \tag{5}$$

Based on Eq. (4), let

$G(Q) = r\delta + s\alpha - w - (r-s)\delta F(\delta Q) - (\lambda-1)(w - s(1-\beta))F(AQ)$, then

$\lim_{Q \to 0^+} G(Q) = r\delta + s\alpha - w$ and $\lim_{Q\to\infty} G(Q) = -\lambda(w - s(1-\beta)) < 0$ are easily obtained. According to Eq. (5), $G'(Q) < 0$ and $G(Q)$ is thus decreasing in Q; therefore, the solution of $G(Q) = 0$ will uniquely exist if $\lim_{Q \to 0^+} G(Q) = r\delta + s\alpha - w > 0$. Consequently, the following is obtained.

Proposition 1 From the standpoint of the retailer's expected utility function,

(1) $E[U(\pi_r(Q))]$ is concave in Q, and

(2) the optimal order quantity, denoted by Q^*, uniquely exists if $r\delta + s\alpha > w$ and satisfies

the following equation.

$$r\delta + s\alpha - w - (r - s)\delta F(\delta Q^*) - (\lambda - 1)(w - s(1-\beta))F(AQ^*) = 0 \quad (6)$$

To realize how the loss aversion level will affect the optimal order quantity, according to Eq. (6), let

$G(Q^*(\lambda),\lambda) = r\delta + s\alpha - w - (r-s)\delta F(\delta Q^*) - (\lambda-1)(w - s(1-\beta))F(AQ^*) = 0$, then

$\dfrac{dG(Q^*(\lambda),\lambda)}{d\lambda} = \dfrac{\partial G(Q^*(\lambda),\lambda)}{\partial Q^*}\dfrac{dQ^*}{d\lambda} + \dfrac{\partial G(Q^*(\lambda),\lambda)}{\partial \lambda} = 0$ implies that

$\dfrac{dQ^*}{d\lambda} = -\dfrac{(w - s(1-\beta))F(AQ^*)}{(r-s)\delta^2 f(\delta Q^*) + (\lambda-1)\dfrac{(w-s(1-\beta))^2}{r-s} f(AQ^*)} < 0$. Therefore, the following is obtained.

Remark 3 The optimal order quantity of a loss-averse retailer decreases with the loss aversion level and is always less than that of a risk-neutral retailer (λ =1).

To understand how the wholesale price will affect the optimal order quantity, according to Eq. (6), let

$G(Q^*(w),w) = r\delta + s\alpha - w - (r-s)\delta F(\delta Q^*) - (\lambda-1)(w - s(1-\beta))F(AQ^*) = 0$, then

$\dfrac{dG(Q^*(w),w)}{dw} = \dfrac{\partial G(Q^*(w),w)}{\partial Q^*}\dfrac{dQ^*}{dw} + \dfrac{\partial G(Q^*(w),w)}{\partial w} = 0$ implies that

$\dfrac{dQ^*}{dw} = -\dfrac{1 + (\lambda-1)(F(AQ^*) + \dfrac{w - s(1-\beta)}{r-s} Q^* f(AQ^*))}{(r-s)\delta^2 f(\delta Q^*) + (\lambda-1)\dfrac{(w-s(1-\beta))^2}{r-s} f(AQ^*)} < 0$, and the following thus is obtained.

Remark 4 The optimal order quantity in a decentralized supply chain decreases with the wholesale price and is always less than that in a centralized supply chain.

Remark 5 In pursuit of a maximal optimal order quantity, the game should be operated as a centralized supply chain, and the manufacturer needs to offer incentives to mitigate the retailer's loss aversion level.

Likely, to realize how the two shrinkages will affect the optimal order quantity, we have

$$\frac{dQ^*}{d\alpha} = -\frac{(r-s)(1-F(\delta Q^*)-\delta Q^* f(\delta Q^*))}{(r-s)\delta^2 f(\delta Q^*)+(\lambda-1)\dfrac{(w-s(1-\beta))^2}{r-s} f(AQ^*)} ,$$

$$\frac{dQ^*}{d\beta} = -\frac{(r-s)(1-F(\delta Q^*)-\delta Q^* f(\delta Q^*))+s+(\lambda-1)(sF(AQ^*)+\dfrac{w-s(1-\beta)}{r-s}Q^* f(AQ^*))}{(r-s)\delta^2 f(\delta Q^*)+(\lambda-1)\dfrac{(w-s(1-\beta))^2}{r-s} f(AQ^*)} \text{ and}$$

$\frac{dQ^*}{d\beta} < \frac{dQ^*}{d\alpha}$ which is consistent with Remark 2 that permanent shrinkage triggers more inventory losses.

As for the manufacturer's expected profit, according to Eq. (3), it turns to $E[\pi_m(w)] = (w-c)Q^*$, and the manufacturer is to determine the wholesale price that maximizes $E[\pi_m(w)]$. Thus, combining $\frac{dE[\pi_m(w)]}{dw} = Q^* + (w-c)\frac{dQ^*}{dw} = 0$ with Eq. (6), the optimal order quantity Q^* and optimal wholesale price w^* satisfy the following equations.

$$\begin{cases} r\delta + s\alpha - w^* - (r-s)\delta F(\delta Q^*) - (\lambda-1)(w^* - s(1-\beta))F(A^*Q^*) = 0 \\ Q^* = (w^*-c)\dfrac{1+(\lambda-1)\left(F(A^*Q^*)+\dfrac{w^*-s(1-\beta)}{r-s}Q^* f(A^*Q^*)\right)}{(r-s)\delta^2 f(\delta Q^*)+(\lambda-1)\dfrac{(w^*-s(1-\beta))^2}{r-s} f(A^*Q^*)} \\ A^* = \dfrac{w^*-s(1-\beta)}{r-s} \end{cases} \tag{7}$$

3. Numerical examples

For convenience, during the examples conducted, the optimal retailer's expected utility function, optimal manufacturer's expected profit and the chain profit would be simplified as π^r, π^m and $\pi^c = \pi^r + \pi^m$, respectively.

Example 1 Parameter values: r =8, c =3, s =1, α =0.1, β =0.1, M =100, $X \sim U[0, M]$. This example is to investigate how the loss aversion level will affect both members' profits. Thus, we increase λ from λ =1.0 to λ =3.0 at a step of 0.2.

Table 1 The results caused by λ

λ	w^*	Q^*	π^r	π^m	π^c
1.0	4.75	39.06	34.18	68.36	102.54
1.2	4.68	37.18	33.79	62.55	96.34
1.4	4.63	35.52	33.26	57.78	91.05
1.6	4.58	34.03	32.66	53.78	86.44
1.8	4.54	32.69	32.03	50.36	82.38
2.0	4.51	31.46	31.37	47.38	78.76
2.2	4.48	3034	30.71	44.77	75.49
2.4	4.44	29.31	30.07	42.46	72.53
2.6	4.42	28.35	29.42	40.39	69.82
2.8	4.40	27.47	28.80	38.53	67.33
3.0	4.38	26.64	28.19	36.85	65.04

Example 2 Parameter values: r =8, c =3, s =1, λ =2.0, β =0.1, M =100, $X \sim U[0,M]$. This example is to study how the temporary shrinkage will influence the profit performances. We thus increase α from α =0.0 to α =0.5 at a step of 0.1.

Table 2 The results caused by α

α	w^*	Q^*	π^r	π^m	π^c
0.0	4.79	31.71	36.95	55.08	92.03
0.1	4.51	31.46	31.37	47.38	78.76
0.2	4.22	31.04	25.03	38.52	63.55
0.3	3.93	30.63	17.96	28.40	46.36
0.4	3.63	27.24	10.45	17.23	27.68

0.5	3.33	18.93	3.52	6.22	9.74

Example 3 Parameter values: r =8, c =3, s =1, λ =2.0, α =0.1, M =100, $X \sim U[0,M]$. This example is to explore how the permanent shrinkage will impact the chain profit. We thus increase β from β =0.0 to β =0.5 at a step of 0.1.

Table 3 The results caused by β

β	w^*	Q^*	π^r	π^m	π^c
0.0	4.83	31.78	39.23	58.21	97.44
0.1	4.51	31.46	31.37	47.38	78.76
0.2	4.18	30.05	22.86	35.43	58.29
0.3	3.85	26.73	14.06	22.67	36.72
0.4	3.51	20.02	5.93	10.17	16.10
0.5	3.15	7.26	0.56	1.06	1.62

Table 1 illustrates that π^r, π^m and π^c all decrease with λ as a result of the decreasing Q^*. Also, high level of loss aversion slashes the optimal wholesale price from w^* =4.75 at λ =1.0 to w^* =4.38 at λ =3.0. Specifically, Table 1 identifies that the loss aversion damages more profit toward the manufacturer, showing that π^m =68.36 at λ =1.0 drastically decreases to π^m =36.85 at λ =3.0 with a decrement by 46.09%. Contrarily, there is only a 17.5% profit drop with respect to the retailer from π^r =34.18 to π^r =28.19.

The reason why only α =0.0 to 0.5 in Table 1 and β =0.0 to 0.5 in Table 2 are presented is because that, whenever $\alpha \geq$0.6 or $\beta \geq$0.6, the optimal wholesale price is less than the production cost, resulting in a negative Q^*. And this is in line with previous analysis that α and β are bounded above.

As analyzed, the optimal order quantity is proven to be more damaged by the permanent shrinkage, showing that Q^*=31.78 at β=0.0 decreases to Q^*=7.26 at β=0.5 with a vast decrement by 77.2%, whereas there is only a 38.4% drop from α=0.0 to 0.5. Nevertheless, Tables 2 and 3 reveal that both members' profits are seriously devastated by the two shrinkages, which consequently suggests that an effort to reduce the inventory errors should be made.

3. Conclusion

This paper studied a decentralized supply chain that is composed of a loss-averse retailer and a risk-neutral manufacturer in a stochastic demand market. With the loss aversion, the retailer places order from the manufacturer, considering inventory inaccuracy that includes contemporary and permanent shrinkages. The manufacturer then, according to the retailer's responses, decides the wholesale price to maximize both members' profits.

In the course of the paper, the following observations are made. (1) Both members' profits are negatively influenced by the loss aversion; especially, a high level of loss aversion seriously deteriorates the manufacturer's profit. Therefore, the manufacturer is advised to offer incentives to reduce the loss aversion level. (2) The permanent shrinkage, compared to the temporary shrinkage, is much more intolerant; thus, the retailer should secure products carefully during the selling period. (3) The two shrinkages dramatically impair both members' profits; therefore, a chain efforts to eliminate the inventory inaccuracy are indispensable during the game.

As analyzed, a maximal optimal order quantity would occur at the situation of a risk-neutral newsvendor and a product wholesale price that is equal to its production cost. Therefore, operating our decentralized supply chain as a centralized supply chain with a return-buyback commitment to mitigate the loss aversion is a direction worthy of exploring. Also, our proposed models could be extended to a loss-averse retailer and a loss-averse manufacturer supply chain, a multiple loss-averse retailers and a risk-neutral manufacturer supply chain, or a loss-averse retailer and a risk-neutral manufacturer supply chain with multiple newsvendor-type items.

References

Atali, A., Lee, H., Ozer, O., 2006. If the inventory manager knew: value of visibility and RFID under imperfect inventory information. Working paper, Standard University.

Bose, I., Anand, P., 2007. On returns policies with exogenous price. European Journal of Operational Research 178, 782-788.

Chen, J., 2011. Returns with wholesale-price-discount contract in a newsvendor problem. International Journal of Production Economics 130, 104-111.

Hollinger, R.C., Adams, A., 2010. 2009 National Retail Security Survey Final Report. University of Florida.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47(2), 263-291.

Liu, W., Song, S., Wu, C., 2013. Impact of loss aversion on the newsvendor game with product substitution. International Journal of Production Economics 141, 352-359.

Schweitzer, M.E., Cachon, G.P., 2000. Decision bias in the newsvendor problem with a known demand distribution: experimental evidence. Management Science 46(3), 404-420.

Tsay, A.A., 1999. The quantity flexibility contract and supplier-customer incentives. Management Science 45, 1339-1358.

Wang, C.X., 2010. The loss-averse newsvendor game. International Journal of Production Economics 124(2), 448-452.

Wang, C.X., Webster, S., 2007. Channel coordination for a supply chain with a risk-neutral manufacturer and a loss-averse retailer. Decision Sciences 38(3), 361-389.

Wang, C.X., Webster, S., 2007. The loss-averse newsvendor problem. Omega 37(1), 93-105.

Xu, J., Jiang, W., Feng, G., Tian, J., 2012. Comparing improvement strategies for inventory inaccuracy in a two-echelon supply chain. European Journal of Operational Research 221, 213-221.

Yao, Z., Leung, S.C.H., Lai, K.K., 2008. Analysis of the impact of price-sensitivity factors on the returns policy in coordinating supply chain. European Journal of Operational Research 187, 275-282.

Yao, Z., Wu, Y., Lai, K.K., 2005. Demand uncertainty and manufacturer returns policies for style-good retailing competition. Production Planning and Control 16(7), 691-700.