STUDY ON MULTI OBJECTIVE OPTIMIZATION METHODS

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ABSTRACT

The majority of problems came across in practice include the optimization of multiple criteria. Usually, few of them are at variance like that no single solution is concomitantly optimal with a particular aspect to all criteria, but alternatively numerous inimitable compromise solutions subsist. At the same time, the search space of such like problems is often very large and complex, so that traditional optimization techniques are not applicable or cannot solve the problem within reasonable time. The process of optimization methodically and concomitantly a collection of objective function are known as multiobjective optimization (MOO) or vector optimization. Optimization mentions to detecting one or numerous attainable solutions which corresponds to utmost values of one or numerous objectives. Necessity for Optimization arrives mostly from the utmost motive of either designing a solution for minimal viable cost of fabrication, or for maximal viable constancy, or others. This paper is a contemplate of different methods for Multi Objective Optimization.

Keywords: Optimization, Multi Objective, Multiple Criteria, Search Space, Pareto Optimal Set, Pareto Optimal Front.

1. Introduction

Optimization [1] mentions to detecting one or numerous realizable solutions which corresponds to utmost values of one or numerous objectives. Necessity for optimization mostly arrives from the utmost motive of either designing a solution for minimal viable cost of fabrication, or for maximal viable constancy, or others. When an optimization problem modeling [2] a physical system includes only one objective the task of detecting the optimal solution is known as single objective optimization.

When an optimization problem modeling a physical system involves more than one objective function, the task of detecting the one or more optimal solutions is known as multi-objective optimization[3].

Virtually, there subsist an infinite number of similar problems. In practice, real-world decision making problems with only one objective are rare. Despite of that, solving single objective optimization [2] problems is far more common than solving multiobjective problems, since there appears to be no generally effective and efficient method [4] available for solving multiobjective problems directly as they are. Typically a multiobjective problem is to be effectively converted to a single objective problem before applying an optimization algorithm. This conversion can be done easily by first deciding the relative importance for each objective a priori. Then, for example, the decision-maker may combine the individual objective functions into a scalar cost function (linear or nonlinear combination), which effectively converts a multiobjective problem into a single objective one.

Anyway, single objective problems are only a subclass of multiobjective problems [5]. Thus finding a method for solving the multiobjective problems as multiobjective problems, without any a priori preference decisions, and without first converting the problem into a single objective one, is one of the most important optimization research objectives at the moment. Justifying more practically, the decision-makers (having multiple objectives), are willing to perform unbiased searches in general.

A wide variety of approaches have been applied for attacking multiobjective optimization problems. A multiobjective optimization problem and its globally optimal solution(s) can be defined in many ways. The most important underlying question is, on how the tradeoff between the conflicting and mutually independent objectives should be done. Which objective should be favored over the others? This report concentrates on the concept of Pareto-optimization originated by the engineer/economist Valfred Pareto [6]. He made one of the most important findings in the field of multiobjective optimization by finding that:

Multiple criteria solutions could be partially ordered without making any preference choices a priori.

2. Background and Objectives

The procedure of optimizing methodically and concomitantly a group of objective function is known as multiobjective optimization (MOO) or vector optimization. This paper is a contemplate of procedure for MOO. In opposite, this contemplates excluded numerous of the technical details and, alternatively, offers a roadmap of presently obtainable ceaseless nonlinear methods and literary text. Generally conceptions are concisely described, and references are contained for further inspection. In addendum, this paper combines apparently contrasting concepts, procedures, and terminology stemming from different uses.

Description

Most optimization problems encountered in practice involve multiple criteria that need to be considered. These so-called *objectives* are thereby mostly conflicting. The decision on a laptop purchase, for instance, amongst other things, maybe influenced by battery life, performance, portability, and the price. None of the lone solution is usually concomitantly optimal with a particular aspect to each these objectives, but rather numerous different designs subsist which is incomparable.

Problem Definition

A multi-objective optimization problem has many objective functions which are to be minimized. Similar single objective optimization problem at this place furthermore the problem normally has numerous limitations which any attainable solution must gratify.

Basic Concepts

Definition 1 (Pareto-optimal set) : The Pareto-optimal set(or Pareto set for short) of the decision space X corresponds to the set of minimal elements of $(X, \preccurlyeq par)$, i.e., the Pareto set consists of all elements in $u \in X$, for which no $x \in X$ exists with $x \prec par\ u$

Definition 2(Pareto-optimal front): The Pareto-optimal front (or Pareto front for short) for a decision space X corresponds to the objective values of the Pareto set—which corresponds to the minimal set of values.

General Form of MOOP

Minimize/Maximize $fm(x)$, $m=1, 2,\ldots, M$

subject to $gi(x) >= 0$ $j=1, 2, \ldots, J$

$hk(x) = 0$, $k=1,2,\ldots,K$

$Xi^{(L)} \leq Xi \leq Xi^{(u)}$ $i=1,2,\ldots,n$

A solution x is a vector of n decision variables: $X= X_1, X_2, ,\ldots\ldots, X_n{}^T$

The endmost set of constraints are called variable bounds, restricting every decision Variable Xi to take a value inside a lower $Xi^{(L)}$ and an upper $Xi^{(u)}$ bound. These Bounds form a decision variables space D, or simply the decision space.

Linear and Nonlinear MOOP

If each of objective functions and restrictions are linear, the resulting MOOP is known as a multi-objective linear problem (MOLP). If some of the objective or limitation functions are nonlinear, the resulting problem is known as a nonlinear multi-objective problem.

Convex and Non Convex MOOP

Convex function is defined as for each of

$$F(\lambda x^{(1)}+(1-\lambda x^{(2)}) \leq \lambda f(x^{(1)})+(1-\lambda)f(x^{(2)}) \qquad \text{for all } 0\leq \lambda \leq 1 \quad (5)$$

For a convex function, a local minimal is always a global minimal. The Hessian matrix of fx is positive definite for each of x. A function gratifying the inequality shown above with > sign alternatively is known as a non convex function.

Objectives in Muti-Objective Optimization

Basic goal in a multi-objective optimization are:

1. To detect a set of linear solutions as close as viable to the pareto optimal front.
2. To detect a set of solutions as diverse as viable.

3. Classification for Multiobjective Optimization Approaches

As discussed, in case of a multiobjective optimization problem, there subsists no single seldom-justified definition of the optimum solution. Basically, the problem is on how the individual objective functions should be weighted in relation to each other. In case of a lone objective problem there is no such problem, since there is only one objective. Because the objective function weighting problem is characteristic property of multiobjective problems, the solution for the weighting problem is a natural basis for the classification. Sooner or later, the decision-maker should finally decide the relative importance of each objective function in order to get a lone unique solution to be used as a solution of his original multidisciplinary decision making problem. This decision can be done applying one of the following approaches [7]:

1. **A Priori Preference Articulation:** The decision-maker selects the weighting before running the optimization algorithm. In practice it means that the decision-maker combines the individual objective functions into a scalar cost function (linear or nonlinear combination). This effectively converts a multiobjective problem into a single objective one.
2. **Progressive Preference Articulation:** Decision-maker interacts with the optimization program during the optimization process. Typically the system provides an updated set of solution and let the decision-maker consider whether or not change the weighting of individual objective functions.
3. **A Posteriori Preference Articulation:** No weighting is specified by the user before or during the optimization process. The optimization algorithm provides a set of efficient candidate solution from which the decision-maker choose the solution to be used.

Currently, in connection with evolutionary algorithms, there exist clearly two mainstream approaches for appropriate definition of multiobjective optimization problem also in case of conflicting objectives:

1. **Weighted Sum of Objective Functions:** Converting the multiobjective problem to a single objective one by using weighted sum of objective functions as a representative objective function, and then solve the problem as a single objective one. Represents a priori preference articulation.
2. **Pareto Optimization:** Solving the multiobjective problem by applying Pareto-optimization approach. Decision-maker selects the solution from the resulting Pareto-optimal set. Represents *a posteriori* preference articulation.

4. Methods with a priori articulation of preferences

The procedure in this section allows the use to specify preferences, which may be articulated in terms of goals or the relative importance of contrast objectives. The majority of these methods incorporate parameters, which are coefficients, exponents, restriction limits, etc. That can be either being set to reflect decision-maker preferences, or be ceaselessly altered in an effort to represent the complete Pareto optimal set.

Considerations of more than single objective function in an optimization problem introduce addendum level of independence. Unless these levels of independences are restricted, mathematical conjecture indicates a set of solution points quite than a lone optimal point. Preferences prescribed by the DM offer restriction. The majority of usual approach to foist like restrictions to grow a useful function as defined priory. Thus the majority of the formulation in this segment is founded on contrasting useful functions.

Weighted global criterion method

One of the most usual common scalrization procedures for multi objective optimization is the global criterion procedure in which each of objective function are integrated to formation a lone function. The word global Criterion technically can mention to any scalarized function but it frequently is retain for the formulations presented in this subsection. In spite of the fact a global criterion may be a mathematical function with no association to predilection, a weighted global criterion is a sort of useful function in which a procedure parameter are employed to model predilections. One of the majorities of common useful function is convey in its easiest shaped as the weighted exponential sum:

$$U = \sum_{i=1}^{k} w_i [F_i(\mathbf{x})]^p \,, \quad F_i(\mathbf{x}) > 0 \, \forall i \,,$$

$$U = \sum_{i=1}^{k} [w_i F_i(\mathbf{x})]^p \,, \quad F_i(\mathbf{x}) > 0 \, \forall i \,.$$

The most common extensions [8, 9] of these equations are:

$$U = \left\{ \sum_{i=1}^{k} w_i [F_i\left(\mathbf{x}\right) - F_i^{\circ}]^p \right\}^{\frac{1}{p}} \,,$$

$$U = \left\{ \sum_{i=1}^{k} w_i^p [F_i\left(\mathbf{x}\right) - F_i^{\circ}]^p \right\}^{\frac{1}{p}} \,.$$

At this spot w is a vector of weights typically set by the decision maker such that $U = \sum_{i=1}^{k} w_i F_i\left(\mathbf{x}\right) .$ and w> 0. As with the mejority of procedures that include objective function weights, setting one or to a greater extent of the weights to zero can result in weak Pareto optimality where Pareto optimality may be achievable. Generally, the relative value of the weights reflects the ralative importance of the objectives. Therefore,global criterion procedures are frequently called utopia point procedures or compromising programming procedure as the DM customarilyhas to settle between the last solution and the utopia point. For computational efficiency or in case where a function's independent minimal may be unachieveable one may imprecisethe utopia point by z, which is called an aspiration point [10, 11] reference point [12] or target point [13] . The solution to these approaches contingents on the value of p. Customarily,p is proportionate to the quantity of emphasis putted on minimizing the function with the immense contrastbetween Fi(x) and F∘I [14]. Nevertheless differing only p (with each of the alternate parameter constants) customarily produce only a finite number of Pareto optimal points in a comparatively little vicinity. One customarily chooses a fixed value for p. Then the user either sets w to reflect predilections a priori or methodically change w t produce a set of Pareto points.

Weighted Sum Method

The most useful approach to multi objective optimization is the weighted sum procedure:

$$U = \sum_{i=1}^{k} w_i F_i (\mathbf{x}) \ .$$

If each of the weights are positive the minimal of this is Pareto optimal ; i.e. minimizing this is enough for Pareto optimality. Nevertheless the formulation does not offer a obligatory stipulation for Pareto optimality [15].

Misinterpretation of the theoretical and practical meaning of the weights can make the process of intuitively selecting non-arbitrary weights an inefficient chore. With ranking methods [16], the contrast objective functions are ordered by significance. The minimum important objective collect a weight of one and integer weights with constant increments are allocate to objectives that are of greater importance. The same approach is employed with categorization procedures in which contrasting objectives are grouped in wide groups analogous as highly important and moderately important. With grading procedure DM allocate independent values of relative importance to every objective function.

Initially, despite the numerous procedure for determining weights a gratify,a priori choice of weights does not obligatory guarantee that the last solution will be adequate ; one may have to fix the problem with novel weights. In fact weights must be functions of the original objectives, not constants, in order for a weighted sum to mimic a predilection function precisely[17].

The second problem with the weighted sum approach is that it is not possible to acquire points on non convex part of the Pareto optimal set in the criterion space [18]. In spite of the fact non convex Pareto optimal sets are comparatively unusual [19, 20, and 21]. The final strenuous with the weighted sum procedure is that differing the weights constantly and ceaselessly may not obligatory outcome in an even distribution of Pareto o[timal points and an precise, entire depiction of the Pareto optimal set.

Lexicographic Method

With the lexicographic method the objective functions are ordered in order of importance. Then the pursuing optimization problems are solved single at the juncture:

$$\underset{\mathbf{x}\in\mathbf{X}}{\text{Minimize}}\ F_i(\mathbf{x})$$
$$\text{subject to } F_j(\mathbf{x}) \leq F_j(\mathbf{x}_j^*)\ , \quad j = 1, 2, \ldots, i-1\,,\ i > 1\,,$$
$$i = 1, 2, \ldots, k\,.$$

At this spot i represent a function's position in the preferred sequence, and Fj represents the optimum of the jth objective function, found in the jth iteration. Some authors distinguish the hierarchical procedure from the lexicographic appproach, as having the pursuing restrictions [22] :

$$F_j(\mathbf{x}) \leq \left(1 + \frac{\delta_j}{100}\right) F_j(\mathbf{x}_j^*)\ , \quad j = 1, 2, \ldots, i\,, \quad i > 1\,.$$

In this instance, δj are positive tolerance determined by the DM, and as they enlarge, the attainable region prescribed by the objective functions enlargs. This lesson the sensitivity of the final solution to the initial objective function grading procedure. δj need not be less than 100 [23].

Weighted min-max method

Weighted min-max formulation or weighted Tchebycheff procedure is stated as pursue:

$$U = \max_i \{w_i [F_i(\mathbf{x}) - F_i^\circ]\}\ .$$

A usual approach for serving is to introduce an addendum not known parameter λ:
Minimize

$$\underset{\mathbf{x}\in\mathbf{X}}{\text{Minimize}}\ F_i(\mathbf{x})$$
$$\text{subject to } F_j(\mathbf{x}) \leq F_j(\mathbf{x}_j^*)\ , \quad j = 1, 2, \ldots, i-1\,,\ i > 1\,,$$
$$i = 1, 2, \ldots, k\,.$$

Nevertheless, enlarging the number of restrictions can enlarge the complexity of the problem. As discussed before incresing the value of p can enlarge the efficaciousness of the weighted global criterion procedure in offering the entire pPareto optimal set.

It is viable to alter the weighted min-max procedure in order to reduce the potential for solutions that are only weakly Pareto optimal using the large weighted Tchebycheff procedure[24] or the altered weighted Tchebycheff procedure[25] as displayed in equations:

$$U = \max_i \{w_i [F_i(\mathbf{x}) - F_i^\circ]\} + \rho \sum_{j=1}^{k} [F_j(\mathbf{x}) - F_j^\circ] \ ,$$

$$U = \max_i \left\{ w_i \left[F_i(\mathbf{x}) - F_i^\circ + \rho \sum_{j=1}^{k} \left(F_j(\mathbf{x}) - F_j^\circ \right) \right] \right\} .$$

ρ, is an adequate little positive scalar allocated by theDM. Minimizing above equations is obligatory and adequate for Pareto optimality with discreet problems and with problems including only linear restrictions [25]. For general problems the two formulations are obligatory and adequate for actual Pareto optoimality [26].

The following modifiction to first modified equation of weighed min max method also offers an obligatory and adequate stipulation for actual Pareto optimality.

$$U = \max_i \{w_i [F_i(\mathbf{x}) - F_i^\circ]\} + \rho \sum_{j=1}^{k} w_j [F_j(\mathbf{x}) - F_j^\circ] \ .$$

Sufficient for actual Pareto optimality implies adequacy for Pareto optimality. The lexicographic weighted Tchiebycheff procedure offer one more alteration that invariably produce Pareto optimal points. This approach stems from first modified equation weighed min max method and optimality in the min-max sense.

In this manner the algorithm remove the viability of non unique solutions and the use of ρ come to be unneeeded. This approach is obligatory and adequate for Pareto optimality.

Exponential weighted criterion

In response to the lack of ability of the weighted sum procedure to captured points on non convex part of the Pareto optimal surface, proposed the exponential weighed criterion[27], as follows:

$$U = \sum_{i=1}^{k} \left(e^{pw_i} - 1\right) e^{pF_i(\mathbf{x})},$$

Where the argument of the summation depicts a single usefull functionfor F1 (x). In spite of the fact large values of p can lead to numerical overflow minimizing the equation given in exponential weighted criteria offers a obligatory and adequate stipulation Pareto optimality.

Weighted product method

To permit function with contrast orders of magnitude to have alike importance and to avoid having to modify objective functions one may consider the pursuing formulation:

$$U = \prod_{i=1}^{k} \left[F_i\left(\mathbf{x}\right)\right]^{w_i},$$

Where wi are weights designate the comparative importance of the objective function.

Goal programming methods

The optimization problem is formulated as follows :

$$\underset{\mathbf{x}\in\mathbf{X},\mathbf{d}^-,\mathbf{d}^+}{\text{Minimize}} \sum_{i=1}^{k} \left(d_i^+ + d_i^-\right)$$

$$\text{subject to } F_j(\mathbf{x}) + d_j^+ - d_j^- = b_j\,, \quad j = 1, 2, \ldots, k\,,$$

$$d_j^+, d_j^- \geq 0\,, \quad j = 1, 2, \ldots, k\,,$$

$$d_j^+ d_j^- = 0\,, \quad j = 1, 2, \ldots, k\,.$$

In the non appearance of any alternate, bj=F ^{o} j, in which instance above equation is is theoretically alike to settle programming and can be consider a kind of global criterion method. This is mainly seeming when a desired point is employed with absolute values signs in equations second, third and fourth in weighted global criteria method. Nevertheless, notwithstanding its popularity and broad range of application, there is no assurance that it priovides a Pareto optimal solution. In addendum ,above equation has addendum variables and nonlinear equality restrictions, both of which can be annoying with greater problems. Archimedean goal programming (or weighted goal programming) compose a subclass of goal programming, in which weights are allocated to the divergence of every objective from its perspective goal. The preemptive (or lexicographic) goal programming approach is alike to the lexicigraphic method in that the divergence |dj |=d+j+d-j for the objective are sequenced in terms of priority and minimized lexicographically. Archimedean goal programming and preemptive goal programming offer Pareto optimal solutions if the goals form a Pareto optimal point or if each of divergence variables, d+j for functions being enlarged and d-j for functions being lessen, have positive values at the optimum. The latter condition propose that each of the goals must be unachievable. Normally, nevertheless, Archemedean and preemtive goal programming can outcome in non-optimal solutions[28]. The goal attainment method which is computationally speedy than classic goal programming methods. it is founded on the weighted min-max approach and is formulated as follows :

$$\underset{\mathbf{x}\in\mathbf{X},\lambda}{\text{Minimize}}\,\lambda$$

$$\text{subject to } F_i(\mathbf{x}) - w_i\lambda \leq b_i\,, \quad i = 1, 2, \ldots, k\,,$$

where wi are weights specifying the comparitive significance of every objective function and λ is an unrestricted scalar, alike to that which is employed in equation given in weighted sum method.

Bounded objective function method

The bounded objective function method minimizes the lone most salient objective function Fs(x). Each of alternate objective functions are employed to formation addendum restriction like that li ≤Fi (x) ≤ ai, I = 1,2………, k, I _ =s. li and ai are the lower and upper bound for the objective function Fi (x), respectively. li is outdated the purpose is to attain a goal or drop within a range of values for fi(x), preferably than to determine a minimum. If it subsist, a solution to the a-constraint formulation is weakly Pareto optimal and an weakly Pareto optima; point can be acquired if the attainable region is convex and if each of the objective functions are explicitly quasi-convex. If the solution is unique, then it is Pareto optimal. of course, uniqueness can be difficult to verify, although if the problem is convex and if Fs(x) is strictly convex, then the solution is necessarily unique[29]. Solutions with active a- constraints(and non-zero Langrange multipliers) are necessary Pareto optimal.

Physical programming

Physical programming maps general classifications of goals and objectives, and verbally expressed preferences to a utility function. It provides a means of incorporating preferences without having to conjure relative weights.

Objective functions, restrictions, and goals are treated equivalently as design metrics. In general, the decision maker customizes an individual utility function, which is called a class function Fi [Fi (x)] , for each design metric.

Specifically, each type of design metric is first associated with a type of individual utility function distinguished by a general form, such as a monotonically increasing, monotonically decreasing, or unimodal function. Then, for each metric the decision-maker specifies the numerical range that corresponds to different degrees of preference (desirable, tolerable, undesirable, etc.) [30].

The requirement that the decision maker quantitatively classify different range of values for each metric can be viewed in two ways. On one hand, it suggests that physical programming requires significant familiarity with each objective and restriction. On the other hand, in a more positive light, it implies that physical programming allows one to make effective use of variable information. The individual utility functions, as non-dimensional unimodal transformations, are combined into a utility function as follows :

$$F_a(\mathbf{x}) = \log\left\{\frac{1}{dm}\sum_{i=1}^{dm}\overline{F}_i\left[F_i(\mathbf{x})\right]\right\},$$

where dm represents the number of design metrics being considered.

Methods for a posteriori articulation of preference

In some case it is difficult for a decision maker to express an explicit approximation of the preference function. Therefore, it can be effective to allow the decision maker to choose from a palette of solutions. To this end, an algorithm is used to determine a representation of the pareto optimal set. Such methods incorporate a posteriori articulation of preferences, and they are called cafeteria or generate-first-choose-later approaches. The use of weighted method is a common means of providing the Pareto optimal set (or subset) . These methods all depend on the solution of multiple sequential optimization problems with a consistent variation in method parameters. When these methods are used to provide only a lone Pareto optimal point, the decision maker's preferences are presumably embedded in the parameter set. On the other hand, when the decision-maker desires a set of Pareto optimal points, the parameters very simply as a mathematical device. In such cases, it is important for the formulation to provide a necessary condition for Pareto optimality, encompassing the ability to yield all of the Pareto optimal points.

Physical programming

Although it was initially developed for a priori articulation of preferences, physical programming can be effective in providing Pareto optimal points that accurately represent the complete Pareto optimal set, even when the Pareto optimal surface is non convex. As explained earlier, when physical programming is used for a priori articulation of preferences, the decision maker specifies a set of constants that delineates numerical ranges of objective function and constraint values. These ranges are associated with different degree of preferences (desirable, tolerable, undesirable, etc). This is done for each metric, resulting in a unique utility function.

Normal boundary intersection (NBI) method

In response to deficiencies in the weighted sum approach there presented the NBI method. This method provides a means for obtaining an even distribution of Pareto optimal points for a consistent variation in the user supplied parameter vector w, even with a non convex Pareto optimal set. The approach is formulated as follows:

$$\underset{\mathbf{x}\in\mathbf{X},\lambda}{\text{Minimize}}\ \lambda$$

$$\text{subject to } \mathbf{\Phi w} + \lambda \mathbf{n} = \mathbf{F}(\mathbf{x}) - \mathbf{F}^{\circ}\,.$$

Φ is a Kxk pay off matrix in which the ith column is composed of the vector F(xi) is the vector of objective functions evaluated at the minimum of the ith objective function. the diagonal elements of Φ are zero. w is a vector of scalar. Since each component of Φ is positive, the negative sign ensure that n points towards the origin of the criterion space. As w is systematically modified, the solution yields an even distribution of Pareto optimal points representing the complete Pareto set. However, the method may yield non Pareto points; it does not provide a sufficient condition for Pareto optimality

Normal constraint (NC) method

The NC method provides an alternative to the NBI method with some improvements. When used with normalized objective functions and with a Pareto filter, which eliminates non Pareto optimal solutions, this approach provides a set of evenly spaced Pareto optimal points in the criterion space. In fact, it always yields Pareto optimal solutions. Its performance is independent of design objective scales. The method as follows:

First the utopia point is determined and its components are used to normalize the objectives with transforming objective functions. The individual minima of the normalized objective functions form the vertices of what is called the utopia hyper plane. A sample of evenly distributed points on the utopia hyper plane is determined from a linear combination of the vertices with consistently varied weights in the criterion space. The user must specify how many points are needed to accurately represent the Pareto optimal set. Then, each sample point is projected onto the Pareto optimal surface by solving a separate single objective problem. This problem entails minimizing one of the normalized objective functions with addendum inequality restrictions.

Methods with no articulation of preferences

Often the decision maker cannot concretely define what they prefer. Consequently, this section describes methods that do not require any articulation preferences.

Global criterion methods

The fundamental idea behind most global criterion methods is the use of an exponential sum, which is formed by setting all of the weights in first and second equations in weighted global criterion method to one. This yields a single function Fg(F).

Techniques for order preference by similarity to ideal solution when forming a measure of distance, it is possible and often necessary to seek a point that only is as close as possible to the utopia point but also is as far away as possible from some detrimental point. The technique for order preference by similarity to ideal solution (TOPSIS) takes this approach and is a form of compromising programming. The utopia point is the positive ideal solution and the vector in the criterion space that is composed of the worst or most undesirable solutions for the objective functions is called the negative ideal. Similarity is developed as a function that is inversely proportional to the distance from the positive ideal and directly proportional to the distance from negative ideal. Then the similarity is maximized.

Objective sum method

When equation of weighted sum method is used with p=1 and w=1 the result is simply the sum of the objective functions. Not only is this a special case of a global criterion method, it is a special case of the weighted sum method and it always provides a Pareto optimal solution.

Min Max method

A basic min-max formulation is derived by excluding the weights in first and second equations in weighted global criterion method and using p=∞. Assuming the weights are excluded in third equation in weighted global criterion method yields an L∞-norm which does not necessarily yield a Pareto optimal point. However, in accordance with the definition of optimality in the min-max sense, if the minimum of the L∞-norm is unique then it is Pareto optimal. If the solution is not unique, the definition of optimality in the min-max sense provides additional theoretical criteria for r min-max algorithm to eventually yield a Pareto optimal point. The basic min-max formulation is posed as follows:

$$\underset{\mathbf{x}\in\mathbf{X}}{\text{Minimize}}\ \max_{i}\left[F_i\left(\mathbf{x}\right)\right] .$$

In order to avoid additional constraints and the discontinuity the following smooth approximations are developed:

$$F_g\left(\mathbf{x}\right) = \frac{1}{c}\ln\left[\sum_{i=1}^{k} e^{cF_i(x)}\right] ,$$

$$F_g\left(\mathbf{x}\right) = c\log\left[\sum_{i=1}^{k} e^{F_i\left(\mathbf{x}\right)/c}\right] .$$

c>0 is called the controlling parameter. Although the physical significance of c is unclear.

Nash arbitration and objective product method

The Nash arbitration scheme is an approach that is derived from game theory. Based on predetermined axioms of fairness suggests that the solution to an arbitration problem be the maximum of the product of the player's utilities. In this case the utility functions always have no negative values and have a value of zero in the absence of cooperation. In terms of a mathematical formulation in which individual objective functions are minimized, the method entails maximizing the following global criterion:

Where si≥Fi(x). si may be selected as an upper limit on each function guaranteeing that F(x)<s. This ensure that above equation yields a Pareto optimal point, considering that if any component of the product in above equation becomes negative the result can be a non Pareto optimal solution. Alternatively si may be determined as the value of objective I at the starting point, in which case the constraint Fi(x) ≤ si must be added to the formulation to ensure Pareto optimality. On fundamental level the Nash arbitration approach simply entails minimizing the product of the objective functions. It is equivalent to equation of weighted product method with w=1.
With a product even objective function values with relatively small orders of magnitude can have a significant effect on the solution. However, a caveat of any product type global criterion is that it can introduce unwieldy nonlinearities.

Rao's method

The following work is based on the use of a product type global criterion shown in equation given in Nash arbitration and objective product method. First, the following super criterion is minimized:

$$SU = \prod_{i=1}^{k} \left[1 - F_i^{\text{norm}}(\mathbf{x})\right],$$

articulation of preferences, which allows one to design a utility function, depends on the type of preferences that the decision maker wishes to articulate and on the amount of preference information that the decision maker has. Where form I (x) is a normalized objective function, with values between zero and one, like that form i=1 is the vanquish viable value. Next, one shaped the Pareto optimal objective FC, which is a bit of scalarized objective function that produce a Pareto optimal solution. The procedure parameters that are absorbed in the scalarized objective function are indulged as design variables. Then, a new objective function, OBJ= FC-SU, is minimized

Conclusion

In general, multi objective optimization needs extra computational endeavour than single-objective optimization. Unless likings are irrelevant or entirely comprehended, solution of various single objective problems may be requisite to acquire an acceptable final solution. Solutions acquired with no articulation of preferences are arbitrary respective to the Pareto optimal set. In this category of methods, the objective sum method is one of the most computationally systematic, easy-to-use, and usual approaches. Therefore, it provides a benchmark approach to multiobjective optimization. Methods with priori articulation of preferences need the user to specify preferences only in terms of objective functions.

ACKNOWLEDGEMENT

We would like to express our gratitude to all those who gave us the possibility to complete this work. We are very grateful to some people who have coordinated us in the effort of achieving the success of my research paper on **“**Study on Multi Objective Optimization methods”.

First and foremost we bear our sincere thanks to **Dr. A.K. Srivastava** who guided us in each & every work related to our research and helped throughout in overcoming the obstacles we encountered.

Second we are very much thankful to **Mr. Deepak chaudhary** for vital encouragement and support. He kindly offered invaluable detailed advices.

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