BASIAN-MARKOVIAN PRINCIPLE IN FITTING OF QUADRATIC CURVE

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ABSTRACT

A method of estimation using both of Basian and Markovian principles has been developed in fitting of quadratic curve to observed data. The method developed has been applied in fitting of quadratic equation of monthly mean extremum temperature on the monthly average length of day at the five cities namely Guwahati, Dhubri, Dibrugarh, Silchar and Tezpur. It has been found that the application of Basian-Markovian principle in fitting of quadratic equation of monthly mean extremum temperature on the monthly average length of day is more accurate than the fitting of the same by applying the Basian principle.

1. Introduction:

In fitting a curve to numerical data by the traditional method of least squares, the parameters involved in the curve are estimated first by solving normal equations of the curve and then the estimated values corresponding to the observed values are obtained from the curve using the estimated values of the parameters and the observed values of the independent variable. The principle of obtaining an estimated value of the dependent variable from the estimates of the parameters (involved) and the corresponding value of the independent variable is nothing but the Basian principle. However, in practice there are situations where a value of the associated variable depends on its just preceding value. In such situations, it seems to be more accurate, if the estimated values of the dependent variable are determined on the basis of the just preceding observed value of it. The principle of obtaining an estimated value of the dependent variable from its just preceding observed value is nothing but the Markovian principle. An attempt has been made to determine estimated value of the dependent variable using the Markovian principle. However, since the values of the parameters involved are unknown, they are to be estimated using the entire observed data (since parameter cannot be estimated from single data). The principle of obtaining estimate of parameter from entire data is the Basian principle. Thus, the fitting of curve where parameters are estimated from entire data and estimate of dependent variable is obtained from its just preceding observed value and estimated values of parameters involved can be termed as Basian-Markovian principle. Rahman and Chakrabarty (2007) have innovated a method of fitting exponential curve by Basian-Markovian principle. In another study, Rahman and Chakrabarty (2008) have innovated similar method of fitting Gompertz curve by the same principle. In the current study, a method of estimation using Basian-Markovian principle has been developed in fitting of quadratic equation to observed data. The method developed has been applied in fitting of quadratic equation of monthly mean extremum temperature on the monthly average length of day at the five cities namely Guwahati, Dhubri, Dibrugarh, Silchar and Tezpur.

2. Fitting of Quadratic curve:

The quadratic curve considered here is of the form

$$Y = a + bX + cX^2 \tag{2.1}$$

where *a. b* and *c* are the parameters of the curve which are to be estimated from observed data on the pair (X, Y) of variables X and Y.

Let the dependent variable 'Y' assume the values

$Y_1, Y_2, \ldots\ldots\ldots\ldots, Y_n$

corresponding to the values

$X_1, X_2, \ldots\ldots\ldots\ldots, X_n$

of the independent variable 'X' respectively.

The objective is to estimate the parameters *a, b* and *c* of the quadratic curve described by the equation (2.1) on the basis of the *n* pairs

$(X_1, Y_1), (X_2, Y_2), \ldots\ldots\ldots\ldots\ldots, (X_n, Y_n)$

of observations.

Since all pairs of observations may not lie on the linear curve defined above, the equation (2.1) yields

$$Y_i = a + bX_i + cX_i^2 \quad , \qquad (i = 1, 2, \ldots\ldots\ldots\ldots, n) \qquad (2.2)$$

Now, from (2.2) we have

$$y_{i+1} = a + bx_{i+1} + cx_{i+1}^2$$

$$\Rightarrow \qquad y_{i+1} - y_i = b(\Delta x_i) + c(\Delta x_i^2)$$

$$\Rightarrow \qquad y_{i+1} = y_i + b(\Delta x_i) + c(\Delta x_i^2) \qquad (2.3)$$

where $$\Delta X_i = X_{i+1} - X_i \qquad (2.4)$$

This is nothing but the recurrence relationship between Y_{i+1} and Y_i.

This relationship can be suitably applied to determine the estimated value of Y_{i+1} from the observed value of Y_i.

However, this recurrence relationship contains two parameters namely '*b*' and '*c*'. This parameters are to be known for estimating Y_{i+1} from Y_i.

In order to know the parameters '*b*' and '*c*', one can apply the principle of least squares to the quadratic curve described by the equation (2.1). On the application of the principle of least squares, the following normal equations are obtained:

$$\sum_{i=1}^{n} Y_i = na + b\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n} Y_i X_i = a\sum_{i=1}^{n} X_i + b\sum_{i=1}^{n} X_i^2$$

$$\sum_{i=1}^{n} X_i^2 Y_i = a\sum_{i=1}^{n} X_i^2 + b\sum_{i=1}^{n} X_i^2$$

Solving these normal equations, one can obtain the estimate of '*b*' and '*c*' as well as of '*a*'.

3. Application of the method to maximum and minimum temperature of five cities of Assam.

The method innovated here has been applied to determine the estimated temperatures of five cities in the context of Assam from the data on temperatures collected from the Regional Meteorological Centre at Borjhar, Guwahati.

3.1. Monthly Mean Extremum Temperature at Guwahati:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.1.1**.

Table - 3.1.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.536	26.226	29.972	30.883	31.363	31.768	31.995	32.470	31.720	30.383	27.731	24.724

The normal equations of the quadratic curve described by the equation (2.1) in this case become

$$352.771 = 12a + 144.131b + 1746.108159c$$

$$4271.610108 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$52148.58542 = 1746.108159\ a + 21331.\ 66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -158.423893, \quad \hat{b} = 29.21089239 \quad \text{and } \hat{c} = -1.120398754$$

Thus, the quadratic curve fitted to the data in **Table: 3.1.1** becomes

$$Y_i = -158.423893 + 29.21089239\ X_i - 1.120398754\ X_i^2 \tag{3.1}$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 29.21089239\ \Delta X_i - 1.120398754\ \Delta X_i^2 \tag{3.2}$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.1) and (3.2) have been presented in the **Table- 3.1.2.**

Table- 3.1.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = -158.4238938 +29.21089239 X_i -1.120398754X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 29.21089239 ΔX_i -1.120308754 ΔX_i^2	$\left\|\widehat{e}_{(B)}\right\| = \left\|Y_i - \widehat{Y}_{(B)}\right\|$	$\left\|\widehat{e}_{(BM)}\right\| = \left\|Y_i - \widehat{Y}_{i+1(BM)}\right\|$
10.553	23.536	25.06453995	-	1.52853995	-
11.105	26.226	27.79434344	26.26580349	1.56834344	0.03980349
11.834	29.972	30.35318110	28.78483766	0.63318110	1.18716234
12.610	30.883	31.76850053	31.38731943	0.88550530	0.50431943
13.266	31.363	31.91246250	31.02696198	0.54946250	0.33603802
13.605	31.768	31.60894139	31.05947888	0.15905861	0.70852112
13.469	31.995	31.76163994	31.920698550	0.23336006	0.07430145
12.921	32.470	31.95696398	32.190324040	0.51303602	0.27967595
12.191	31.720	31.17189360	31.684929620	0.54810640	0.03507038
11.424	30.383	29.06059125	29.608697650	1.32240875	0.77430235
10.755	27.731	26.14270197	27.465156540	1.58829803	0.26584346
10.398	24.724	24.17524015	25.763538180	0.54875985	1.03953818

Mean of the absolute deviations in column 5 = 0.839838334

& Mean of the absolute deviations in column 6 = 0.571283123

(B) Estimation of Mean Minimum Temperature (Guwahati)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.1.3.**

Table - 3.1.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	10.815	14.849	16.087	20.488	22.793	25.045	25.660	25.635	24.650	22.057	16.982	12.226

The normal equations of the quadratic described by the equation (2.1) in this case become

$$237.287 = 12a + 144.131b + 1746.108159c$$

$$2909.964292 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$35956.50357 = 1746.108159\ a + 21331.66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -168.964747,\ \hat{b} = 27.635085140 \text{ and } \hat{c} = -0.984021793$$

Thus, the quadratic curve fitted to the data in **Table – 3.1.3** becomes

$$Y_i = -168.964747 + 27.635085140\ X_i - 0.984021793X_i^2 \qquad (3.3)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 27.635085140\ \Delta X_i - 0.984021793\Delta\ X_i^2 \qquad (3.4)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.3) and (3.4) have been presented in the **Table- 3.1.4.**

Table- 3.1.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -168.9647479 +27.63508514 X_i -0.984021793X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 27.63508514 ΔX_i -0.984021793 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	10.815	13.08192253	-	2.26692253	-
11.105	14.849	16.57229644	14.30537391	1.72329644	0.54362609
11.834	16.087	20.26293857	18.53964213	4.17593857	2.45264213
12.610	20.488	23.04230397	18.86636539	2.55430397	1.62163461
13.266	22.793	24.46748838	21.91318442	1.67448838	0.87981558
13.605	25.045	24.87206303	23.19757465	0.17293697	1.84742535
13.469	25.660	24.73692267	24.90985964	0.92307733	0.75014036
12.921	25.635	23.82354366	24.74662099	1.81145634	0.88837901
12.191	24.650	21.68878285	23.50023919	2.96121715	1.14976081
11.424	22.057	18.31596900	21.27718615	3.74103100	0.77981385
10.755	16.982	14.42876738	18.16979828	2.55323262	1.18779838
10.398	12.226	11.99400163	14.54723424	0.23199837	2.32123424

Mean of the absolute deviations in column 5 = 2.065824973

& Mean of the absolute deviations in column 6 = 1.311115492

3.2. Monthly Mean Extremum Temperature at Dibrugarh:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.2.1**.

Table - 3.2.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.089	23.891	26.689	27.206	29.823	31.123	31.057	31.714	30.746	29.820	27.209	24.140

The normal equations of the quadratic curve described by the equation (2.1) in this case become

$$336.507 = 12a + 144.131b + 1746.108159c$$

$$4074.145399 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$49737.85797 = 1746.108159\ a + 21331.66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -63.77051619,\ \hat{b} = 13.22310356 \quad \text{and} \quad \hat{c} = -0.460513251$$

Thus, the quadratic curve fitted to the data in **Table – 3.2.1** becomes

$$Y_i = -63.77051619 + 13.22310356\ X_i - 0.460513251\ X_i^2 \qquad (3.5)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 13.22310356\ \Delta X_i - 0.460513251\ \Delta X_i^2 \qquad (3.6)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.5) and (3.6) have been presented in the **Table- 3.2.2.**

Table- 3.2.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = -63.77051619 +13.22310356 X_i -0.460513251X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 13.22310356 ΔX_i -0.460513251 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	23.089	24.48746493	-	1.39844930	-
11.105	23.891	26.28108270	24.88261778	2.39008270	0.99161778
11.834	26.689	28.21977808	25.82969538	1.53077808	0.85930462
12.610	27.206	29.74564058	28.21486250	2.53964058	1.00886250
13.266	29.823	30.60294250	28.06330192	0.77994250	1.75969808
13.605	31.123	30.89063552	30.11069303	0.23236448	1.01230697
13.469	31.057	30.78793270	31.02029718	0.26906730	0.03670282
12.921	31.714	30.20148564	30.47055294	1.51251436	1.24344706
12.191	30.746	28.99063844	30.50296948	1.75536156	0.24303052
11.424	29.820	27.18965867	28.94499566	2.63034133	0.87500434
10.755	27.209	25.17638334	27.80672467	2.03261666	0.59772467
10.398	24.140	23.93335691	25.96597357	0.20664309	1.82597357

Mean of the absolute deviations in column 5 = 1.439816828

& Mean of the absolute deviations in column 6 = 0.950333902

(B) Estimation of Mean Minimum Temperature (Dibrugarh)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.2.3.**

Table: 3.2.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	9.374	12.434	16.089	18.914	21.951	24.277	24.746	25.034	23.963	20.843	14.983	10.229

The normal equations of the quadratic described by the equation (2.1) in this case become

$$222.837 = 12a + 144.131b + 1746.108159c$$

$$2741.911569 = 144.131\,a + 1746.108159\,b + 21331.66706\,c$$

$$33984.03941 = 1746.108159\,a + 21331.66706\,b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -203.4115382,\ \hat{b} = 32.846126930 \text{ and } \hat{c} = -1.185705279$$

Thus, the quadratic curve fitted to the data in **Table: 3.2.3** becomes

$$Y_i = -203.4115382 + 32.846126930\,X_i - 1.185705279\,X_i^2 \tag{3.7}$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 32.846126930\,\Delta X_i - 1.185705279\,\Delta X_i^2 \tag{3.8}$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.7) and (3.8) have been presented in the **Table- 3.2.4.**

Table- 3.2.4

X_i	Y_i	$\hat{Y}_{(B)}$ = -203.4115382 +32.84612693 X_i -1.185705279X_i^2	$\hat{Y}_{i+1(BM)}$ = Y_i+ 32.84612693 ΔX_i -1.185705279 ΔX_i^2	$\lvert\hat{e}_{(B)}\rvert = \lvert Y_i - \hat{Y}_{(B)}\rvert$	$\lvert\hat{e}_{(BM)}\rvert = \lvert Y_i - \hat{Y}_{i+1(BM)}\rvert$
10.553	9.374	11.16661166	-	1.79261166	-
11.105	12.434	15.12231100	13.32969934	2.68831100	0.89569934
11.834	16.089	19.23914425	16.55083325	3.15014425	0.46183325

12.610	18.914	22.23663599	19.08649174	3.32263599	0.17249174
13.266	21.951	23.65756030	20.33412004	1.70575603	1.61687996
13.605	24.277	23.99068472	22.28492869	0.28631528	1.99207131
13.469	24.746	23.88945418	24.17576946	0.85654582	0.57023054
12.921	25.034	23.03711437	23.89366019	1.99688563	1.14033981
12.191	23.963	20.79550631	22.79239195	3.16749369	1.17060805
11.424	20.843	17.07885690	20.24635058	3.76414310	0.59664942
10.755	14.983	12.69799767	16.46214077	2.28500233	1.47914077
10.398	10.229	9.925927239	12.21092957	0.303072761	1.98192957

Mean of the absolute deviations in column 5 = 2.109909795

& Mean of the absolute deviations in column 6 = 1.097988524

3.3. Monthly Mean Extremum Temperature at Dhubri:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.3.1**.

Table: 3.3.1.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	22.362	25.748	28.486	30.605	30.900	30.932	31.031	31.696	31.027	29.459	26.486	23.276

The normal equations of the quadratic described by the equation (2.1) in this case become

$$342.008 = 12a + 144.131b + 1746.108159c$$

$$4144.870466 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$50642.11038 = 1746.108159\ a + 21331.66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -178.3101091,\ \hat{b} = 32.22917606 \quad \text{and} \quad \hat{c} = -1.239037831$$

Thus, the quadratic curve fitted to the data in **Table: 3.3.1** becomes

$$Y_i = -178.3101091 + 32.22917606\ X_i - 1.239037831\ X_i^2 \tag{3.9}$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 32.22917606\ \Delta X_i - 1.239037831\ \Delta X_i^2 \tag{3.10}$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.9) and (3.10) have been presented in the **Table- 3.3.2.**

Table- 3.3.2

X_i	Y_i	$\hat{Y}_{(B)} = -178.3101091 + 32.22917606\ X_i - 1.239037831 X_i^2$	$\hat{Y}_{i+1(BM)} = Y_i + 32.22917606\ \Delta X_i - 1.239037831\ \Delta X_i^2$	$\lvert\hat{e}_{(B)}\rvert = \lvert Y_i - \hat{Y}_{(B)}\rvert$	$\lvert\hat{e}_{(BM)}\rvert = \lvert Y_i - \hat{Y}_{i+1(BM)}\rvert$
10.553	22.362	23.81793543	-	1.45593543	-
11.105	25.748	26.79547571	25.33954028	1.04747571	0.40845972
11.834	28.486	29.57069652	28.52322081	1.08469652	0.03722081
12.610	30.605	31.07779353	29.99309701	0.47279353	0.61190299
13.266	30.900	31.18789207	30.71509854	0.28789207	0.18490146
13.605	30.932	30.82685385	30.53896178	0.10514615	0.39303822
13.469	31.031	31.0059025	31.11104864	0.02509750	0.08004864
12.921	31.696	31.2629322	31.28802970	0.43306780	0.40797030
12.191	31.027	30.44937783	30.88244563	0.57762217	0.14455437
11.424	29.459	28.17192651	28.74954868	1.28707349	0.70945132
10.755	26.486	24.99514254	26.28221603	1.49085746	0.20378397
10.398	23.276	22.84607079	24.33692825	0.42992921	1.06092825

Mean of the absolute deviations in column 5 = 0.724798928

& Mean of the absolute deviations in column 6 = 0.385660004

(B) Estimation of Mean Minimum Temperature (Dhubri)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.3.3.**

Table: 3.3.3.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	11.938	14.281	18.105	21.600	23.010	24.668	25.568	25.738	24.982	22.645	18.214	13.619

The normal equations of the quadratic described by the equation (2.1) in this case become

$$244.368 = 12a + 144.131b + 1746.108159c$$

$$2989.765122 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$36859.2407 = 1746.108159\ a + 21331.66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -186.3085113,\ \hat{b} = 31.005579160 \quad \text{and} \quad \hat{c} = -1.138987287$$

Thus, the quadratic curve fitted to the data in **Table: 3.3.3** becomes

$$Y_i = -186.3085113 + 31.005579160 \quad X_i - 1.138987287 X_i^2 \tag{3.11}$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 31.005579160 \quad \Delta X_i - 1.138987287 \Delta X_i^2 \tag{3.12}$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.11) and (3.12) have been presented in the **Table- 3.3.4**

Table- 3.3.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -186.3085113 +31.00557916 X_i -1.138987287X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 31.00557916 ΔX_i -1.138987287 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	11.938	14.04912492	-	2.11112492	-
11.105	14.281	17.54736558	15.43624066	3.26636558	1.15524066
11.834	18.105	21.10368257	17.83731699	2.99868257	0.26768301
12.610	21.600	23.55908153	20.56039896	1.95908153	1.03960104
13.266	23.010	24.56482407	22.60574254	1.55482407	0.40425746
13.605	24.668	24.70037382	23.14554975	0.03237382	1.52245025
13.469	25.569	24.67743914	24.64506532	0.89056086	0.92293468
12.921	25.738	24.15809699	25.04865785	1.57990301	0.68934215
12.191	24.982	22.40366579	23.98356880	2.57833421	0.99843120
11.424	22.645	19.25252731	21.83086151	3.39247269	0.81413849
10.755	18.214	15.4098046	18.8022773	2.80419540	0.58827730
10.398	13.619	12.94201316	15.74620856	0.67698684	2.12720856

Mean of the absolute deviations in column 5 = 1.987075458

& Mean of the absolute deviations in column 6 = 0.957233163

3.4. Monthly Mean Extremum Temperature at Silchar:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table: 3.4.1.**

Table: 3.4.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	24.548	26.692	29.904	30.607	31.093	31.582	31.629	32.104	31.715	30.965	28.688	25.962

The normal equations of the quadratic described by the equation (2.1) in this case become

$$355.489 = 12a + 144.131b + 1746.108159\ c$$

$$4297.161584 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$52375.65783 = 1746.108159\ a + 21331.66706\ b + 262712.8879\ c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of a, b and c respectively as

$$\hat{a} = -125.3393282,\ \hat{b} = 24.17260292 \quad \text{and} \quad \hat{c} = -0.930332113$$

Thus, the quadratic curve fitted to the data in **Table: 3.4.1** becomes

$$Y_i = -125.3393282 + 24.17260292\ X_i - 0.930332113\ X_i^2 \qquad (3.13)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 24.17260292\ \Delta X_i - 0.930332113 \Delta X_i^2 \qquad (3.14)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.13) and (3.14) have been presented in the **Table: 3.4.2.**

Table: 3.4.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = -125.3393282 +24.17260292 X_i -0.930332113X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 24.17260292 ΔX_i -0.930332113 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	24.548	26.14696201	-	1.59896201	-
11.105	26.692	28.36791757	26.76895545	1.67591757	0.07695545
11.834	29.904	30.43223739	28.75631993	0.52823739	1.14768007
12.610	30.607	31.54313164	31.01489425	0.93613164	0.40789425
13.266	31.093	31.60829157	30.67215993	0.51529157	0.42084007
13.605	31.582	31.32815848	30.81286691	0.25384152	0.76913309
13.469	31.629	31.46622687	31.72006838	0.16277313	0.09106838
12.921	32.104	31.67384299	31.83661613	0.43015701	0.26738387
12.191	31.715	31.08246787	31.51262488	0.63253213	0.20237512
11.424	30.965	29.39291255	30.02544468	1.57208745	0.93955532
10.755	28.688	27.02547744	28.59756489	1.66252256	0.09043511
10.398	25.962	25.42137372	27.08389628	0.54062628	1.12189628

Mean of the absolute deviations in column 5 = 0.875756688

& Mean of the absolute deviations in column 6 = 0.503201546

(B) Estimation of Mean Minimum Temperature (Silchar)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.4.3.**

Table - 3.4.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	12.112	13.980	17.600	21.139	23.164	24.761	25.200	25.267	24.811	23.172	18.152	13.728

The normal equations of the quadratic described by the equation (2.1) in this case become

$$243.086 = 12a + 144.131b + 1746.108159\, c$$

$$2973.124095 = 144.131\, a + 1746.108159\, b + 21331.66706\, c$$

$$36644.18864 = 1746.108159\, a + 21331.66706\, b + 262712.8879\, c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -166.6107954,\ \hat{b} = 27.76247702 \quad \text{and} \quad \hat{c} = -1.007393512$$

Thus, the quadratic curve fitted to the data in **Table: 3.4.3** becomes

$$Y_i = -166.6107954 + 27.76247702\, X_i - 1.007393512\, X_i^2 \qquad (3.15)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 27.76247702\, \Delta X_i - 1.007393512\, \Delta X_i^2 \qquad (3.16)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.15) and (3.16) have been presented in the **Table: 3.4.4.**

Table: 3.4.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -166.6107954 +27.76247702 X_i -1.007393512X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 27.76247702 ΔX_i -1.007393512 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	12.112	14.17743115	-	2.23943115	-
11.105	13.980	17.45871143	15.21928028	3.17771143	0.93828028
11.834	17.600	20.85138794	17.67367651	2.74638794	0.43132349
12.610	21.139	23.28628195	20.53989401	1.68628195	1.06010599
13.266	23.164	24.39830856	22.71202660	1.38830856	0.29797340
13.605	24.761	24.63316978	23.24486122	0.03483022	1.42313878
13.469	25.200	24.56676029	24.60159051	1.00123971	0.96640949
12.921	25.267	23.92156578	24.92280549	1.81643422	0.81519451
12.191	24.811	22.12225364	23.93868786	2.85974636	1.04331214
11.424	23.172	19.07505527	21.93480163	3.56994473	0.71019837
10.755	18.152	15.44941223	19.01935696	2.76458777	0.80535696
10.398	13.728	13.14566194	15.91024970	0.47333806	2.29124970

Mean of the absolute deviations in column 5 = 1.979853508

& Mean of the absolute deviations in column 6 = 0.980231191

3.5. Monthly Mean Extremum Temperature at Tezpur:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table: 3.5.1.**

Table: 3.5.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.559	25.976	29.397	30.015	30.895	31.867	31.900	32.197	31.587	30.680	28.195	24.716

The normal equations of the quadratic described by the equation (2.1) in this case become

$$350.984 = 12a + 144.131b + 1746.108159c$$

$$4248.336627 = 144.131\ a + 1746.108159\ b + 21331.66706\ c$$

$$51846.71968 = 1746.108159\ a + 21331.66706\ b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of *a, b* and *c* respectively as

$$\hat{a} = -134.3453199,\ \hat{b} = 25.26319768 \quad \text{and} \quad \hat{c} = -0.961041329$$

Thus, the quadratic curve fitted to the data in **Table: 3.5.1** becomes

$$Y_i = -134.3453199 + 25.26319768\ X_i - 0.961041329\ X_i^2 \qquad (3.17)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 25.26319768\ \Delta X_i - 0.961041329\ \Delta X_i^2 \qquad (3.18)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.17) and (3.18) have been presented in the **Table: 3.5.2.**

Table: 3.5.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = -134.3453199 +25.26319768 X_i -0.961041329X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 25.26319768 ΔX_i -0.961041329 ΔX_i^2	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	23.559	25.23006013	-	1.63106013	-
11.105	25.976	27.68588858	26.01482845	1.70988858	0.03882845
11.834	29.397	30.03171627	28.32182769	0.63471627	1.07517231
12.610	30.015	31.40640293	30.77168667	1.39140293	0.75668667
13.266	30.895	31.66571465	30.27431172	0.77071465	0.62068828
13.605	31.867	31.47555468	30.70484003	0.39144532	1.16215997
13.469	31.900	31.57837547	31.96982080	0.32162453	0.06982080
12.921	32.197	31.63245375	31.95407828	0.56454625	0.24292172
12.191	31.587	30.80789844	31.37244469	0.77910156	0.21455531
11.424	30.680	28.83808391	29.61718546	1.84191609	1.06281454
10.755	28.195	26.19669660	28.03861269	1.9983034	0.15638731
10.398	24.716	24.43515491	26.43345831	0.28084509	1.71745831

Mean of the absolute deviations in column 5 = 1.026297067

& Mean of the absolute deviations in column 6 = 0.647044879

(B) Estimation of Mean Minimum Temperature (Tezpur)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.5.3.**

Table: 3.5.3.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	11.424	13.651	17.075	19.828	22.377	24.603	25.132	25.264	24.497	21.665	16.808	12.610

The normal equations of the quadratic described by the equation (2.1) in this case become

$$234.934 = 12a + 144.131b + 1746.108159c$$

$$2878.797313 = 144.131\,a + 1746.108159\,b + 21331.66706\,c$$

$$35544.74592 = 1746.108159\,a + 21331.66706\,b + 262712.8879c$$

which yield the least squares estimates $\hat{a}$, $\hat{b}$ and $\hat{c}$ of a, b and c respectively as

$$\hat{a} = -161.3371675,\ \hat{b} = 26.518196580 \quad \text{and} \quad \hat{c} = -0.945596739$$

Thus, the quadratic curve fitted to the data in **Table: 3.5.3** becomes

$$Y_i = -161.3371675 + 26.518196580\,X_i - 0.945596739\,X_i^2 \tag{3.19}$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 26.518196580\,\Delta X_i - 0.945596739\,\Delta X_i^2 \tag{3.20}$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.19) and (3.20) have been presented in the **Table: 3.5.4.**

Table: 3.5.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -161.3371675 +26.51819658 X_i -0.945596739X_i^2	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 26.51819658 ΔX_i -0.945596739 ΔX_i^2	$\left\lvert\widehat{e}_{(B)}\right\rvert = \left\lvert Y_i - \widehat{Y}_{(B)}\right\rvert$	$\left\lvert\widehat{e}_{(BM)}\right\rvert = \left\lvert Y_i - \widehat{Y}_{i+1(BM)}\right\rvert$
10.553	11.424	13.20221518	-	1.77821518	-
11.105	13.651	16.53544643	14.757233125	2.88444643	1.10623125
11.834	17.075	20.05444096	17.169994530	2.97944096	0.09499453
12.610	19.828	22.69596815	19.716527200	2.86796815	0.11147280
13.266	22.377	24.04072575	21.172757600	1.66372575	1.20424240
13.605	24.603	24.41669933	22.752973580	0.18630067	1.85002642
13.469	25.132	24.29197231	24.478272980	0.84002769	0.65372702
12.921	25.264	23.43495585	24.274983550	1.82904415	0.98901645
12.191	24.497	24.41112482	23.240168970	0.08587518	1.25683103
11.424	21.665	18.19898283	21.284858010	3.46601717	0.38014199
10.755	16.808	14.48883828	17.954855454	2.31916172	1.14685545
10.398	12.610	12.16263029	14.481792010	0.44736971	1.87179201

Mean of the absolute deviations in column 5 = 1.778966063

& Mean of the absolute deviations in column 6 = 0.969575577

4. Conclusion:

In the fitting of quadratic equation of monthly mean extremum (maximum and minimum) temperature on the monthly average length of day at the five cities in Assam, the computed values of the mean absolute deviations of the estimated values from the respective observed values, obtained in both types of fitting (namely fitting by Basian principle and fitting by Basian-Markovian principle) have been shown in the following table (**Table: 4.1 and Table: 4.2**).

Table: 4.1- Maximum Temperature

City	Mean of absolute deviation		Comparison
	Basian Principle	Basian – Markovian Principle	
Guwahati	**0.839838334**	**0.571283123**	Basian › Basian - Markovian
Dibrugarh	**1.439816828**	**0.950333902**	Basian › Basian - Markovian
Dhubri	**0.724798928**	**0.385660004**	Basian › Basian - Markovian
Silchar	**0.875756688**	**0.503201546**	Basian › Basian - Markovian
Tezpur	**1.026297067**	**0.647044879**	Basian › Basian - Markovian

Table: 4.2- Minimum Temperature

City	Mean of absolute deviation		Comparison
	Basian Principle	Basian – Markovian Principle	
Guwahati	2.065824973	1.311115492	Basian › Basian - Markovian

Dibrugarh	2.109909795	1.097988524	Basian › Basian - Markovian
Dhubri	1.987075458	0.957233163	Basian › Basian - Markovian
Silchar	1.979853508	0.980231191	Basian › Basian - Markovian
Tezpur	1.778966063	0.969575577	Basian › Basian - Markovian

It is found that the mean absolute deviation of estimated values from the respective observed values in the case of fitting by Basian-Markovian principle is less than the corresponding mean absolute deviation in the case of fitting by Basian principle in the case of both monthly mean maximum and monthly mean minimum temperature at all the five cities of Assam. Thus one can conclude that the application of Basian-Markovian principle in fitting of quadratic equation of monthly mean extremum temperature on the monthly average length of day is more accurate than the fitting of the same by applying the Basian principle.

5. Reference:

1. Adrian, A. (1808). The Analyst. No.-IV. Pq. 93-109

2. Bassel (1838). On untersuchungen ueber die Wahrscheinlichkeit der Beobachtungsfehler.

3. Crofton (1870). On the proof of the law of error of observations. Phil. Trans, London, pp 175-188

4. Donkin (1844). An essay on the theory of the combination of observations. Joua. Math., XV: 297-322

5. Gauss, C.F. (1809). Theory of Motion of Heavenly bodies. Humburg. Pp 205-224

6. Hagen (1837). Grandzuge der Wahrscheinlichkeitsrechnung, Berlin.

7. Herschel, J. (1850). Quetelet On Probabilities. Edinburgh Review, Simpler XCII: 1-57.

8. Ivory (1825). On the method of least squares, Phil. Mag., LXV: 3-10

9. Koppelmann, R. (1971). The calculus of operation, History of Exact Sc., 8: 152-242

10. Laplace (1810), Theoretic Analytique des prob. Ch-IV.

11. Merriman, M. (1877). The Analyst. Vol.-IV, NO.-2

12. Rahman A. and Chakrabarty D (2007): "Exponential Curve: Estimation – using the just preceding observation in fitted curve", Int. J. Agricult. Stat. Sci., 5(2), 415–424.

13. Rahman A. and Chakrabarty D (2008): "Gompartz Curve: Estimation – using the just preceding observation in fitted curve", Int. J. Agricult. Stat. Sci., Vol. 4, No. 2, Pp 421-424