ESTIMATION OF VARIANCE OF THE TIME TO RECRUITMENT IN A SINGLE GRADED MANPOWER SYSTEM WITH TWO SOURCES OF DEPLETION AND TWO THRESHOLDS

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ABSTRACT

In this paper the problem of time to recruitment is studied for a single grade manpower system with two sources of depletion and two thresholds using a univariate policy of recruitment based on shock model approach. The variance of the time to recruitment is estimated when the inter-policy decision times are exchangeable and constantly correlated exponential random variables and the number of transfer decisions are governed by a Poisson process.

KEYWORDS - Single grade manpower system, two sources of depletion, two thresholds, exchangeable and constantly correlated exponential inter-policy decision times, Poisson process, univariate policy of recruitment and variance of time to recruitment.

1. INTRODUCTION

Early studies related to manpower planning were reported by a number of researchers namely Bartholomew(1971, 1973), Grinold and Marshal(1977), Mehlmann(1977), Mukherjee and Chattopadhyay(1985) in the context of providing elementary theory of wastages and their measures for a manpower system. For a single grade manpower system which is subjected to loss of manpower due to the policy decisions taken in the system, Sathiyamoorthi and Elangovan (1998) have initiated the study on finding the time to recruitment in this system using a univariate policy of recruitment based on shock model approach. Kasturi and Srinivasan(2005) have studied this problem when the inter-policy decision times are exchangeable and constantly correlated exponential random variables. Venkat Lakshmi(2007) studied this problem using a bivariate policy of recruitment. In 2012, Esther clara has studied this problem for the first time by considering an optional and mandatory thresholds for the cumulative loss of manpower when the inter-policy decision times are either independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables using univariate and bivariate policies of recruitment. Like policy decisions which form one source of depletion, transfer decisions also produces loss of manpower independently and hence, a second source of depletion can be considered to make manpower planning models more realistic. In this context, Elangovan et.al(2011) have studied the problem of time to recruitment for a single grade manpower system with two sources of depletion when the inter-policy decisions and inter-transfer decisions form two ordinary renewal processes by considering only a mandatory threshold. Usha et.al(2013) have extended this work for correlated inter-policy decision times. Recently Mercy Alice and Srinivasan(2014) have studied the work of Elangovan et.al for a manpower system with optional and mandatory thresholds. The present paper extends the work of Mercy Alice and Srinivasan(2014) for correlated inter-policy decision times. For a single grade manpower system, the present work also studies the work of Elangovan et.al(2011) for a system with two thresholds and that of Esther Clara(2012) for a system with two sources of depletion.

2. MODEL DESCRIPTION

Consider an organization taking decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let $X_{iP}, i = 1,2, \ldots.$ be the continuous random variables representing the amount of depletion of manpower due to the i[th] policy decision in the organization. It is assumed that X_{iP} form a sequence of independent and identically distributed random variables with density function $f_P(.)$. Let $\bar{X}_{mP}$ be the cumulative loss of manpower in the first m policy decisions. For $j = 1,2, \ldots.$, let X_{jT} be continuous random variables representing the amount of depletion of manpower in the organization caused due to the j[th] transfer decision. It is assumed that X_{jT} form a sequence of independent and identically distributed random variables with probability density function $f_T(.)$. Let $\bar{X}_{nT}$ be the cumulative loss of manpower in the first n transfer decisions. For each i and j, let X_{iP} and X_{jT} be statistically independent. Let Y (Z) be a continuous random variable denoting the optional(mandatory) threshold level such that Z › Y. It is assumed that Y and Z are independent.

Let U_{iP} be the time between $(i-1)^{th}$ and $(i)^{th}$ inter policy decision times forming a sequence of exchangeable and constantly correlated exponential random variables with distribution function $G_P(.)$ and mean $\frac{1}{\lambda_P}$ $(\lambda_P > 0)$. It is assumed that the number of transfer decisions form a poisson process with rate λ_T. Let $g_{mP}^{*}(.)$ be the Laplace transform of $g_{mP}(.)$ and R be the correlation between U_{iP} and $U_{jP}, i \neq j$, and b = $\frac{1}{\lambda_P}$ (1 – R). Let $G_{mP}(.)$ be the distribution of the waiting time upto m policy decisions and $G_{nT}(.)$ be the n-fold convolution of $G_T(.)$. Let W be the time to recruitment in the organization with distribution L(.), mean E(W) and variance V(W). Let $N_P(t)$ be the number of policy decisions and $N_T(t)$ be the number of transfer decisions until the time recruitment W. The univariate CUM policy of recruitment employed in this paper is stated as follows: If the cumulative loss of manpower in the organization exceeds the optional threshold level the organization may or may not go for recruitment but if the cumulative loss of man hours exceeds the mandatory threshold recruitment is necessary. Let q be the probability that the organization is not going for recruitment whenever the total loss of manpower exceeds the optional threshold level.

3. MAIN RESULT

The tail distribution of the time to recruitment can be written as follows using the recruitment policy.

$$P[W > T] = P\left[\left(\bar{X}_{N_P(t)} + \bar{X}_{N_T(t)}\right) \leq Y\right] + qP\left[Y < \left(\bar{X}_{N_P(t)} + \bar{X}_{N_T(t)}\right) \leq Z\right] \quad (1)$$

By the law of total probability, we get

$$P[W > t] =$$

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[G_{mP}(t) - G_{(m+1)P}(t)\right]\left[G_{nT}(t) - G_{(n+1)T}(t)\right] P[(\bar{X}_{mP} + \bar{X}_{nT}) \leq Y] +$$
$$q \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[G_{mP}(t) - G_{(m+1)P}(t)\right]\left[G_{nT}(t) - G_{(n+1)T}(t)\right] \times$$
$$P[(\bar{X}_{mP} + \bar{X}_{nT}) > Y] P[(\bar{X}_{mP} + \bar{X}_{nT}) \leq Z] \quad (2)$$

Invoking to laws of probability it can be shown that

$$P[(\bar{X}_{mP} + \bar{X}_{nT}) \leq Y] = [f_P{}^{*}(\alpha_1)]^m [f_T{}^{*}(\alpha_1)]^n \quad (3)$$

From (1) (2), (3) and on simplification, we get

$P[W > t] =$

$$1 + [f_P{}^*(\alpha_1) - 1] \sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\alpha_1)]^{m-1} + [f_T{}^*(\alpha_1) - 1] \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\alpha_1)]^{n-1}$$

$$+ [f_P{}^*(\alpha_1) - 1][f_T{}^*(\alpha_1) - 1] \sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\alpha_1)]^{m-1} \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\alpha_1)]^{n-1}$$

$$+ q[f_P{}^*(\beta_1) - 1] \sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\beta_1)]^{m-1} + q[f_T{}^*(\beta_1) - 1] \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\beta_1)]^{n-1}$$

$$+q[f_P{}^*(\beta_1) - 1][f_T{}^*(\beta_1) - 1] \sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\beta_1)]^{m-1} \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\beta_1)]^{n-1}$$

$$- q\, [f_P{}^*(\alpha_1) f_P{}^*(\beta_1) - 1] \sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\alpha_1) f_P{}^*(\beta_1)]^{m-1}$$

$$-q[f_T{}^*(\alpha_1) f_T{}^*(\beta_1) - 1] \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\alpha_1) f_T{}^*(\beta_1)]^{n-1}$$

$$-q\, [f_P{}^*(\alpha_1) f_P{}^*(\beta_1) - 1][f_T{}^*(\alpha_1) f_T{}^*(\beta_1) - 1] \times$$

$$\sum_{m=1}^{\infty} G_{mP}(t)[f_P{}^*(\alpha_1) f_P{}^*(\beta_1)]^{m-1} \sum_{n=1}^{\infty} G_{nT}(t)[f_T{}^*(\alpha_1) f_T{}^*(\beta_1)]^{n-1} \qquad (4)$$

Since $l(t) = \frac{d}{dt}[1 - P(W > t)]$, from (4) we get

$$l^*(s) = \frac{A_1}{s+A_1} + B_1 \sum_{m=1}^{\infty} (1 - B_1)^{m-1} g_{mP}^*(s + A_1) \left(\frac{s}{s+A_1}\right)$$

$$+q\left(\frac{A_2}{s+A_2} + B_2 \sum_{m=1}^{\infty} (1 - B_2)^{m-1} g_{mP}^*(s + A_2) \left(\frac{s}{s+A_2}\right)\right)$$

$$-q\left(\frac{A_3}{s+A_3} + B_3 \sum_{m=1}^{\infty} (1 - B_3)^{m-1} g_{mP}^*(s + A_3) \left(\frac{s}{s+A_3}\right)\right) \qquad (5)$$

where, $A_1 = \lambda_T[1 - f_T{}^*(\alpha_1)] = \frac{\lambda_T \alpha_1}{\mu_T + \alpha_1}$

$$A_2 = \lambda_T[1 - f_T{}^*(\beta_1)] = \frac{\lambda_T \beta_1}{\mu_T + \beta_1}$$

$$A_3 = \lambda_T[1 - f_T{}^*(\alpha_1) f_T{}^*(\beta_1)] = \frac{\lambda_T[\mu_T(\alpha_1 + \beta_1) + \alpha_1 \beta_1]}{(\mu_T + \alpha_1)(\mu_T + \beta_1)}$$

$$B_1 = 1 - f_P{}^*(\alpha_1) = \frac{\alpha_1}{\mu_P + \alpha_1}$$

$$B_2 = 1 - f_P{}^*(\beta_1) = \frac{\beta_1}{\mu_P + \beta_1}$$

and $B_3 = 1 - f_P{}^*(\alpha_1) f_P{}^*(\beta_1) = \frac{\mu_P(\alpha_1+\beta_1)+\alpha_1\beta_1}{(\mu_P+\alpha_1)(\mu_P+\beta_1)}$ (6)

From (6) we get

$$E[W] = \frac{1}{A_1} - \frac{B_1}{A_1} \sum_{m=1}^{\infty} (1 - B_1)^{m-1} \, g_{mP}^*(A_1)$$

$$+ q\left(\frac{1}{A_2} - \frac{B_2}{A_2} \sum_{m=1}^{\infty} (1 - B_2)^{m-1} g_{mP}^*(A_2)\right)$$

$$- q\left(\frac{1}{A_3} - \frac{B_3}{A_3} \sum_{m=1}^{\infty} (1 - B_3)^{m-1} \, g_{mP}^*(A_3)\right) \tag{7}$$

$$E[W^2] = \frac{2}{A_1}\left(\frac{1}{A_1} + B_1\right) \sum_{m=1}^{\infty} (1 - B_1)^{m-1} \left[{g_{mP}^*}'(A_1) - \frac{g_{mP}^*(A_1)}{A_1}\right]$$

$$+ \frac{2q}{A_2}\left(\frac{1}{A_2} + B_2\right) \sum_{m=1}^{\infty} (1 - B_2)^{m-1} \left[{g_{mP}^*}'(A_2) - \frac{g_{mP}^*(A_2)}{A_2}\right]$$

$$- \frac{2q}{A_3}\left(\frac{1}{A_3} + B_3\right) \sum_{m=1}^{\infty} (1 - B_3)^{m-1} \left[{g_{mP}^*}'(A_3) - \frac{g_{mP}^*(A_3)}{A_3}\right] \tag{8}$$

We now obtain $g_{mP}^*(.)$.

Gurland[1955] has obtained the expression for the cumulative distribution function $G_{mP}(x)$ of the partial sum $S_{mP} = U_{1P} + U_{2P} + \dots\dots + U_{mP}$ when $U_{iP}'s$ $(i = 1,2,\dots.,m)$ are exchangeable and constantly correlated exponential random variables, Gurland has shown that

$$G_{mP}(x) = (1 - R) \sum_{i=0}^{\infty} \frac{(mR)^i}{R_1^{i+1}} \frac{\varphi\left(m+i, \frac{x}{b}\right)}{(m+i-1)!} \tag{9}$$

Where $\varphi(n, x) = \int_0^{\infty} e^{-x} x^{n-1} dx$, $b = \frac{1}{\lambda_P}(1 - R)$ and $R_1 = 1 - R + mR$

From (9) we obtain

$$g_{mP}^*(s) = \frac{1}{(1+bs)^m \left(1 + \frac{mRbs}{(1-R)(1+bs)}\right)} \tag{10}$$

and $${g_{mP}^*}'(s) = \frac{bm(R-1)[1+bsR_1]}{(1+bs)^m (1-R+bsR_1)^2} \tag{11}$$

By using (10) and (11), we get

$$g_{mP}^{*}(A_1) = \frac{(\mu_T+\alpha_1)^m (1-R)[\mu_T+\alpha_1+b\lambda_T\alpha_1]^{1-m}}{(1-R)(\mu_T+\alpha_1)+b\lambda_T\alpha_1R_1} \quad (12)$$

$$g_{mP}^{*}(A_2) = \frac{(\mu_T+\beta_1)^m (1-R)[\mu_T+\beta_1+b\lambda_T\beta_1]^{1-m}}{(1-R)(\mu_T+\beta_1)+b\lambda_T\beta_1R_1} \quad (13)$$

$$g_{mP}^{*}(A_3) = \frac{[(\mu_T+\alpha_1)(\mu_T+\beta_1)]^m (1-R)[(\mu_T+\alpha_1)(\mu_T+\beta_1)+b\lambda_T(\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1)]^{1-m}}{(1-R)(\mu_T+\alpha_1)(\mu_T+\beta_1)+b\lambda_T(\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1)R_1} \quad (14)$$

$$g_{mP}^{*}{}'(A_1) = \frac{(R-1)bm\,[\mu_T+\alpha_1+b\lambda_T\alpha_1R_1][\mu_T+\alpha_1]^{m+1}}{[\mu_T+\alpha_1+b\lambda_T\alpha_1]^m\,[(1-R)(\mu_T+\alpha_1)+b\lambda_T\alpha_1R_1]^2} \quad (15)$$

$$g_{mP}^{*}{}'(A_2) = \frac{(R-1)bm\,[\mu_T+\beta_1+b\lambda_T\beta_1R_1][\mu_T+\beta_1]^{m+1}}{[\mu_T+\beta_1+b\lambda_T\beta_1]^m\,[(1-R)(\mu_T+\beta_1)+b\lambda_T\beta_1R_1]^2} \quad (16)$$

$$g_{mP}^{*}{}'(A_3) = \frac{(R-1)bm\,[(\mu_T+\alpha_1)(\mu_T+\beta_1)+b\lambda_T(\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1)R_1][(\mu_T+\alpha_1)(\mu_T+\beta_1)]^{m+1}}{[(\mu_T+\alpha_1)(\mu_T+\beta_1)+b\lambda_T(\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1)]^m\,[(1-R)(\mu_T+\alpha_1)(\mu_T+\beta_1)+b\lambda_T(\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1)R_1]^2}$$

(17)

From (7) to (17), we can obtain E[W] and V[W] in closed form for specific choices for m as we can also have finite occasions of decision making and transfer of personnels as a special case. For example, if m = 1, we get

$$E[W] = \frac{1}{\lambda_T}\left[\frac{\mu_T+\alpha_1}{\alpha_1} + q\left(\frac{\mu_T+\beta_1}{\beta_1}\right) - q\left(\frac{(\mu_T+\alpha_1)(\mu_T+\beta_1)}{\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1}\right)\right] -$$

$$\frac{\lambda_P}{\lambda_T}\left\{\frac{(\mu_T+\alpha_1)^2}{(\mu_P+\alpha_1)(\lambda_P(\mu_T+\alpha_1)+\lambda_T\alpha_1)} + q\left(\frac{(\mu_T+\beta_1)^2}{(\mu_P+\beta_1)(\lambda_P(\mu_T+\beta_1)+\lambda_T\beta_1)}\right) -\right.$$

qμPα1+β1+α1β1μT+α1μT+β12μP+α1μP+β1μTα1+β1+α1β1λPμT+α1μT+β1+λTμTα1+β1+α1β1 (18)

and

$$E[W^2] = \frac{2}{\lambda_T^2}\left\{\left[\frac{\mu_T+\alpha_1}{\alpha_1}\right]^2 + q\left[\frac{\mu_T+\beta_1}{\beta_1}\right]^2 - q\left[\frac{(\mu_T+\alpha_1)(\mu_T+\beta_1)}{\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1}\right]^2 - \lambda_P\left[\frac{(\mu_T+\alpha_1)^3(\lambda_P(\mu_T+\alpha_1)+2\lambda_T\alpha_1)}{\alpha_1(\mu_P+\alpha_1)(\lambda_P(\mu_T+\alpha_1)+\lambda_T\alpha_1)^2}\right.\right. +$$

qμT+β13λPμT+β1+2λTβ1β1μP+β1λPμT+β1+λTβ12− qμPα1+β1+α1β1μT+α1μT+β13λPμT+α1μT+β1+2λTμTα1+β1+α1β1λTμP+α1μP+β1μTα1+β1+α1β12λPμT+α1μT+β1+λTμTα1+β1+α1β12 (19)

Equation (18) gives the mean time to recruitment and we get the variance of time to recruitment from equations (18) and (19).

4. CONCLUSION

The manpower planning model discussed in this paper can be used to plan for the adequate provision of manpower in any marketing organisation in which depletion of manpower is induced by attrition. The applicability of the model can further be studied using simulation. One can also test the goodness of fit for the distributions assumed in this paper by collecting relevant data

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