FUZZY β-IRRESOLUTEMAPPING

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ABSTRACT

Somepropertiesoffuzzyβ-irresolutemapping(formerlyknownasfuzzy$M\beta$-continuousmapping[8])havebeenstudiedhere.Alsoithasbeenshownthatfuzzyirresolutemapping [11]andfuzzyβ-irresolutemappingareindependentnotions.Inthelastsectionsomeapplicationsoffuzzyβ-irresolutemappinghavebeendiscussed.

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INTRODUCTION

Throughoutthepaper,by(X,τ)orsimplyby$X$wemeanafuzzytopologicalspace(fts,forshort)inthesenseofChang[4].Afuzzyset[16]Aisamappingfromanonemptyset$X$intoaclosedinterval$I=[0,1]$.Thesupport[13]ofafuzzyset$A$in$X$willbedenotedby$suppA$andisdefinedby$suppA=\{x\in X:A(x)\neq 0\}$. Afuzzypoint[13]withthesingletonsupport$x\in X$andthevalue$t$$(0<t\leq 1)atx$willbedenotedby$x_t$.$0_Xand1_X$aretheconstantfuzzysetstakingvalues0and1in$X$respectively.Thecomplement[16]ofafuzzy set$A$ in Xwillbedenoted by$1_X\setminus A$ andisdefinedby $(1_X\setminus A)(x)=1-A(x)$,forall$x\in X$.Fortwofuzzysets$AandBinX$,wewrite$A\leq B$ifandonlyif$A(x)\leq B(x)$,foreach$x\in X$,and$AqB$means$A$isquasi-coincident(q-coincident,forshort)with$B$[13]if$A(x)+B(x)>1$,forsome$x\in X$.Thenegationofthesetwostatementswillbedenotedby $A\not\leq B$and$A\bar{q}B$respectively.clA and$intA$ of a fuzzyset A in Xrespectively

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standforthefuzzyclosure[4]andfuzzyinterior[4]of$A$in$X$.Afuzzyset$A$in$X$willbecalledfuzzy semiopen[2](resp.,fuzzyβ-open[1],fuzzypreopen[12])if$A\leq clintA$(resp.,$A\leq clintclA$,$A\leq intclA$). Thesetofallfuzzysemiopen(resp.,β-open)setsof$X$willbedenotedby$SO(X)$(resp.,$\beta O(X)$).Thecomplementofafuzzysemiopen(resp.,fuzzyβ-open,fuzzypreopen)set$A$in$X$iscalledfuzzysemiclosed[2](resp.,fuzzyβ-closed[1],fuzzypreclosed[12]).Thesmallestfuzzysemiclosed(resp.,fuzzyβ-closed,fuzzypreclosed)setcontainingafuzzyset$A$in$X$iscalledfuzzysemiclosure[2](resp.,fuzzyβ-closure[1],fuzzypreclosure[12])of$A$andisdenotedby$sclA$(resp.,βclA,$pclA$).Afuzzyset$B$in$X$issaidtobeaβ-nbd[1]ofafuzzypointx_t in$X$ifthereexistsafuzzyβ-openset$U$ in$X$suchthat$x_t\leq U\leq B$.Afuzzyset$B$iscalledafuzzyβ-q-nbd[8]ofafuzzypoint$x_t$in$X$ifthereisafuzzyβ-openset$U$in$X$suchthat$x_t qU\leq B$.

1. FUZZY β-IRRESOLUTE MAPPING : SOME CHARACTERIZATIONS

Inthissectionfuzzyβ-irresolutemappinghasbeencharacterizedindifferentways.

DEFINITION1.1.Afuzzy mapping$\boldsymbol{f}:X\rightarrow Y$is saidto befuzzy β-irresolute(fuzzy$M\beta$-continuousmapping[8])if$\boldsymbol{f}^{-1}(A)$isfuzzyβ-openin$X$foreachfuzzyβ-openset$A$in$Y$.

THEOREM1.2.Let$\boldsymbol{f}:X\rightarrow Y$beamapping.Thenthefollowingareequivalent:

(a) $\boldsymbol{f}$isfuzzyβ-irresolute,

(b) foreachfuzzypoint$x_t$in$X$andeachfuzzyβ-openset$A$in$Y$suchthat$\boldsymbol{f}(x_t)\leq A$,thereexistsafuzzy$\beta$-openset$BinX$suchthat$x_t\leq Band\boldsymbol{f}(B)\leq A$,

(c) $\boldsymbol{f}^{-1}(B)$isfuzzyβ-closedin$X$foreachfuzzyβ-closedset$B$in$Y$,

(d) foreachfuzzypoint$x_t$in$X$,theinverseofeachfuzzyβ-nbd$B$of$\boldsymbol{f}(x_t)$in$Y$isafuzzy β-nbdof $x_t$in X,

(e) foreachfuzzypoint$x_t$in$X$andeachfuzzyβ-nbd$B$of$\boldsymbol{f}(x_t)$,thereexistsafuzzyβ-nbd$C$of$x_t$in$X$suchthat$\boldsymbol{f}(C)\leq B$,

(f) foreachfuzzyset$D$in$X$,$\boldsymbol{f}(\beta clD)\leq\beta clf(D)$,

(g) foreachfuzzyset$B$in$Y$,$\beta cl(f^{-1}(B))\leq\boldsymbol{f}^{-1}(\beta clB)$.

PROOF.(b)$\Rightarrow$(a).Let$A$beafuzzyβ-opensetin$Y$andx_t,afuzzypointin$\boldsymbol{f}^{-1}(A)$.Then

$x_t \leq f^{-1}(A)$,i.e.,$f(x_t) \leq A$.By(b),thereexistsafuzzyβ-opensetBinXsuchthat$x_t \leq B$ and$f(B) \leq A$.Thus$B \leq f^{-1}(A)$.Wehavetoshowthat$f^{-1}(A) \leq clintclf^{-1}(A)$.As $B \in \beta O(X)$,$x_t \leq B \leq clintclB \leq clintclf^{-1}(A)$.As$x_t \leq f^{-1}(A)$,$f^{-1}(A) \leq clintclf^{-1}(A)$.

(a) ⇒(c). Let$B$beanyfuzzyβ-closedsetin$Y$. Then$1_Y \setminus B \in \beta O(Y)$. By(a), $f^{-1}(1_Y \setminus B) = 1_X \setminus f^{-1}(B) \in \beta O(X)$andso$f^{-1}(B)$isfuzzy$\beta$-closedin$X$.

(c)⇒(a).Straightforward.

(a)⇒(d). Let$x_t$beafuzzypointin$X$and$B$,afuzzyβ-nbdof$f(x_t)$in$Y$. Thenthereexists$U \in \beta O(Y)$suchthat$f(x_t) \leq U \leq B$.Then$x_t \leq f^{-1}(U) \leq f^{-1}(B)$.Since$U \in \beta O(Y)$,by(a)$f^{-1}(U) \in \beta O(X)$andhencetheresult.

(d) ⇒(e).Since$ff^{-1}(B) \leq B$,theresultfollowsbytaking$C = f^{-1}(B)$.

(e) ⇒(b).Let $x_t$beafuzzypointin$X$and A,anyfuzzy β-opensetin Ysuchthat$f(x_t) \leq A$.Then$A$isfuzzyβ-nbdof$f(x_t)$in$Y$.By(e),thereexistsafuzzyβ-nbd$C$ of$x_t$in$X$suchthat$f(C) \leq A$.Therefore,thereexists$U \in \beta O(X)$suchthat$x_t \leq U \leq C$andso$f(U) \leq f(C) \leq A \Rightarrow f(U) \leq A$.

(c)⇒(f).Let$D$beanyfuzzysetin$X$.Then$\beta clf(D)$isfuzzyβ-closedin$Y$.By(c),$f^{-1}(\beta clf(D))$isfuzzyβ-closedin$X$.Now$D \leq f^{-1}f(D) \leq f^{-1}(\beta clf(D))$,i.e.,$\beta clD \leq \beta clf^{-1}(\beta clf(D)) = f^{-1}(\beta clf(D))$. Therefore,$f(\beta clD) \leq \beta clf(D)$.

(f)⇒(c).Let$B$beanyfuzzyβ-closedsetin$Y$.Put$D = f^{-1}(B)$.By(f),$f(\beta clD) \leq \beta clf(D) = \beta cl(f(f^{-1}(B))) \leq \beta clB = B$.Thus$\beta clD \leq f^{-1}(f(\beta clD)) \leq f^{-1}(B) = D$.Hence$D = f^{-1}(B)$isfuzzy$\beta$-closedin$X$.

(f) ⇒(g). Let$B \in I^Y$. Againlet$D = f^{-1}(B)$. By(f),$f(\beta clD) \leq \beta clf(D)$,i.e., $\beta clD \leq f^{-1}(\beta clf(D))$,i.e.,$\beta clf^{-1}(B) \leq f^{-1}(\beta cl(f(f^{-1}(B)))) \leq f^{-1}(\beta clB)$.

(g) ⇒(f).Let$D \in I^X$.By(g),$\beta cl(f^{-1}f(D)) \leq f^{-1}(\beta clf(D)) \Rightarrow \beta clD \leq f^{-1}(\beta clf(D)) \Rightarrow f(\beta clD) \leq \beta clf(D)$.

THEOREM1.3.Amapping$f$:$X\rightarrow Y$isfuzzyβ-irresoluteiffforeachfuzzypoint$x_t$in$X$andanyfuzzyβ-openβ-q-nbd$V$of$f(x_t)$in$Y$,thereexistsafuzzyβ-openβ-q-nbd$U$ofx_t in$X$suchthat$f(U)\leq V$.

PROOF.Let$f$:$X\rightarrow Y$befuzzyβ-irresoluteand$x_t$beafuzzypointin$X$.Let$V$beafuzzyβ-openβ-q-nbdof$f(x_t)$in$Y$.Then$f^{-1}(V)(=U$,say)isafuzzyβ-openβ-q-nbdof$f(x_t)$in$X$suchthat$f(U)\leq V$.
Conversely,letx_t beanyfuzzypointin$X$and$V$beanyfuzzyβ-opensetcontaining$f(x_t)$.Letm_t beapositiveintegersuchthat$1/m_t$ $<t$.Then$0<1-t+1/n=\beta_n$ (say)<1,forall$n\geq m_t$.Now$y_{\beta_n}qV$foreach$n\geq m_t$,where$y=f(x)$.Thenbyhypothesis,thereexistsafuzzy β-openset$U_n$in$X$suchthat$x_{\beta_n}qU_n$and$f(U_n)\leq V$,forall$n\geq m_t$.Put
Then$U\in\beta O(X)$s uchthat$f(U)\leq V$. Also$\beta_n+U_n(x)>1$,for
all$n\geq m_t \Rightarrow 1-t+1/n+U_n(x)>1$,forall$n\geq m_t \Rightarrow t<U_n(x)+1/n$,forall
$n\geq m_t\Rightarrow t\leq\sup_{n\geq m_t}U_n(x)=U(x)\Rightarrow x_t\leq U$.HencebyTheorem1.2,fisfuzzyβ-irresolute.

2. FUZZYIRRESOLUTEANDFUZZYβ-IRRESOLUTEMAPPING

Inthissectionithasbeenshownthatfuzzyirresolutemapping[11]andfuzzyβ-irresolutemappingareindependentnotions.

Firstwerecallthedefinitionfrom[11]forreadyreference.

DEFINITION2.1.Amapping$f$:$X\rightarrow Y$issaidtobefuzzyirresoluteif$f^{-1}(A)$isfuzzysemiopenin$X$for eachfuzzysemiopenset$A$in$Y$.

REMARK2.2.Itisclearfromthefollowingtwoexamplesthatfuzzyirresolutemappingandfuzzyβ-irresolutemappingareindependentnotions.

EXAMPLE 2.3. Let$X=\{a,b\}$,$\tau=\{0_X,1_X,A\}$,$\tau_1=\{0_X,1_X,C\}$where$A(a)=$
1.5,$A(b)=0.4$,$C(a)=0.6$,$C(b)=0.5$.Then$(X,\tau)$and(X,τ_1)arefts's.Considerthefuzzymapping$f$:$(X,\tau)\rightarrow(X,\tau_1)$definedby$f(a)=b$,$f(b)=a$.Weclaimthat$f$isfuzzy
β-irresolutebutnotfuzzyirresolutemapping. Thecollectionofallfuzzysemiopensetsin

(X,τ) is $\{0_X, 1_X, A, U\}$ where $A \leq U \leq 1_X \setminus A$ and that of in (X,τ_1) is $\{0_X, 1_X, C, V\}$ where $V \geq C$. Again any fuzzy set in (X,τ) is fuzzy β-open in (X,τ) and the collection of all fuzzy β-open sets in (X,τ_1) is $\{0_X, 1_X, C, W\}$ where $W \not\leq 1_X \setminus C$.
Let B be a fuzzy semiopen set in (X,τ_1) defined by $B(a)=B(b)=0.6$. Now $[f^{-1}(B)](a) = Bf(a)=B(b)=0.6$, $[f^{-1}(B)](b)=Bf(b)=B(a)=0.6$ and so $f^{-1}(W) \notin SO(X,\tau)$. Therefore, f is not fuzzy irresolute mapping. Since any fuzzy set in (X,τ) is fuzzy β-open in (X,τ), f is fuzzy β-irresolute.

EXAMPLE 2.4. Let $X=\{a,b\}$, $\tau=\{0_X, 1_X, B\}$ where $A(a)=0.4$, $A(b)=0.7$, $B(a)= 1.6$, $B(b)=0.7$. Then (X,τ) and (X,τ_1) are fts's. Now fuzzy semiopen sets in (X,τ) are $0_X, 1_X, A, V$ where $V \geq A$ and that of fuzzy β-open sets in (X,τ) are $0_X, 1_X, A, U$ where $U \not\leq 1_X \setminus A$. Again fuzzy semiopen sets in (X,τ_1) are $0_X, 1_X, B, C$ where $C \geq B$ and that of fuzzy β-open sets in (X,τ_1) are $0_X, 1_X, B, W$ where $W \not\leq 1_X \setminus B$. Consider the fuzzy identity mapping $i:(X,\tau)\to(X,\tau_1)$. We claim that i is fuzzy irresolute but not fuzzy β-irresolute mapping.
$[i^{-1}(C)](a)=C(i(a))=C(a)\geq B(a)$ and $[i^{-1}(C)](b)=C(i(b))=C(b)\geq B(b)$ and $B \geq A$ $i^{-1}(C)\geq A$ which shows that i is fuzzy irresolute. But $W(a)=0.6$, $W(b)=0.3$ being a fuzzy β-open set in (X,τ_1) and $i^{-1}(W)=W \notin \beta O(X,\tau)$ and so i is not fuzzy β-irresolute mapping.

3. APPLICATIONS

Let us recall some definitions for ready references.

DEFINITION 3.1 [4]. Let A be a fuzzy set in an fts X. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\sup_{U\in\mathcal{U}} U(x)=1$, for each $x \in supp A$. In particular, if $A=1_X$, we get the definition of fuzzy cover of the fts X.

DEFINITION 3.2 [6]. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts X is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$. In particular, if $A=1_X$, then the requirement on $\mathcal{U}_0$ is $\bigcup \mathcal{U}_0 = 1_X$.

DEFINITION 3.3 [9]. An fts X is said to be a fuzzy semicompact space if every cover of

Xbyfuzzysemiopensetshasafinitesubcover.

DEFINITION3.4.Anfts$X$issaidto befuzzy$S$-closed[10](resp.,fuzzys-closed[15])ifeveryfuzzycover$U$of$X$byfuzzysemiopensetsin$X$hasafinitesubfamilyU_0suchthat $\bigvee_{U\in U_0} clU=1_X$(resp., $\bigvee_{U\in U_0} sclU=1_X$).

DEFINITION3.5[3].Anfts$X$issaidtobefuzzyβ-compactspaceifeveryfuzzycover $U$of$X$byfuzzyβ-opensetsin$X$hasafinitesubcover.

DEFINITION3.6[8].Anfts$X$issaidtobefuzzyβ-closedifforeveryfuzzycover$U$of$X$ byfuzzyβ-opensetsin$X$,thereexistsafinitesubfamily$U_0$of$U$suchthat $\bigvee_{U\in U_0} \beta clU=1_X$.

DEFINITION3.7. Anfts$X$issaidtobefuzzystronglycompact[12](resp.,fuzzy$P$-closed[17])ifeveryfuzzycoverof$X$byfuzzypreopensetsin$X$hasafinitesubcover(resp., subfamily$U_0$of$U$suchthat $\bigvee_{U\in U_0} pclU=1_X$).

THEOREM3.8.If $X$is a fuzzy β-compactspaceand f:$X\rightarrow Y$is fuzzy β-irresolute surjectivemapping,then$Y$isfuzzysemicompact.

PROOF.Let$V=\{V_\alpha:\alpha\in\Lambda\}$beafuzzycoverof$Y$byfuzzysemiopensetsof$Y$.Thenasfuzzysemiopensetsarefuzzyβ-open$V$isafuzzycoverof$X$byfuzzyβ-opensetsof$Y$.Now$f$beingfuzzyβ-irresolutesurjectivemapping,$\{f^{-1}(V_\alpha):\alpha\in\Lambda\}$isafuzzycoverof$X$byfuzzy$\beta$-opensetsof$X$. As$X$isfuzzy$\beta$-compact,thereexistsafinitesubfamilyΛ_0 ofΛsuchthat$\{f^{-1}(V_\alpha):\alpha\in\Lambda\}$alsocovers$X$,i.e.,$1_X=\bigvee_{\alpha\in\Lambda_0} f^{-1}(V_\alpha)\Rightarrow 1_Y=f(1_X)=f(\bigvee_{\alpha\in\Lambda_0} f^{-1}(V_\alpha))=ff^{-1}(\bigvee_{\alpha\in\Lambda_0} V_\alpha)\le\bigvee_{\alpha\in\Lambda_0} V_\alpha$.Hence$Y$ isfuzzysemicompactspace.

REMARK3.9.Sincefuzzysemicompactspaceisfuzzy$S$-closedspace,wecanstatethefollowingtheorem.

THEOREM3.10.If Xisfuzzy β-compact spaceand f: $X\rightarrow Y$is fuzzy β-irresolute,then$Y$isfuzzy$S$-closedspace.

PROOF.TheproofissameasthatofTheorem3.8andhenceomitted.

REMARK 3.11. Sincefuzzypreopensetisfuzzyβ-open,wecanstatethefollowingtheorem.

THEOREM3.12.If$X$is fuzzy β-compact space and$f$:$X\rightarrow Y$is fuzzy β-irresolute,then$Y$isfuzzystronglycompact(resp.,fuzzy$P$-closed).

REMARK3.13.Sinceforafuzzyset$A$in$X$,$\beta clA\leq sclA$,$\beta clA\leq pclA$,$\beta clA\leq clA$,wecaneasilystatethefollowingtheorem.

THEOREM3.15. IfXisfuzzyβ-closedspaceand$f$:$X\rightarrow Y$ isfuzzyβ-irresolutesurjectivemapping,then$Y$isfuzzy$S$-closed(resp.,fuzzy$s$-closed,fuzzy$P$-closed)space.

NOTE3.16.InsteadofspacewecanstatetheTheorem3.8,Theorem3.10,Theorem3.12,Theorem3.14for afuzzyset$A\in I^X$also.

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