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PRICING AND INVENTORY POLICY FOR DETERIORATING ITEMS WITH PRICE AND TIME DEPENDENT DEMAND, SHORTAGES AND PARTIAL BACKLOGGING

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ABSTRACT

This paper presents inventory model for perishable items with price and time dependent demand rate. It is assumed that shortages are allowed and partially backlogged. It is assumed that the deterioration of the item does not begin at the instant of the arrivals in stock. It begins after some fixed time. Profit function has been established and objective is to maximize the total average profit per unit time. The results are illustrated with the help of numerical examples. The sensitivity of the solution with the change of the values of the parameters associated with the model is also discussed.

KEYWORDS: INVENTORY, PRICE AND TIME DEPENDENT DEMAND, SHORTAGES, DETERIORATION, BACKLOGGING

1. INTRODUCTION

It is observed that the demand of many seasonal products, such as fruits, fashion apparels, hi-tech products, Christmas items, over the entire selling season may vary with time and price. In the meantime, such types of seasonal products suffer from depletion by direct spoilage while kept in storage. Deterioration of these products is a realistic phenomenon. The value of seasonal perishable products decreases due to deterioration during their normal storage period. As a result, firms face the problem of determining the price and the order/production quantity of their products simultaneously.

Existing deterministic inventory models concentrating on deteriorating seasonal products. These can be classified into three main types of models according to their demand characteristic: stock dependent demand, time-dependent demand, and price dependent demand. Since we mainly focus on the last two demand patterns, we refer the readers to Urban (2005) for a review of inventory models with stock-dependent demand. Deteriorating items are common in our daily life; however, academia has not reached a consensus on the definition of the deteriorating items. According to the study of Wee HM , deteriorating items refer to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons.

In the classical EOQ model, demand is assumed to be constant. However, in reality, demand for a product may vary with the time. Since Silver and Meal (1973) proposed a heuristic solution to determine lot size quantities for the general case of a time-dependent demand, numerous inventory control papers have investigated the time-dependent demand pattern. Among different time dependent demand models, we are typically interested in the price and time-dependent demand pattern in which demand increases up to a point of time then it becomes steady. Hill (1995) first proposed a ramp-type time-dependent demand pattern by considering it as the combination of two different types of demand in two successive time periods. Wu (2001) considered an EOQ model with ramp-type demand, Weibull distribution deterioration, and partial backlogging. Manna and Chaudhuri (2006) developed a production–inventory model with a ramp-type demand pattern where the finite production rate depends on the demand. Panda, Senapati, and Basu (2008) also developed an inventory model for deteriorating seasonal products with a ramp-type time-dependent function over the season. Cheng, Zhang, and Wang (2011) considered an inventory model for deteriorating items using a ramp-type time-dependent demand rate with three time periods. Readers can refer to Skouri, Konstantaras, Manna, and Chaudhuri (2011) for more references on ramp-type time-dependent demand models. Since demand is usually price sensitive, many researchers have developed inventory models for deteriorating items with price dependent demand. Dave, Fitzpatrick, and Baker (1995) presented a deterministic production lot size inventory model for the items with price-dependent demand. Urban and Baker (1997) investigated a deterministic inventory model in which demand is dependent on price, time, and inventory level. They also extended the model to the case with a single price markdown. Khouja (2000) dealt with the newsvendor problem with multiple discount prices, which are set at equal intervals on a price domain. Shinn and Hwang (2003) investigated the problem of determining the order quantity in which demand is a convex function of price and the delay in payments is order-size dependent. Teng and Chang (2005) established an economic production quantity model for deteriorating items when the demand is a function of price and on-display stock level. Banerjee and Sharma (2010) presented a deterministic inventory model for the product with a price and time-dependent demand rate. In the above mentioned inventory models with pricing strategy, pricing decision is determined under the assumption that the times of changing price are pre-specified. You (2005) relaxed this assumption and investigated the problem of jointly determining the number of price settings and optimal prices for a perishable inventory system in which demand is time and price-dependent. However, You (2005) did not consider the deterioration of the inventory and the time-dependent demand used was not ramp-type. Maihami and Kamalabadi (2012) developed a joint pricing and inventory control model for non-instantaneous deteriorating items with price- and time-dependent demand, in which the demand is an increasing/decreasing exponential function of time and shortages are allowed.

In this paper we analyze a deterministic inventory model assuming that demand is price and time dependent. In addition shortages are allowed and partially backlogged and the products have a deterioration process after a certain period of time. The organization of the paper is as follows: In section 2, we introduce the notation used throughout the paper and the basic assumptions of the inventory system, in section 3, we develop the mathematical model that describes the evolution of the inventory system and a procedure to solve the inventory problem is presented, in section 4, numerical example is provided to illustrate the solution procedure, in section 5, we present a sensitivity analysis of the inventory policy and in last the conclusion is discussed.

2. ASSUMPTIONS AND NOTATIONS

The following notations are used throughout the paper.

2.1 Notations:

$I_1(t)$	:	the stock level at time t when deterioration does not take place.
$I_2(t)$	:	the stock level at time t when deterioration takes place.
$I_3(t)$	:	the stock level at time t when shortages and backlogging take place.
D(p,t)	:	the demand rate which is a function of the price and time.
c_1	:	the inventory holding cost per unit per unit time.
c_2	:	the purchasing cost per unit.
c_3	:	the shortage cost per unit per unit time.
c_4	:	the opportunity cost due to lost sale per unit.
c_5	:	the ordering cost.
t_1	:	the time when deterioration starts.
t_2	:	the time when inventory level falls to zero due to demand and deterioration.
T	:	the length of each ordering cycle.
$I_{\max}$	:	the maximum inventory level for each ordering cycle.
I_θ	:	the inventory level when deterioration starts.
Q	:	the order quantity for each ordering cycle.
B_Q	:	total amount of backorder i.e. demand backlogged at the end of cycle.
p	:	the selling price of item/unit.

2.2 Assumptions:

1. The inventory system involves only one deteriorating item.
2. Planning horizon is infinite.
3. The replenishment rate is infinite but replenishment size is finite.
4. The deterioration rate $\theta(0 \le \theta < 1)$ is constant.
5. There is no replacement or repair of deteriorated units during the period under consideration.
6. The demand rate $D(p,t) = (\alpha - \beta p)e^{\gamma . t}$, where a>0, b>0 and $\alpha > 0, \beta > 0 and \gamma$ are constants.

7. The deterioration function is assumed to be $\theta(t) = \theta.\psi(t - t_1)$ where θ (0< θ <<1)is constant and $\psi(t-t_1)$ is defined as $\psi(t - t_1) = \begin{cases} 1 & , \quad t > t_1 \\ 0 & , \quad t \le t_1 \end{cases}$, where t is the time measured from the instant of arrival of a fresh replenishment indicating that the deterioration of the items begins after a time from the instant of that arrival in stock.

8. Shortages are allowed and partially backlogged and fraction of shortage backorder is $B(t) = ae^{-bt}$ Where $0 < a \le 1, b > 0$

3. MODEL FORMULATION AND DEVELOPMENT

let I(t) the inventory level at any time t. The inventory is depleted partly to meet the demand and partly for deterioration during the period of positive inventory. The behavior of inventory system at any time is depicted in figure:

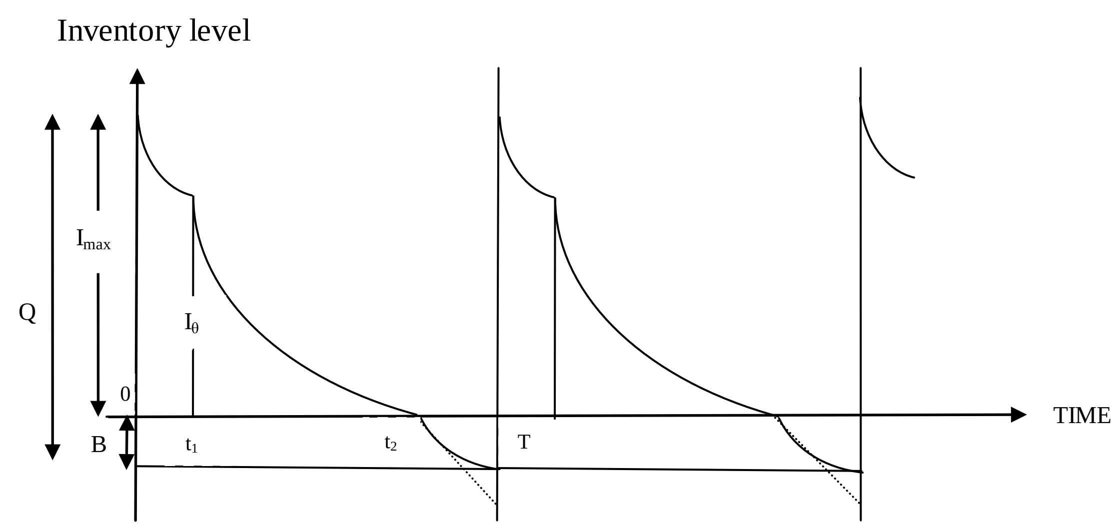

Figure : Graphical representation for the inventory system

Replenishment is made at a time t=0 and inventory level is at its maximum I_{max}. The inventory level decreases gradually during the period $[0,t_1]$, due to demand. After time t_1 inventory level decreases gradually due to both demand and deterioration and ultimately falls to zero at $t = t_2$ .Thereafter shortage occurs due to demand and partial backlogging during the time interval $[t_2,T]$. The rate of change of inventory during positive stock period $[0, t_1]$ then $[t_1, t_2]$ and during negative stock period [t_2, T] i.e. shortage period, can be described by the following differential equations:

$$\frac{d}{dt}I_1(t) = -(\alpha - \beta p)e^{\gamma.t}, \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{d}{dt}I_2(t) + \theta.I_2(t) = -(\alpha - \beta p)e^{\gamma.t}, \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{d}{dt}I_3(t) = -(\alpha - \beta p)e^{\gamma.t}.B(T-t), \qquad t_2 \le t \le T \qquad (3)$$

The solution of differential equation (1), using boundary conditions, $I_1(0) = I_{\max}$ and $I_1(t_1) = I_\theta$

$$I_1(t) = \frac{\alpha - \beta p}{\gamma}\left(1 - e^{\gamma.t}\right) + I_{\max}, \quad 0 \le t \le t_1 \qquad (4) \qquad \text{and} \qquad I_\theta = \frac{\alpha - \beta p}{\gamma}\left(1 - e^{\gamma.t_1}\right) + I_{\max} \quad (5)$$

The solution of differential equation (2), using boundary conditions, $I(t_2) = 0$ and $I_2(t_1) = I_\theta$

$$I_2(t) = \frac{(\alpha - \beta p)e^{-\theta t}}{\gamma + \theta}\left[e^{(\gamma+\theta)t_2} - e^{(\gamma+\theta)t}\right], t_1 \le t \le t_2 \text{ (6)and } I_\theta = \frac{(\alpha - \beta p)e^{-\theta t}}{\gamma + \theta}\left[e^{(\gamma+\theta)t_2} - e^{(\gamma+\theta)t_1}\right] \quad (7)$$

From equations (5) and (7) we get, $I_{\max} = (\alpha - \beta p)\left[\frac{e^{-\theta t_1}}{\gamma + \theta}\left\{e^{(\gamma+\theta)t_2} - e^{(\gamma+\theta)t_1}\right\} - \frac{1 - e^{\gamma.t_1}}{\gamma}\right]$ (8)

The solution of differential equation (3), using boundary conditions, $I_3(t_2) = 0$

$$I_3(t) = \frac{(\alpha - \beta p)e^{-bT}}{\gamma + b}\left[e^{(\gamma+b)t_2} - e^{(\gamma+b)t}\right], \qquad t_2 \le t \le T \qquad (9)$$

Maximum amount of demand backlogged

$$B_Q = -I_3(T) = -\frac{(\alpha - \beta p)e^{-bT}}{\gamma + b}\left[e^{(\gamma+b)t_2} - e^{(\gamma+b)T}\right] \qquad (10)$$

Order quantity per cycle

$$Q = B_Q + I_{\max} = (\alpha - \beta p)\left[-\frac{e^{-bT}}{\gamma + b}\left[e^{(\gamma+b)t_2} - e^{(\gamma+b)T}\right] + \frac{e^{-\theta t_1}}{\gamma + \theta}\left[e^{(\gamma+\theta)t_2} - e^{(\gamma+\theta)t_1}\right] - \frac{1 - e^{\gamma.t_1}}{\gamma}\right] \qquad (11)$$

The total profit per unit time per cycle is

TPF=TPF(p,t_2,T) =[The sales revenue-(ordering cost+ inventory holding cost + shortage cost+ opportunity cost+ purchase cost)]/Length of ordering cycle

Total sale revenue

$$R = p\left[B_Q + \int_0^{t_2} (\alpha - \beta p) e^{\gamma . t} dt \right]$$

$$R = p(\alpha - \beta p)\left[\frac{e^{-bT}}{\gamma + b} \left\{ e^{(\gamma + b)T} - e^{(\gamma + b)t_2} \right\} - \frac{1 - e^{\gamma . t_2}}{\gamma} \right] \tag{12}$$

Calculation of variable costs:

Holding cost

$$C_h = c_1 \int_0^{t_2} I(t) dt = c_1 \left[\int_0^{t_1} I_1(t) dt + \int_{t_1}^{t_2} I_2(t) dt \right]$$

$$= c_1 \left[\begin{array}{l} \dfrac{e^{-\theta t_1}(\alpha - \beta p)}{\gamma(\gamma + \theta)} \left\{ \dfrac{\gamma + \theta}{\gamma} e^{\theta t_1}(1 - e^{\gamma t_1}) + \theta . e^{(\theta + \gamma)t_1} t_1 + \gamma . e^{(\theta + \gamma)t_2} t_1 \right\} \\ + \dfrac{e^{-\theta t_1}(\alpha - \beta p)}{\gamma \theta(\gamma + \theta)} \left\{ \theta . e^{(\theta + \gamma)t_1} + \gamma . e^{(\theta + \gamma)t_2} - (\theta + \gamma) e^{\gamma . t_2 + \theta t_1} \right\} \end{array} \right] \tag{13}$$

Shortage cost

$$C_{Sh} = -c_3 \int_{t_2}^{T} I_3(t) dt = \frac{c_3 e^{-bT}(\alpha - \beta p)}{(b + \gamma)^2} \left[e^{(b + \gamma)T} - e^{(b + \gamma)t_2} \left\{ 1 + (b + \gamma)(T - t_2) \right\} \right] \tag{14}$$

Opportunity cost

$$C_{Op} = c_4 \int_{t_2}^{T} (\alpha - \beta p) e^{\gamma . t} \left\{ 1 - B(T - t) \right\} dt$$

$$= \frac{c_4(\alpha - \beta p)}{\gamma . (b + \gamma)} \left[b(e^{\gamma . T} - e^{\gamma . t_2}) - \gamma . e^{\gamma . t_2} \left\{ 1 - e^{b.(t_2 - T)} \right\} \right] \tag{15}$$

Purchase cost

$$C_p = c_2 Q = c_2(\alpha - \beta p)\left[-\frac{e^{-bT}}{\gamma + b}\left[e^{(\gamma + b)t_2} - e^{(\gamma + b)T} \right] + \frac{e^{-\theta t_1}}{\gamma + \theta}\left[e^{(\gamma + \theta)t_2} - e^{(\gamma + \theta)t_1} \right] - \frac{1 - e^{\gamma . t_1}}{\gamma} \right] \tag{16}$$

Now, TPF(p,t_2,T)=(R-C_h-C_{Sh}-C_{Op}-C_p-c_5)/T

Replacing all values, we get

$$TPF(p,t_2,T)=\frac{1}{T}(\alpha-\beta p)\left[\begin{array}{l} p\left[\frac{e^{-bT}}{\gamma+b}\left\{e^{(\gamma+b)T}-e^{(\gamma+b)t_2}\right\}-\frac{(1-e^{\gamma.t_2})}{\gamma}\right] \\ -c_1\left[\begin{array}{l}\frac{e^{-\theta t_1}}{\gamma(\gamma+\theta)}\left\{\frac{\gamma+\theta}{\gamma}e^{\theta t_1}(1-e^{\gamma t_1})+\theta.e^{(\theta+\gamma)t_1}t_1+\gamma.e^{(\theta+\gamma)t_2}t_1\right\} \\ +\frac{e^{-\theta t_1}}{\gamma\theta(\gamma+\theta)}\left\{\theta.e^{(\theta+\gamma)t_1}+\gamma.e^{(\theta+\gamma)t_2}-(\theta+\gamma)e^{\gamma.t_2+\theta t_1}\right\}\end{array}\right] \\ -\frac{c_3e^{-bT}}{(b+\gamma)^2}\left[e^{(b+\gamma)T}-e^{(b+\gamma)t_2}\left\{1+(b+\gamma)(T-t_2)\right\}\right] \\ -\frac{c_4}{\gamma.(b+\gamma)}\left[b(e^{\gamma.T}-e^{\gamma.t_2})-\gamma.e^{\gamma.t_2}\left\{1-e^{b.(t_2-T)}\right\}\right] \\ -c_2\left[\begin{array}{l}-\frac{e^{-bT}}{\gamma+b}\left[e^{(\gamma+b)t_2}-e^{(\gamma+b)T}\right]+\frac{e^{-\theta t_1}}{\gamma+\theta}\left[e^{(\gamma+\theta)t2}-e^{(\gamma+\theta)t_1}\right] \\ -\frac{1-e^{\gamma.t_1}}{\gamma}\end{array}\right]-c_5\end{array}\right] \quad (17)$$

Now, TPF(p,t_2,T) is a function of p,t_2 and T. Keep p fixed and differentiating equation (17) partially w.r.t. t_2 and T, we get

$$\frac{\partial TPF}{\partial t_2}=-\frac{(\alpha-\beta p)e^{-bT+\gamma.t_2-\theta t_1}}{\theta T}\left[\begin{array}{l}\theta c_2\left(e^{bT+\theta t_2}-e^{bt_2+\theta t_1}\right)-e^{bT+\theta t_1}\left\{c_1+\theta(c_4+p)\right\} \\ +\theta e^{bt_2+\theta t_1}\left(c_4+p-Tc_3+c_3t_2\right)+e^{bT+\theta t_2}c_1(1+\theta t_1)\end{array}\right] \quad (18)$$

$$\frac{\partial TPF}{\partial T}=\frac{(\alpha-\beta p)}{T^2}\left[\begin{array}{l} p\left[\dfrac{e^{-bT}}{\gamma+b}\left[e^{(\gamma+b)T}-e^{(\gamma+b)t_2}\right]-\dfrac{(1-e^{\gamma.t_2})}{\gamma}\right] \\ -c_1\left[\begin{array}{l}\dfrac{e^{-\theta t_1}}{\gamma(\gamma+\theta)}\left\{\dfrac{\gamma+\theta}{\gamma}e^{\theta t_1}(1-e^{\gamma t_1})+\theta.e^{(\theta+\gamma)t_1}t_1+\gamma.e^{(\theta+\gamma)t_2}t_1\right\} \\ +\dfrac{e^{-\theta t_1}}{\gamma\theta(\gamma+\theta)}\left\{\theta.e^{(\theta+\gamma)t_1}+\gamma.e^{(\theta+\gamma)t_2}-(\theta+\gamma)e^{\gamma.t_2+\theta t_1}\right\}\end{array}\right] \\ -\dfrac{c_3e^{-bT}}{(b+\gamma)^2}\left[e^{(b+\gamma)T}-e^{(b+\gamma)t_2}\left\{1+(b+\gamma)(T-t_2)\right\}\right] \\ -\dfrac{c_4}{\gamma.(b+\gamma)}\left[b(e^{\gamma.T}-e^{\gamma.t_2})-\gamma.e^{\gamma.t_2}\left\{1-e^{b.(t_2-T)}\right\}\right] \\ -c_2\left[\begin{array}{l}-\dfrac{e^{-bT}}{\gamma+b}\left[e^{(\gamma+b)t_2}-e^{(\gamma+b)T}\right]+ \\ \dfrac{e^{-\theta t_1}}{\gamma+\theta}\left[e^{(\gamma+\theta)t2}-e^{(\gamma+\theta)t_1}\right]-\dfrac{1-e^{\gamma.t_1}}{\gamma}\end{array}\right]-\dfrac{c_5}{(\alpha-\beta p)}+ \\ \dfrac{1}{(b+\gamma)^2}e^{-bT}T\left[\begin{array}{l}-c_2(b+\gamma)\left\{be^{(b+\gamma)t_2}+\gamma e^{(b+\gamma)T}\right\} \\ +\gamma\left\{(\gamma.p-c_3)e^{(b+\gamma)T}+c_3e^{(b+\gamma)t_2}\right\} \\ b^2\left\{-c_4e^{(b+\gamma)T}+e^{(b+\gamma)t_2}(c_4+p-T.c_3+c_3t_2)\right\} \\ +b\gamma\left\{(-c_4+p)e^{(b+\gamma)T}+e^{(b+\gamma)t_2}(c_4+p-T.c_3+c_3t_2)\right\}\end{array}\right]\end{array}\right] \tag{19}$$

Now for maximum TPF, we must have

$$\frac{\partial TPF}{\partial t_2}=0 \quad \text{and} \quad \frac{\partial TPF}{\partial T}=0 \tag{20}$$

Satisfying,

$$\frac{\partial^2 TPF}{\partial t_2{}^2}<0 \;,\; \frac{\partial^2 TPF}{\partial T^2}<0 \quad \text{and} \quad \left(\frac{\partial^2 TPF}{\partial t_2{}^2}\right)\left(\frac{\partial^2 TPF}{\partial T^2}\right)-\left(\frac{\partial^2 TPF}{\partial t_2\partial T}\right)^2>0 \tag{21}$$

Equations (18) and (19) are non- linear equations in p, t_2 and T, for given the value of p these can be solved for t_2 and T with the help of computer software. The obtained values of t_2 and T must satisfy inequations (21) to maximize the total profit per unit time. After satisfying the conditions (21) the optimal solutions are obtained. For any fixed value of p, let t_2*and T*be the optimal values of TPF.

Now to optimize the selling price, differentiate TPF(p,t_2*,T*) partially w.r.t. p and equate to zero.

$$\frac{\partial TPF(p,t_2^*,T^*)}{\partial p}=\frac{1}{T^*}\left[\left(\frac{e^{\gamma T^*}-e^{-bT^*+bt_2^*+\gamma.t_2^*}}{b+\gamma}\right)\{\beta c_2-\beta p+(\alpha-\beta p)\}\right]+$$

$$\left[\begin{array}{l}-\frac{1}{T^*}\left(\beta c_2\left(\frac{1-e^{\gamma.t_1}}{\gamma}+\frac{e^{\gamma.t_1}-e^{\theta t_2^*+\gamma.t_2^*-\theta t_1}}{\theta+\gamma}\right)\right)-\beta\left(\frac{1-e^{\gamma.t_2^*}}{\gamma}\right)p\\ -\frac{c_4\beta}{\gamma.(b+\gamma)}\left[b(e^{\gamma.T^*}-e^{\gamma.t_2^*})-\gamma.e^{\gamma.t_2^*}\left\{1-e^{b.(t_2^*-T^*)}\right\}\right]+\left(\frac{1-e^{\gamma t_2^*}}{\gamma}\right)(\alpha-\beta p)\\ -\frac{c_3e^{-bT^*}\beta\left\{e^{(b+\gamma)T^*}-e^{(b+\gamma)t_2^*}\left\{1+(b+\gamma)(T^*-t_2^*)\right\}\right\}}{(b+\gamma)^2}\\ -\frac{\beta c_1e^{-\theta t_1}}{\theta(\theta+\gamma)\gamma^2}\left\{\theta(\theta+\gamma)e^{\theta t_1}-\gamma(\theta+\gamma)e^{\gamma.t_2^*+\theta t_1}+\gamma^2(\theta t_1+\gamma)e^{(\theta+\gamma).t_2^*}+\theta^2(\gamma.t_1-1)e^{(\theta+\gamma).t_1}\right\}\end{array}\right]=0 \tag{22}$$

Differentiate (22) partially w.r.t. p , we get

$$\frac{\partial^2 TPF(p,t_2^*,T^*)}{\partial p^2}=-\frac{2\beta}{T^*}\left[\left(\frac{e^{\gamma.t_2^*}-1}{\gamma}\right)+\left(\frac{e^{\gamma T^*}-e^{-bT^*+bt_2^*+\gamma.t_2^*}}{b+\gamma}\right)\right]<0 \tag{23}$$

Now corresponding to the values of t_2* and T*, from the equation (22) by using $\beta c_2-\beta p+(\alpha-\beta p)=0$,there is unique value of p, satisfying (23).

4. NUMERICAL EXAMPLE

Suppose that parameters of the inventory system are c_1=40, c_2=200, c_3=80, c_4=120, c_5=250,θ=0.08,t_1=0.04,α=500,β=0.5,ɤ=-0.98,b=0.1. Using these values of parameters we get value of p=600, t_2= 0.0585267 and T=0.0785638,Q=15.1005 and TP=73578.1 and then after taking iterations we obtain optimal values $p^*=600.5681$ $t_2^*=0.05857$ and optimal length of ordering cycle $T^*=0.08757$, $Q^*=15.1018$, $TP^*=73587.7$.

5. SENSITIVITY ANALYSIS

In this section, we study that if there is change in one parameter at a time while keeping remaining unchanged, how it affects the optimal shortage point, the optimal length of ordering cycle, the optimal order quantity, the optimal maximum inventory level and the maximum average total profit per unit time. The sensitivity analysis has been performed by changing each of the parameters by -50%, -25%, +25%, +50%. In the following tables:

Sensitivity analysis on deterioration rate

%change	Deterioration rate (θ)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TPF^*)
-50%	0.04	598.947	0.723	0.906	17.412	76764.2
-25%	0.06	599.727	0.646	0.837	16.076	75057.2
25%	0.1	601.229	0.541	0.744	14.297	72401.0
50%	0.12	601.889	0.500	0.718	13.803	71047.2

On the basis of the results shown in above table, the following observations can be made.

Higher the value of deterioration rate results lower value of order quantity, decrease in time without shortages and replenishment time interval, lower the optimal total profit but increase in optimal selling price.

Sensitivity analysis on inventory holding cost

%change	Inventory holding cost (c_1)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TPF^*)
-50%	20	599.307	0.652	0.846	16.511	75697.4
-25%	30	600.028	0.617	0.814	15.727	74571.6
25%	50	601.229	0.554	0.763	14.558	72761.6
50%	60	601.769	0.533	0.741	14.128	71856.6

On the basis of the results shown in above table, the following observations can be made.

Higher the value of inventory holding cost results lower value of order quantity, decrease in time without shortages and replenishment time interval, lower the optimal total profit but increase in optimal selling price.

Sensitivity analysis on purchasing cost

%change	purchasing cost (c_2)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TPF^*)
-50%	100	519.792	0.620	0.770	20.117	146339.9
-25%	150	560.330	0.593	0.767	17.502	107755.4
25%	250	641.407	0.592	0.824	13.021	44470.7
50%	300	685.248	0.618	0.909	9.594	25428.7

On the basis of the results shown in above table, the following observations can be made.

Higher the value of purchasing cost results lower value of order quantity, decrease the replenishment time interval, lower the optimal total profit but increase in optimal selling price.

Sensitivity analysis on backorder cost

%change	backorder cost (c_3)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TP^*)
-50%	40	599.727	0.562	0.821	15.727	74637.8
-25%	60	600.208	0.576	0.802	15.353	73990.3
25%	100	600.868	0.578	0.774	14.899	73217.8
50%	120	601.169	0.598	0.765	14.735	72938.2

On the basis of the results shown in above table, the following observations can be made.

Higher the value of back order cost results lower value of order quantity, increase in time without shortages but decrease in replenishment time interval, lower the optimal total profit but increase in optimal selling price.

Sensitivity analysis on lost sale cost

%change	lost sale cost (c_4)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TP^*)
-50%	60	600.208	0.576	0.801	15.336	73990.3
-25%	90	600.388	0.581	0.792	15.203	73747.5
25%	150	600.688	0.589	0.780	15.001	73416.4
50%	180	600.868	0.593	0.774	14.906	73247.2

On the basis of the results shown in above table, the following observations can be made.

Higher the value of lost sale cost results lower value of order quantity, increase in time without shortages but decrease in replenishment time interval, lower the optimal total profit but increase in optimal selling price.

Sensitivity analysis on ordering cost

%change	ordering cost (c_5)	Optimal selling price (p^*)	Optimal time without shortage (t_2^*)	Optimal replenishment time interval (T^*)	EOQ (Q^*)	Optimal total Profit (TP^*)
-50%	125	595.583	0.429	0.574	11.225	81348.2
-25%	187.5	598.406	0.518	0.693	13.389	77264.6
25%	312.5	602.610	0.641	0.866	16.526	69649.2
50%	375	604.592	0.712	0.965	18.399	67522.8

On the basis of the results shown in above table, the following observations can be made.

Higher the value of ordering cost results increase in value of order quantity, increase in time without shortages but decrease in replenishment time interval, lower the optimal total profit but increase in optimal selling price.

6. CONCLUSION AND FUTURE RESEARCH

In this proposed model, we present a deterministic inventory model with price and time dependent demand rate, allowing shortages with partial backlogging. Deterioration of items starts after some time and rate of deterioration is constant. This type of model is useful for all such items where deterioration of items begin after a specific time from the instant of their arrival in stock. Profit function has been established and objective is to maximize the total average profit per unit time.Different results of calculus have been used to establish the model.

The present model may be extended in several ways. For instance, we may extend the model to allow for a varying rate of deterioration. Additionally, we could consider stock dependent demand, time varying holding cost. Also, we could extend this model for finite planning horizon.

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