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AN INTERVAL-VALUED INTUITIONISTIC FUZZY WEIGHTED ENTROPY METHOD FOR FINDING OPTIMAL SEQUENCE OF FLOW SHOP SEQUENCING PROBLEMS

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ABSTRACT

This paper proposes the interval valued intuitionistic fuzzy weighted entropy (IVIFWE) for solving flow shop sequencing problems. The processing time of n jobs and m machines flow shop sequencing problem is modeled as fuzzy numbers by using similarity relation and then, we consider the neighboring points are interval of these fuzzy numbers. Used the calculated intervals in interval valued intuitionistic fuzzy weighted entropy for solving the sequencing problem. The proposed entropy method gives the optimal sequence for minimize the makespan C_j *of flow shop sequencing problem without any condition and minimum idle time of the machines.*

Keywords— Flow shop Sequencing Problem, Intuitionistic fuzzy set, Intuitionistic fuzzy interval value, weighted entropy.

I. INTRODUCTION

The flow shop sequencing problem has paying a great role in all production and assembly areas. Assignment of job operations to machines for processing is big task, and sometimes assignment may be imperfectly because of differences in machine type. Further, the time for each operation required by different machines is different, and some machines can execute only particular operations. Therefore, an effective scheduling method is required for growing the productivity of the organizations with reduced cost. In the recent year, fuzzy sets are used to model the uncertain processing times for the flow shop sequencing problems. Zadeh[1] has introduced the membership of an element between zero and one to a fuzzy set as single valued functions. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov[2] as an extension of fuzzy set and the author was define the characterization of IFS by two functions as degree of membership function and non-membership function respectively. Li[3] used intuitionistic fuzzy set theory for solving LPP models and multi criteria decision making problems. Xu[7] developed some optimization models to determine the weights of attributes using IFS based score matrix. Vlachos and Sergiadis[5] derived the De Luca–Termini non-probabilistic entropy for intuitionistic fuzzy sets. This entropy is useful for connecting the notation of fuzzy set entropy and intuitionistic fuzzy sets in terms of fuzziness and intuitionism. In this paper, we consider flow shop sequencing problems, and use interval valued fuzzy numbers to represent the processing time of the problem. Section 2 present the various basic definitions on Interval valued intuitionistic fuzzy weighted entropy. Section 3 details the flow shop sequencing problem using IVIFWE. Section 4 gives the numerical example followed by the conclusion in section 5.

2. DEFINITIONS FOR INTERVAL-VALUED INTUITIONISTIC FUZZY WEIGHED ENTROPY

Let M [0, 1] be the set of all closed subinterval of the set [0, 1], let X be an ordinary finite non-empty set. An interval-valued intuitionistic fuzzy set (IVIFS) in a universe X is defined by $A=\{x, \mu_A(x),\gamma_A(x):x\epsilon X\}$,where $\mu_A: X\rightarrow M[0,1]$, $\gamma_A: X\rightarrow M[0,1]$. The interval $\mu_A(x)$ and $\gamma_A(x)$ denote the membership degree and non-membership degree of the element x in A respectively. Then, for each $x\epsilon X$, $\mu_A(x)$ and $\gamma_A(x)$ are closed intervals and $\mu_A^L(x)$, $\gamma_A^L(x)$ & $\mu_A^R(x)$, $\gamma_A^R(x)$ are lower and upper end points respectively. Hence, in the condition $0< \mu_A^L(x) + \gamma_A^L(x) \leq 1$, and $\mu_A^L(x) \geq 0$, $\gamma_A^L(x) \geq 0$; A is denoted by $A=[\mu_A^L(x), \gamma_A^L(x)$ and $\mu_A^R(x) , \gamma_A^R(x) /x\epsilon X]$. The interval-valued intuitionistic index of *x* in A is called as hesitancy degree, denoted by $\pi_A(x)$ and abbreviated by $[\pi_A^L(x), \pi_A^R(x)]$ can be calculated using the following formula

$$\pi_A(x)=1- \eta_A(x) - \gamma_A(x)$$
$$= [1- \eta_A^R(x) - \gamma_A^R(x), 1- \eta_A^L(x) - \gamma_A^L(x)].$$

Definition2.1.

Let A, B are two IVIFS. A subset relation is defined by $A\subset B$ if and only if $\mu_A^L(x)\leq \mu_B^L(x)$, $\mu_A^U(x)\leq \mu_B^U(x)$and $\gamma_A^L(x) \geq \gamma_B^L(x) ,\gamma_A^U(x))\geq \gamma_B^U(x))$, $\forall$ x $\in$ X.

The complement A^c of A = $\{x,\mu_A^L(x), \gamma_A^L(x): x \in X \} : x \in X\}$ is defined by $A^c = \{x,\gamma_A^L(x)$ $\mu_A^L(x): x \in X \}$.

From now the interval value of membership and non-membership degree is defined by A = ([Δa, Δb], [Δc, Δd]) and the same time the hesitancy degree is [Δe, Δf] for our best practice.

Definition 2.2.

Let A = ([Δa, Δb], [Δc, Δd]) be an interval valued intuitionistic fuzzy number. Then the score function S of an interval valued intuitionistic fuzzy value is given by

S (A) = (Δa + Δb – Δc − Δd)/2.

Here, the score function is applied to compare the grades of the interval valued intuitionistic fuzzy set.

Definition 2.3

Let A_i be the interval valued intuitionistic fuzzy sets, the degree of the interval valued fuzziness is measured by the entropy E (A_i) is defined by

$$E(A_i) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min\{\Delta a_i, \Delta c_i\} + \min\{\Delta b_i, \Delta d_i\} + \Delta e_i + \Delta f_i}{\max\{\Delta a_i, \Delta c_i\} + \max\{\Delta b_i, \Delta d_i\} + \Delta e_i + \Delta f_i}$$

In interval valued intuitionistic fuzzy weighted entropy method (IVIFWE), the hesitancy degree of the intuitionistic set is preferred to be as small as possible to get more stable results. Meanwhile, the bigger value of entropy gives the smaller weight to each corresponding criteria.

Definition 2.4

Let $w_i = (w_1, w_2, w_3, ..., w_n)$ be a subjective weighting vector of decision makers with the condition of $w_i > 0, \sum_{i=1}^{n} w_i = 1, i = 1,2,..., n.$then the weight vector can be calculated by

$$w_i = (1 - E(A_i)) / \sum_{i=1}^{n} (1 - E(A_i)), i = 1,2,...n.$$

Definition 2.5

Let $A_{ij} = \lfloor \Delta a_{ij}, \Delta b_{ij} \rfloor, \lfloor \Delta c_{ij}, \Delta d_{ij} \rfloor, i = 1,2,..., n, j = 1,2,...m$ be interval valued intuitionistic fuzzy sets, the interval valued intuitionistic fuzzy weighted averaging operator can be represented as follows,

$$A_w(A_1, A_2, A_3, ..., A_n)$$
$$= \sum_{i=1}^{n} w_i A_{ij}$$
$$= \left(\begin{array}{l} \left[1 - \prod_{i=1}^{n} (1 - \Delta a_{ij})^{w_i}, 1 - \prod_{i=1}^{n} (1 - \Delta b_{ij})^{w_i} \right] \\ \prod_{i=1}^{n} \Delta c_{ij}^{\ w_i} (x_i), \prod_{i=1}^{n} \Delta d_{ij}^{\ w_i} \end{array} \right)$$

Where $w_i = (w_1, w_2, w_3, ..., w_n)$ is the weight of $A_{ij}, and \sum_{i=1}^{n} w_i = 1, i = 1,2,..,n.$

It is used to aggregate the interval valued intuitionistic fuzzy information corresponding to each alternate towards the final ranking.

3. JOB SEQUENCING PROBLEM USING IVIFWE

In flow shop sequencing problem, the determination of order in which the operations in the behavioural description will execute. It includes optimal schedule under various constrains, machine environments and characteristics of the jobs. The possible sequence of n jobs and m machines in the flow-shop scheduling problem is $(n!)^m$. Among these finding the optimal solution to the problem is very difficult. Therefore to find the optimal sequence of jobs on each machine in order to complete all the jobs on all the machines in the minimum total time is important.

There are few methods based on interval- valued intuitionistic fuzzy set (IVIFS) in measuring job sequences. In this study the job system consisting of n jobs and m machines. The machines are not identical and perform different operations. The following assumptions are made: a) each machine can perform one operation at a time on any job, b) each machine continuously available for processing, c) no interception is allowed during any operation, once the current operation is over then that machine is available for next operation and d) there is no alternate route for operations of jobs. The main objective is to minimize the total completion time C_j (makespan) and reduce the idle time for each machines. This paper, propose a new method to obtain an optimal sequence for flow-shop scheduling problems with m machines. The proposed method is simple to handle and easy to understand also. With the help of the numerical examples, the proposed method is illustrated.

The computational procedure for solving flow shop sequencing problem based on interval valued intuitionistic fuzzy weighted entropy is being presented as follows.

Consider J_1, J_2, $J_3 \ldots J_n$ are jobs and M_1, M_2, $M_3, .. M_m$ are machines. The given job sequencing problem is fuzzifing for processing time according to similarity relation to create fuzzy number data $d_k = (Pt - Pt_{lower})/(Pt_{upper} - Pt_{lower})$ where Pt is the processing time. An interval number can be defined based on fuzzy similarity relation. The interval fuzzy number should be an extension of the concept of a real number and also as a subset of the real line. The following algorithm is used for finding the optimal sequence of the job.

Step 1: Construct an interval valued intuitionistic fuzzy decision matrix D = (d_{ij}); i=1,2,...m; j=1,2,3...n; such that

$$D = \begin{pmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{m1} & \cdots & d_{mn} \end{pmatrix}$$

$$where\, d_{ij} = \{[\Delta a_{ij}, \Delta b_{ij}], [\Delta c_{ij}, \Delta d_{ij}], [\Delta e_{ij}, \Delta f_{ij}]\}, i = 1,2,...m; j = 1,2,...n,$$

$d_{ij} = [\Delta a_{ij}, \Delta b_{ij}]$ Represent the closed interval degree that the alternative M_i satisfied the attribute C_j and $[\Delta c_{ij}, \Delta d_{ij}]$ represent the closed interval degree that alternate M_i does not satisfy the attribute C_j and $[\Delta e_{ij}, \Delta f_{ij}]$ represent the closed interval degree of hesitation associated with the alternative M_i and attribute C_j.

Step 2: Calculate the entropy value corresponding to the criteria C_j, j = 1,2,...n. using definition 2.3

Step3: Calculate the weight vector $J_w = (w_1, w_2, w_3, ..., w_n)$ of the criteria by using definition2.4.

Step 4: Aggregate the evaluation of the alternatives by IVIFWE J_w operator and get the synthetic evaluation M_i , i = 1,2,..,m using definition 2.5.

Step 5: Apply the definition 2.2 to obtain a crisp score function J_{M1}, $J_{M2} \ldots J_{Mm}$ for various alternatives. The smallest value of J_{Mi} processed first and next smallest second and so on.

4. NUMERICAL EXAMPLE

Four jobs 1, 2, 3, and 4 are to be processed on each of the four machines M_1, M_2, M_3 and M_4 in the order of ABCD. The processing times in minutes are given in the Table I.

	M_1	M_2	M_3	M_4
J_1	58	14	14	48

J_2	30	10	18	32
J_3	28	12	16	44
J_4	64	16	12	42

Table I: Job sequencing Problem

An optimal crisp sequence for Table I, obtained by Johnson's algorithm is $J_3 \rightarrow J_2 \rightarrow J_1 \rightarrow J_4$ and the C_j(Makespan) is 250 minutes and idle time of the machines M_1, M_2, M_3 and M_4 are 70, 198, 190 and 84 respectively.

Step 1: Construct the interval valued intuitionistic fuzzy decision matrix. The interval valued intuitionistic fuzzy decision matrix of the processing time has been constructed in table 2.

Job	M_1	M_2
1	[0.795,0.835][0.0,0.1][0.205,0.065]	[0.118,0.150][0.6,0.7][0.282,0.142]
2	[0.365,0.405][0.4,.5][0.235,0.095]	[0.057,0.097][0.7,0.8][0.243,0.103]
3	[0.334,0.374][0.3,0.4][.0366,0.266]	[0.088,0.128][0.7,0.8] [0.212,0.072]
4	[0.888,0.928][0.0,0.1][0.112,0]	[0.149,0.189][0.6,.7][0.251,0.111]

Job	M_3	M_4
1	[0.118,0.158][0.6,0.7][0.285,0.142]	[0.62,0.682][0.1,0.2][0.258,0.118]
2	[0.18,0.22][0.9,0.8][0.120,0.0]	[0.395,0.435][0.2,0.3][0.405,0.265]
3	[0.149,0.189][0.6,0.7][0.251,0.111]	[0.58,0.62][.1,0.2][0.320,0.180]
4	[0.038,0.128][0.7,0.8][0.212,0.072]	[0.549,0.589][0.1,0.2][0.357,.211]

Table 2: Decision Matrix

Step 2: Applying definition2.3 to calculate the entropy value corresponding to the criteria Cj, J=1,2,3,4. The entropy value for each criteria is calculated as given below:

$E(M_1)= 0.740$, $E(M_2) = 0.600$, $E(M_3) = 0.605$, $E(M_4) = 0.878$

Step 3: Applying definition2.4 to calculate the weight vector w= (w_1, w_2, w_3, w_4) of the criteria. The weight vector values are calculated as given below.

$w_1 = 0.221$, $w_2 = 0.34$, $w_3 = 0.336$, $w_4=0.104$

Step 4: Applying definition2.5 to aggregate the evaluation of the alternatives by IVIFWE J_w operator and get the synthetic evaluation M_i, i= 1,2,3,4 then the collection of interval intuitionistic fuzzy value M_i, J_{M1} = 0.481, J_{M2}= 0.284, J_{M3}= 0.287, J_{M4}= 0.512

Step 5: Rank the alternative. Apply equation 1to obtain a crisp score function value J_{M1} ... for the values alternatives given below.

By comparing the score functions values, the alternatives are ranked from the lowest to highest. Thus ranking as follows $J_{M2}<J_{M3}<J_{M1}<J_{M4}$. That is the sequence order is $J_2 \to J_3 \to J_1 \to J_4$ without any conditions. In this order the total completion (makespan) time is same as Johnson's algorithm 250 but the idle times of each machines are 70, 144, 158 and 84 respectively.

In another example we consider five jobs and four machines, the processing times are (7,15,14,21: 11,18,18,6: 2,13,11,16: 14,4,27,14: 18,11,32,16) and the optimal sequence is $J_3 \to J_4 \to J_2 \to J_1 \to J_5$. The makespan is 133 and the idle time of machines on the above example are 81, 100, 31, and 58. further we consider three machines and four jobs, the processing times are (8,2,4: 5,4,5: 6,1,3: 7,3,2). Here the sequence is $J_1 \to J_2 \to J_4 \to J_3$, the makespan is 30 and the idle times are 4, 22, and 16.

5. CONCLUSION

This paper discussed the flow shop sequencing problem and the processing time with on interval valued intuitionistic fuzzy weighted entropy. Minimize the total completion time of each job and idle time for each machine are considered as the objective. The experiment instances show IVIFWE can effectively solved the flow shop sequencing problem with interval processing time. The results could be tested for large scale problems. There are also some works need to be done to obtain better results. This algorithm may apply for Job Shop Scheduling problems and multi criteria job shop scheduling problems. IVIFWE has good local search ability, but its global search ability can be strengthened by hybridizing with other algorithms. More simple and more accurate interval comparison method also needs to be researched.

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