International Research Journal of Natural and Applied Sciences
ISSN: (2349-4077)
Impact Factor- 5.46, Volume 4, Issue 12, December 2017
Website- www.aarf.asia, **Email** : editor@aarf.asia , editoraarf@gmail.com

SQUARE DIFFERENCE PRIME LABELING –MORE RESULTS ON CYCLE RELATED GRAPHS

Sunoj B S[1], Mathew Varkey T K[2]

[1]Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India.

[2]Department of Mathematics, TKM College of Engineering, Kollam, Kerala, India.

ABSTRACT

*Square difference prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the edges with absolute difference of the squares of the labels of the incident vertices .The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits square difference prime labeling. Here we investigate, duplicating an edge in a cycle, duplicating a vertex by an edge in a cycle, duplicating an edge by a vertex in cycle, switching a vertex in cycle, strong duplicate graph of cycle, crown graph, prism graph and two copies of cycle sharing a common vertex, for square difference prime labeling.*

KEYWORDS - Graph labeling, Square difference, Prime labeling, Prime graphs, Cycle related graphs.

INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In [5] , we introduced the concept, square difference prime labeling and proved that some snake graphs admit this kind of labeling. In [6], [7], [8] , [9] and [10] we extended our study and proved that the result is true for some path related graphs, cycle related graphs, some planar graphs, some tree graphs, fan graph, helm graph, umbrella graph, gear graph, friendship and wheel graph. In this paper we investigated square difference prime labeling of duplicating an edge in a cycle, duplicating a vertex by an edge in a cycle, duplicating an edge by a vertex in cycle, switching a vertex in cycle, strong duplicate graph of cycle, crown graph, prism graph and two copies of cycle sharing a common vertex.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,------------------, p-1} by $f(v_i) = i - 1$, for every i from 1 to p and define a 1-1 mapping f_{sdp}^{*} : E(G) → set of natural numbers N by $f_{sdp}^{*}(uv) = | f(u)^2 - f(v)^2 |$. The induced function f_{sdp}^{*} is said to be a square difference prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits square difference prime labeling is called a square difference prime graph.

Definition 2.3 Duplication of an edge e = ab of a graph G produces a new graph H by adding an edge f = xy such that N(x) = N(a) U {y}- {b} and N(y) = N(b) U {x}- {a}

Definition 2.4 Duplication of a vertex v by a new edge e = ab in a graph G produces a new graph H such that N(a) = {v,b) and N(b) = {v,a}.

Definition 2.5 Duplication of an edge e = ab by a vertex v in a graph G produces a new graph H such that N(v) = {a,b}

Definition 2.6 A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 2.7 Let C_n be a cycle with n vertices and K_1 be a complete graph of one vertex. The crown graph $C_n \odot K_1$ is obtained by taking one copy of C_n and n copies of K_1; and by joining each vertex of the i-th copy of K_1 to the i-th vertex of C_n.

Definition 2.8 A prism graph Y_n is a graph corresponding to the skeleton of an n- prism.

Definition 2.9 Consider two copies of a cycle with n vertices. Let $u_1,u_2,---,u_n$ are the vertices of the first cycle C_n and let $v_1,v_2,---,v_n$ are the vertices of the second cycle C_n'. Remove all the edges of both cycles and joing u_i with v_j, if u_iu_j is an edge of C_n. The new graph is called duplicate graph of cycle C_n and is denoted by $D(C_n)$. Strong duplicate graph of cycle Cn is obtained from D(Cn) by adding edges b/w u_i and v_i for every i and is denoted by $S\{D(C_n)\}$.

Theorem 2.1 Let G be the graph obtained by duplicating an edge in cycle C_n (n is a natural number greater than 4). G admits square difference prime labeling, if $(n-2) \not\equiv 0$ (mod5) and $(n-1) \not\equiv 0$ (mod3).

Proof: Let G be the graph and let $v_1,v_2,$----------------$,v_{n+2}$ are the vertices of G.

Here $|V(G)| = n+2$ and $|E(G)| = n+3$

Define a function $f : V \rightarrow \{0,1,2,$-----------------$,n+1 \}$ by

$$f(v_i) = i-1 , \quad i = 1,2,\text{------},n+2$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\ v_{i+1}) = 2i-1,\quad i = 1,2,$------------$,n+1$

$f_{sdp}^*(v_1\ v_n) = (n-1)^2.$

$f_{sdp}^*(v_3\ v_{n+2}) = n^2+2n-3.$

Clearly f_{sdp}^* is an injection.

gcin of (v_{n+2}) $= \gcd$ of $\{f_{sdp}^*(v_{n+1}\ v_{n+2}), f_{sdp}^*(v_3\ v_{n+2})$

$= \gcd$ of $\{2n+1, n^2+2n-3\} = 1.$

gcin of (v_{i+1}) $= \gcd$ of $\{f_{sdp}^*(v_i\ v_{i+1}), f_{sdp}^*(v_{i+1}\ v_{i+2})$

$= \gcd$ of $\{ 2i-1, 2i+1)\}$

$= \gcd$ of $\{2, 2i-1\} = 1,\quad i = 1,2,$------------$,n.$

gcin of (v_1) $= \gcd$ of $\{f_{sdp}^*(v_1\ v_2), f_{sdp}^*(v_1\ v_n)$

= gcd of $\{1, (n\text{-}1)^2\}$= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.2 Let G be the graph obtained by duplicating a vertex by an edge in cycle C_n (n is a natural number greater than 2). G admits square difference prime labeling, if (n-2) ≢ 0 (mod5) and (n-1) ≢ 0 (mod3).

Proof: Let G be the graph and let v_1,v_2,----------------,v_{n+2} are the vertices of G.

Here |V(G)| = n+2 and |E(G)| = n+3

Define a function f : V → {0,1,2,-----------------,n+1 } by

$f(v_i)$ = i-1 , i = 1,2,------,n+2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^{*} is defined as follows

$f_{sdp}^{*}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n+1

$f_{sdp}^{*}(v_1\ v_3)$ = 4.

$f_{sdp}^{*}(v_3\ v_{n+2})$ = n^2+2n-3.

Clearly f_{sdp}^{*} is an injection.

gcin of (v_{n+2}) = gcd of $\{f_{sdp}^{*}(v_{n+1}\ v_{n+2}), f_{sdp}^{*}(v_3\ v_{n+2})$

= gcd of $\{2n+1, n^2+2n-3\}$= 1.

gcin of (v_{i+1}) = gcd of $\{f_{sdp}^{*}(v_i\ v_{i+1}), f_{sdp}^{*}(v_{i+1}\ v_{i+2})$

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,n.

gcin of (v_1) = gcd of $\{f_{sdp}^{*}(v_1\ v_2), f_{sdp}^{*}(v_1\ v_3)$

= gcd of {1, 4}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.3 Let G be the graph obtained by duplicating an edge by a vertex in cycle C_n (n is a natural number greater than 2). G admits square difference prime labeling.

Proof: Let G be the graph and let v_1, v_2,----------------,v_{n+1} are the vertices of G.

Here |V(G)| = n+1 and |E(G)| = n+2

Define a function f : V → {0,1,2,-----------------,n } by

$f(v_i)$ = i-1 , i = 1,2,------,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n.

$f_{sdp}^* (v_1\ v_{n+1})$ = n^2.

$f_{sdp}^* (v_2\ v_{n+1})$ = n^2-1.

Clearly f_{sdp}^* is an injection.

gcin of (v_{n+1}) = gcd of {$f_{sdp}^* (v_1\ v_{n+1})$, $f_{sdp}^* (v_2\ v_{n+1})$

= gcd of {n^2, n^2-1}= 1.

gcin of (v_{i+1}) = gcd of {$f_{sdp}^* (v_i\ v_{i+1})$, $f_{sdp}^* (v_{i+1}\ v_{i+2})$

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,n-1.

gcin of (v_1) = gcd of {$f_{sdp}^* (v_1\ v_2)$, $f_{sdp}^* (v_1\ v_{n+1})$

= gcd of {1, n^2}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.4 Let G be the graph obtained by switching a vertex in cycle C_n (n is a natural number greater than 5). G admits square difference prime labeling.

Proof: Let G be the graph and let v_1, v_2,----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = 2n-5

Define a function f : V → {0,1,2,-----------------,n-1 } by

$f(v_i) = i-1$, i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n-2

$f_{sdp}^* (v_{i+1}\ v_n)$ = $(n-1)^2 - i^2$, i = 1,2,------------,n-3

Clearly f_{sdp}^* is an injection.

gcin of (v_n) = 1.

gcin of (v_{i+1}) = gcd of $\{f_{sdp}^* (v_i\ v_{i+1}), f_{sdp}^* (v_{i+1}\ v_{i+2})$

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,n-3.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.5 Strong duplicate graph of cycle C_n (n is a natural number greater than 2) admits square difference prime labeling, if n is odd.

Proof: Let G be the graph and let v_1, v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,-----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^* (v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,2n-1.

$f_{sdp}^* (v_i\ v_{n+i})$ = $(n+i-1)^2 - (i-1)^2$, i = 1,2,------------,n.

$f_{sdp}^* (v_1\ v_{2n})$ = $4n^2-4n+1$.

Clearly f_{sdp}^* is an injection.

gcin of (v_{2n}) = gcd of $\{f_{sdp}^* (v_1\ v_{2n}), f_{sdp}^* (v_{2n-1}\ v_{2n})\}$

= gcd of $\{(2n-1)^2, 4n-3\}$

= gcd of {2n-1,4n-3)}

= gcd of {2n-2, 2n-1} = 1.

gcin of (v_{i+1}) = gcd of {$f^*_{sdp}(v_i\ v_{i+1})$, $f^*_{sdp}(v_{i+1}\ v_{i+2})$}

= gcd of { 2i-1, 2i+1)}

= gcd of {2, 2i-1} = 1, i = 1,2,------------,2n-2.

gcin of (v_1) = gcd of {$f^*_{sdp}(v_1\ v_2)$, $f^*_{sdp}(v_1\ v_{n+1})$

= gcd of {1, n^2}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.6 Crown graph C_n Θ K_1 (n is a natural number greater than 2) admits square difference prime labeling.

Proof: Let G = C_n Θ K_1 and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n

Define a function f : V → {0,1,2,-----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(v_{2i-1}\ v_{2i})$ = 4i-3, i = 1,2,------------,n.

$f^*_{sdp}(v_{2i-1}\ v_{2i+1})$ = 8i-4, i = 1,2,------------,n-1.

$f^*_{sdp}(v_1\ v_{2n-1})$ = $4n^2$-8n+4.

Clearly f^*_{sdp} is an injection.

gcin of (v_{2n-1}) = gcd of {$f^*_{sdp}(v_{2n-1}\ v_{2n})$, $f^*_{sdp}(v_{2n-3}\ v_{2n-1})$}

= gcd of {4n-3, 4(2n-3)}

= gcd of {2n-3,4n-3)}

= gcd of {2n, 2n-3} = 1.

gcin of (v_{2i-1}) = gcd of {$f^*_{sdp}(v_{2i-1}\ v_{2i+1})$, $f^*_{sdp}(v_{2i-1}\ v_{2i})$ }

= gcd of { 8i-4, 4i-3)}

= gcd of {4i-1, 4i-3}

= gcd of {2, 4i-3}= 1, i = 1,2,------------,n-1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits square difference prime labeling.

Theorem 2.7 Prism graph Y_n (n is a natural number greater than 2) admits square difference prime labeling.

Proof: Let G =Y_n and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,-----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$f^*_{sdp}(v_{2i-1}\, v_{2i})$ = 4i-3, i = 1,2,------------,n.

$f^*_{sdp}(v_{2i-1}\, v_{2i+1})$ = 8i-4, i = 1,2,------------,n-1.

$f^*_{sdp}(v_{2i}\, v_{2i+2})$ = 8i, i = 1,2,------------,n-1.

$f^*_{sdp}(v_1\, v_{2n-1})$ = $4n^2$-8n+4.

$f^*_{sdp}(v_2\, v_{2n})$ = $4n^2$-4n.

Clearly f^*_{sdp} is an injection.

gcin of (v_{2n-1}) = gcd of {$f^*_{sdp}(v_{2n-1}\, v_{2n}), f^*_{sdp}(v_{2n-3}\, v_{2n-1})$}

= gcd of {4n-3, 4(2n-3)}

= gcd of {2n-3,4n-3)}

= gcd of {2n, 2n-3} = 1.

gcin of (v_{2i-1}) = gcd of {$f^*_{sdp}(v_{2i-1}\, v_{2i+1}), f^*_{sdp}(v_{2i-1}\, v_{2i})$ }

= gcd of { 8i-4, 4i-3)}

= gcd of {4i-1, 4i-3}

= gcd of {2, 4i-3}= 1, i = 1,2,------------,n-1.

gcin of (v_{2i+2}) = gcd of $\{f^*_{sdp}(v_{2i}\, v_{2i+2}), f^*_{sdp}(v_{2i+1}\, v_{2i+2})\}$

= gcd of { 8i, 4i+1}

= gcd of {4i-1, 4i+1}

= gcd of {2, 4i-1}= 1, i = 1,2,------------,n-1.

gcin of (v_2) = gcd of $\{f^*_{sdp}(v_1\, v_2), f^*_{sdp}(v_2\, v_4)\}$

= gcd of {1,8} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Y_n , admits square difference prime labeling.

REFERENCES

1. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
2. F Harary, Graph Theory, Addison-Wesley,Reading, Mass, (1972)
3. Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics(2015), #DS6, Pages 1 – 389.
4. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.
5. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling of Some Snake Graphs, Global Journal of Pure and Applied Mathematics, Volume 13, Issue 3, March 2017, pp 1083-1089.
6. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling of Some Planar Graphs, Global Journal of Pure and Applied Mathematics, Volume 13, Issue 6, March 2017, pp 1993-1998.
7. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling of Wheel Graph, Fan Graph, Friendship Graph,Gear Graph, Helm Graph and Umbrella Graph, IJMMS, Volume 13, Issue 1, (January-June) 2017, pp 1-5.
8. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling of Some Path Related Graphs, International Review of Pure and Applied Mathematics, Volume 13, Issue 1, (January-June) 2017, pp 95-100.
9. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling for Some Tree Graphs, International of Engineering Development and Research, Volume 5, Issue 4, (November) 2017, pp 875-878.

10. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling for Some Cycle Related Graphs, International Journal of Computational Engineering Research, Volume 7, Issue 11, (November) 2017, pp 22-25.