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NONLINEAR STRETCHING SHEET WITH HEAT SOURCE IN POROUS MEDIUM

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ABSTRACT

The presence of porous media in a boundary layer flow can significantly alter the field of flow and, as a consequence, affect the heat transfer rate at the surface.A Porous medium is generally modeled by using the classical Darcy formulation, which implies that the velocity of mean filter is proportional to the summation of the pressure gradient and the gravitational force. The model is empirical and it cannot be derived via a momentum analytically on a small element of porous medium. Sheikholeslami et al. [11] studied the effects of heat transfer in flow of nanofluids over a permeable stretching surface in presence of a porous medium.The flow of Natural convective boundary layer over a horizontal plate embedded in porous medium with a nanofluid was proposed by Gorla and Chamkha [12].

1.Introduction

The presence of porous media in a boundary layer flow can significantly alter the field of flow and, as a consequence, affect the heat transfer rate at the surface.A Porous medium is generally modeled by using the classical Darcy formulation, which implies that the velocity of mean filter is proportional to the summation of the pressure gradient and the gravitational force. The model is empirical and it cannot be derived via a momentum analytically on a small element of porous medium. Sheikholeslami et al. [11] studied the effects of heat transfer in flow of nanofluids over a permeable stretching surface in presence of a porous medium.The flow of Natural convective boundary layer over a horizontal plate embedded in porous medium with a nanofluid was proposed by Gorla and Chamkha [12].

The presence of magnetic field in fluid flow is of great importance because their combination is used in many devices such as electromagnetic propulsion, MHD pump, nuclear reactors and MHD generators. Thus significant number of studies are available in literature which addressed MHD flows. Keshtkar and Amiri [13] derived MHD flow and heat transfer of a nanofluid over a stretching sheet which is permeable. Effects of heat and mass transfer on MHD slip fluid in nanofluids was carried out by Noghrehabadi and Ghalambaz [14]. Pal and Mandal [15] proposed the importance of MHD heat transfer effect of nanofluid over a non-linear stretching or shrinking sheet. Gnaneswara Reddy [16] proposed the Influence of magnetohydrodynamic and thermal radiation boundary layer flow of a nanofluid past a stretching sheet.

The effects of radiation on temperature have become very important industrially. Many processes in engineering areas which occur at high temperature and acknowledge radiation heat transfer have become very important for designing pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for making air-craft, missiles, satellites and space vehicles are examples of such engineering areas. Rashidi et al. [17] proposed heat and mass transfer effects on MHD fluid flow over a permeable vertical stretching sheet with radiation and buoyancy. Thermal radiation effects on steady MHD free convective boundary layer flow of nanofluid along a nonlinear stretching sheet was studied by Poornima and Bhaskar Reddy [18]. Shateyi and Prakash [19] developed a new numerical approach for MHD laminar boundary layer flow and heat transfer of a nanofluid over a moving surface in the presence of radiation. Radiation effects on MHD boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions were formulated by Gbadeyan et al. [20]

The problem of free convection boundary layer flow of nanofluids over a nonlinear stretching sheet in the presence of MHD and heat source/sink was investigated by Ranga Rao et al. [21]. Chamkha and Aly [22] derived the effects of heat generation on MHD free convection flow of a nanofluid past a vertical plate. MHD boundary layer flow past a wedge moving in a nanofluid in presence of radiation and heat generation was studied by Khan et al. [23].

Gireesha and Rudraswamy [24] proposed chemical reaction and heat source effects on MHD flow of a nanofluid near the stagnation point over a permeable stretching surface. Chemical reaction effect on MHD nanofluid flow due to a stretching or shrinking sheet was studied by Kameswaran et al. [25]. Rosmila Abdul et al. [26] derived the boundary layer flow of a nanofluid past a porous vertical stretching surface with chemical reaction and radiation. The importance of chemical reaction effect in nanofluid flows were studied by many researchers such as Rosca [27], Kasmani et al [28], Venkateswarlu and Satya Narayana [29], and Yohannes and Daniel [30].

Dissipation is the process of converting mechanical energy of downward-flowing water into energy which is thermal and acoustical . Viscous dissipation is of interest for many applications: Rises in significant temperature are observed in flows of polymer processing such as injection modeling or extrusion at high rates. Aerodynamic heating in the boundary layer which is thin around high speed aircraft raises the temperature of the skin. Pal and Mandal [31] found that the effects of MHD boundary layer flow of nanofluids over a nonlinear stretching or shrinking sheet in presence of viscous ohmic dissipation and radiation. Krishnamurthy et al. [32] carried out experiments to study the effect of viscous dissipation on hydromagnetic flow and heat transfer of nanofluids over an exponentially stretching sheet with fluid particle suspension. Heat and mass transfer effects on MHD flow of nanofluids over porous media through a stretching sheet in presence of chemical reaction and viscous dissipation were studied by Yohannes and Shankar [33]. Motsumi and Makinde [34] investigated the effects of radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate. Habibi Matin et al. [35] derived the MHD flow of nanofluid over a nonlinear stretching sheet in presence of viscous dissipation. MHD laminar boundary layer flow with heat and mass transfer of an electrically conducting water based nanofluid over a nonlinear stretching sheet in presence of viscous dissipation was studied numerically by Mabood et al. [36]. Ganga et al. [37] illustrated the effects of radiation and heat generation on MHD boundary layer flow of a nanofluid past a vertical plate with viscous and ohmic dissipation. Machireddy Gnaneswara Reddy, Polarapu Padma, Bandari Shankar et al. [38] illustrated the effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet.

2.Mathematical Formulation

A two-dimensional, steady and incompressible viscous boundary layer flow of an incompressible electrically conducting and radiative past over a nonlinear stretching surface is considered under the assumptions that the external pressure on nonlinear stretching sheet in the x- direction is having diluted nanoparticles and y-axis normal to it. The sheet is extended with velocity $u_w(x) = ax^n$ with fixed origin location, here n is a nonlinear stretching parameter, a is a constant. The fluid is considered to be a gray, absorbing and emitting radiation but nonscattering medium. A uniform magnetic field was applied in the transverse direction to the flow. We assumed that the variable magnetic field B(x) is of the form $B(x) = B_0 x^{\frac{(n-1)}{2}}$. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity and hence the magnetic field which is induced is negligible in comparison with the applied magnetic field.By using the Oberbeck-Boussinesq approximation, the governing equations of the flow field can be written in the dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\rho_f \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \left(1 - C_\infty\right)\rho_{f_\infty}\beta_T g\left(T - T_\infty\right) - \left(\rho_p - \rho_{f_\infty}\right)\beta_C g\left(C - C_\infty\right) - \sigma B_0^2 u - \frac{\mu}{Kp}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\left(\rho c_p\right)_f}\frac{\partial q_r}{\partial y} + \tau\left[D_B\left(\frac{\partial T}{\partial y}\frac{CT}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q'}{\left(\rho c_p\right)_f}\left(T - T_\infty\right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_1\left(C - C_\infty\right) \tag{4}$$

Here, u and v are the velocity components in the x and y directions, respectively, g is the acceleration due to gravity, μ is the viscosity, σ is the electrical conductivity, p is the pressure, ρ_f is the density of the base fluid, ρ_f is the density of nanoparticle, β_T is the coefficient of volumetric thermal expansion, β_C the coefficient of volumetric concentration expansion, Kp is permeability of the porous medium, T - the temperature of the nanofluid, C - the concentration of the nanofluid, T_w and C_w are the temperature and concentration along the stretching sheet, T_∞ and

C_∞ are the ambient temperature and concentration, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis coefficient, B_0 is the magnetic induction, q_r is the radiative heat flux, k - the thermal conductivity, c_p is specific heat at constant pressure, $\left(\rho c_p\right)_p$ is the heat capacitance of the nanoparticles, $\left(\rho c_p\right)_f$ is the capacitance of heat of the base fluid, $\alpha = k/\left(\rho c_p\right)_f$ is the

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

Assuming that the external pressure on the plate, in the direction having diluted nanoparticles, to be constant, the similarity transformations are taken as (Rana and Bhargava [8])

$$\eta = y\sqrt{\frac{(n+1)ax^{n-1}}{2\upsilon}} \qquad \psi = \sqrt{\frac{2\upsilon ax^{n+1}}{n+1}}\, f(\eta), \qquad u = ax^n f'(\eta),$$

$$v = -\sqrt{\frac{av(n+1)}{2}}\, x^{(n-1)/2}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right), \qquad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \qquad \phi(\eta) = \frac{C-C_\infty}{C_w - C_\infty},$$

$$Nr = \frac{kk_e}{4\sigma_s T_\infty^3}, \qquad R = \frac{4}{3Nr}, \qquad \lambda = \frac{Gr}{\mathrm{Re}_x^{\frac{3}{2}}}, \qquad \delta = \frac{Gm}{\mathrm{Re}_x^{\frac{3}{2}}}, \qquad M = \frac{2\sigma B_0^2}{\rho_f a(n+1)x^{n-1}}$$

$$K = \frac{Kpa(n+1)}{2\upsilon} x^{n-1} \qquad \Pr = \frac{\upsilon}{\alpha}, \qquad \upsilon = \frac{\mu}{\rho_f} \qquad Le = \frac{\upsilon}{D_B}, \qquad (10)$$

$$Nb = \frac{\tau D_B}{\upsilon}\left(C_w - C_\infty\right), \qquad Q = \frac{2Q'}{(n+1)\left[\rho c_p\right]_f ax^{n-1}}, \qquad Ec = \frac{a^2}{c_p(T_w - T_\infty)} \qquad Nt = \frac{\tau D_T}{\upsilon T_\infty}\left(T_w - T_\infty\right),$$

$$Gr = \frac{\left(1-C_\infty\right)\left(\dfrac{\rho_{f\infty}}{\rho_f}\right) gn\left(T_w - T_\infty\right)}{\upsilon^2 \mathrm{Re}_x^{\frac{1}{2}}}, Gm = \frac{\left(\dfrac{\rho_P - \rho_{f\infty}}{\rho_f}\right) gn_1\left(C_w - C_\infty\right)}{\upsilon^2 \mathrm{Re}_x^{\frac{1}{2}}}, \qquad \gamma = \frac{2k_l}{\left(n+1\right)ax^{n-1}},$$

$$\mathrm{Re}_x = \frac{u_w(x)x}{\upsilon},$$

In view of the above similarity transformations, the equations (2), (9) and (4) reduce to

$$f'''(\eta)+f(\eta)f''(\eta)-\frac{2n}{n+1}\left(f'(\eta)\right)^2+\frac{2}{n+1}\left(\lambda\theta-\delta\phi\right)-(M+K)f'=0 \tag{11}$$

$$\left(1+R\right)\theta''+\Pr f(\eta)\theta'(\eta)+\Pr Nb\theta'(\eta)\phi'(\eta)+\Pr Nt\left(\theta'(\eta)\right)^2+\Pr Q\theta(\eta)+E_c \Pr f''^2=0 \tag{12}$$

$$\phi''\left(\eta\right)+Lef\left(\eta\right)\phi'\left(\eta\right)+\frac{Nt}{Nb}\theta''\left(\eta\right)-Le\gamma\phi\left(\eta\right)=0 \tag{13}$$

where λ is the buoyancy parameter, δ is the solutal buoyancy parameter, n is nonlinear stretching parameter, Pr is the Prandtl number, Le is the Lewis number, υ is the kinematic viscosity of the nanofluid, Nb is the Brownian motion parameter, Nt is the thermophoresis

3.Solution of the Problem

The reduced equations (11) – (13) are nonlinear and coupled, and thus their exact analytical solutions are not possible. They can be solved numberically using Runge-Kutta Fehlberg fourth fifth order method for different values of parameters such as porous parameter, magnetic field parameter, Prandtl number, radiation parameter, heat source, Eckert number, Lewis number, the Brownian motion parameter, the thermpohoresis parameter and chemical reaction parameter. The effects of the emerging parameters on the dimensional velocity, concentration,temperature skin friction coefficient, Nusselt number and Sherwood number are studied. The step size and convergence criteria were chosen to be 0.001 and 10^{-6}, respectively. The asymptotic boundary conditions in (14) were approximated by using a value of 8 for $\eta_{\max}$. This ensures that all numerical solutions approached the asymptotic values correctly.

4. Results and Discussion

Computations were carried out with Runge-Kutta Fehlberg fourth fifth order method for several non-dimensional parameters. We have compared our numerical computation results of Nusselt and Sherwood numbers with those which are available in open literature in tables (1) – (3), which show an excellent agreement. The values of skin friction, Nusselt and Sherwood numbers for different values of parameters are displayed in table 3.

To analyze the results, numerical computation are carried out for variation in physical parameters. In the present case, the following default parameter values are taken for computations: $\lambda = 0.5$, δ =0.5, *M = 0.5, K = 0.5, n = 2.0, Nb = 0.1, Nt = 0.1, Pr = 0.71, R = 1.0, Q = 0.1, Ec = 0.01, Le = 0.01 and* γ *= 0.5*. All graphs therefore correspond to these values unless specifically indicated in the appropriate graph.

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