



## HEAT AND MASS TRANSFER FOR VISCO-ELASTIC MHD BOUNDARY LAYER FLOW PAST A VERTICAL POROUS PLATE OF SLIP FLOW REGION IN THE PRESENCE CHEMICAL REACTIVE SPICES

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### ABSTRACT

*The study of heat and mass transfer for visco-elastic MHD boundary layer flow past a vertical porous plate of slip flow region in the presence chemical reactive spices has been investigated. The suction at the plate is assumed to be constant. The perturbation technique has been used to solve the governing equations of motion. The expressions for velocity field, temperature field, concentration field and skin friction both for Newtonian fluid and visco-elastic fluid have been obtained. The results are discussed in detailed with the help of graphs and tables to observe the effect of different parameters.*

### 1. Introduction

The analysis of visco-elastic fluid flow is one of the important field of fluid dynamics. The complex stress-strain relationships of visco-elastic fluid flow mechanisms are used in geophysics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil-physics, bio-physics, paper and pulp technology. Walters liquid (Model B/) is a type of visco-elastic fluid which resists shear flow and strains linearly with time under the application of an applied stress but when the stress is removed it quickly returns to its original position. The viscosity of the visco-elastic fluid enables the physics of the energy dissipated during the flow and its elasticity analyses the energy stored during the flow. As Walters liquid (Model B/) contains both viscosity and elasticity so it is different from Newtonian fluid because the Newtonian fluid does not have the concept of elasticity as it discusses the concept of energy dissipation. The mechanisms of elasticoviscous boundary layer flow are used in various manufacturing processes such as fabrication of adhesive tapes, extrusion of plastic sheets, coating layers into rigid surfaces etc. Various blood flow problems are also explained by using the visco-elastic boundary layer theory. Analytical studies of forced, free and mixed convection flow of a viscous incompressible fluid along a vertical surface in the absence of magnetic field have been conducted by Merkin[1] Sparrow[2]. The influence of magnetic field on an electrically conducting viscous incompressible fluid is extensively used in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, textile industry etc. Because of its application for MHD natural convection flow in the nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer

flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors such as Crammer [4], Hossain[5] and Kuiken[3]. In our daily life, the combined heat and mass transfer phenomenon is observed in the formation of fog. The convective flow associated with the combined heat and mass transfer has many applications in various branches of science and engineering. Singh and Singh[6] discussed the MHD free convection flow and mass transfer past a flat plate. Al-Qadat and Al-Azab[7] studied the influence of chemical reaction on transient MHD freeconvective flow over a moving vertical plate. The study of combined heat and mass transfer in mixed convective MHD flow along a vertical plate in presence of heat source has been shown by Zueco and Ahmed[19]. Palani and Srikanth[8] explained the mass transfer effects on MHD flow past a semi infinite vertical plate. Chaudhary and Jain[9] analyses the combined heat and mass diffusion in a MHD free convective flow past a surface embedded in a porous medium. Effects of chemical reaction on transient MHD free convective flow over a vertical plate in slip-flow regime are explained by Sahin[10]. It is proposed to study the effect of Heat and Mass Transfer For Visco-Elastic MHD Boundary Layer Flow Past a Vertical Porous Plate Of Slip Flow Region In The Presence Chemical Reactive Spices

## 2. Formulation of the problem

The steady two dimensional free convective boundary layer flow with heat and mass transfer of an electrically conducting elastico-viscous fluid past a vertical porous plate of slipflow region in the presence of chemically reactive spices is analyzed. A magnetic field of uniform strength  $B$  is applied in the normal to the plate. It is also assumed that the interaction of induced magnetic field with the flow is of negligible order in comparison with the interaction of the imposed magnetic field. The electrical conductivity of the fluid is also assumed to be of smaller order of magnitude. Let  $x'$ -axis be taken along the vertical plate and  $y'$ -axis is taken normal to the plate. Let  $T'_w$  and  $C'_w$  be respectively the temperature and the molar species concentration of the fluid at the plate and  $T'_\infty$  and  $C'_\infty$  be respectively the equilibrium temperature and equilibrium molar species concentration of the fluid. The fluid has constant properties and the variation in density and mass concentration is considered only in the body force term. Then neglecting viscous dissipation and ohmic dissipation by usual Boussinesq's approximation the steady flow is governed by the following equations:

The equation of continuity gives suction velocity  $v' = -U_0$

$$-U_0 \frac{du'}{dy'} = \nu \frac{d^2u'}{dy'^2} - \nu k_0 U_0 \frac{d^3u'}{dy'^3} + g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty) - \frac{\nu u'}{K} - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$-U_0 \frac{dT'}{dy'} = \frac{\alpha}{\rho c_p} \frac{d^2T'}{dy'^2} \quad (2)$$

$$-U_0 \frac{dC'}{dy'} = D \frac{d^2C'}{dy'^2} - R'(C' - C'_\infty) \quad (3)$$

with the following boundary conditions

$$u' = h' \frac{du'}{dy}, T' = T'_w, C' = C'_w \quad \text{at } y' = 0 \quad (4)$$

$$u' = 0, T' = T'_\infty, C' = C'_\infty \quad \text{at } y' \rightarrow \infty,$$

where  $g$  is the acceleration due to gravity,  $T'$  is the temperature of the fluid within the boundary layer,  $C'$  is the species concentration,  $\rho$  is the fluid density of boundary layer,  $\nu$  is the kinematic viscosity,  $\alpha$  is the thermal conductivity,  $C_p$  is the specific heat at constant pressure,  $D$  is the mass diffusivity,  $K$  is the permeability of porous medium,  $R'$  is the chemical reaction parameter,  $h'$  is refraction parameter and  $k_0$  is the elasto-viscous parameter.

Let us introduce the non-dimensional variables

$$u = \frac{u'}{U_0}, y = \frac{y'U_0}{\nu}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, K = \frac{K'U_0^2}{\nu^2}, Pr = \frac{\rho\nu C_p}{\alpha}, k = \frac{k_0 U_0^2}{\nu^2} \quad (5)$$

$$Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3}, R = \frac{R'\nu}{U_0^2}, h = h' \frac{U_0^2}{\nu}$$

Equations (1) to (3) are transformed to their corresponding non dimensional forms as

$$k \frac{d^3 u}{dy^3} + \frac{d^2 u}{dy^2} + \frac{du}{dy} - \left( M + \frac{1}{K} \right) u = -GrT - GmC \quad (6)$$

$$\frac{d^2 T}{dy^2} + Pr \frac{dT}{dy} = 0 \quad (7)$$

$$\frac{d^2 C}{dy^2} + Sc \frac{dC}{dy} - ScRC = 0 \quad (8)$$

where  $Pr$  is the Prandtl number,  $Gr$  is the Grashof number,  $k$  is the non-dimensional elasto-viscous parameter,  $Sc$  is the Schmidt number,  $M$  is the magnetic parameter,  $K$  is the non dimensional permeability parameter of porous medium,  $R$  is the non dimensional chemical reaction parameter and  $h$  is the non dimensional refraction parameter.

with the following boundary conditions

$$\left. \begin{aligned} u = h \frac{\partial u}{\partial y}, T = 1, C = 1 \quad \text{at } y = 0 \\ u = 0, T = 0, C = 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (9)$$

### 3. Method of Solution

The solution of the equations (7) and (8) subject to the boundary conditions (9) are given as

$$T = e^{-Pr y} \quad (10)$$

$$C = e^{-A_{11} y} \quad (11)$$

To solve the equation (6), we use the perturbation technique, where the velocity profile can be expressed as:

$$u = u_0 + k u_1 + o(k^2) \quad (12)$$

where the elastico-viscous parameter is assumed to be small ( $k \ll 1$ ).

Then substituting the equation (12) into the equation (6), and equating the like powers of the parameter  $N$ , we get the following two equations:

$$\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} - \left(M + \frac{1}{K}\right) u_0 = -GrT - GmC \quad (13)$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(M + \frac{1}{K}\right) u_1 = -\frac{d^3 u_0}{dy^3} \quad (14)$$

With the boundary conditions

$$\left. \begin{aligned} u_0 = h \frac{\partial u_0}{\partial y}, u_1 = h \frac{\partial u_1}{\partial y} \quad \text{at } y = 0 \\ u_0 = 0, \quad u_1 = 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (15)$$

By solving the equation (13)- (14) using the boundary conditions (15) and (12) by neglecting the higher power of  $k$ , we get

$$u = (B_{11} e^{-A_{12}y} - Gre^{-Pr y} - Gme^{-A_{11}y}) + k \left( (B_{12} - B_{11} A_{12}^3) e^{-A_{12}y} + GrPr^3 e^{-Pr y} + Gm A_{11}^3 e^{-A_{11}y} \right)$$

The non-dimensional shearing stress at the plate is given by

$$\tau = (-A_{12} B_{11} + PrGr + A_{11} Gm) + k(-A_{12}(B_{12} - B_{11} A_{12}^3) - GrPr^4 - Gm A_{11}^4)$$

$$\text{where } A_{11} = \frac{Sc + \sqrt{Sc^2 + 4ScR}}{2}, A_{12} = \frac{1 + \sqrt{1 + 4(M + \frac{1}{K})}}{2}, B_{11} = \frac{Gr + Gm + h(GrPr + Gm A_{11})}{(1 + hA_{12})}$$

$$B_{12} = \frac{B_{11} A_{12}^3 - GrPr^3 - Gm A_{11}^3 + h(A_{12}^4 B_{11} - Pr^4 Gr - A_{11}^4 Gm)}{1 + hA_{12}}$$

#### 4. Results and discussion

In this paper we have studied the Heat and mass transfer for visco-elastic MHD boundary layer flow past a vertical porous plate of slip flow region in the presence chemical reactive species. The effect of the parameters  $Gr$ ,  $Gm$ ,  $M$ ,  $K$ ,  $R$ ,  $Pr$ ,  $k$ , and  $Sc$  on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w.r.t.  $y$  and the values of Skin friction shown in the table for different values of flow parameters. The elastic-viscous effect is exhibited through the nondimensional parameter  $k$ . The nonzero values of the parameter  $k$  characterize the visco-elastic fluid and  $k=0$  represents the Newtonian fluid flow phenomenon.

*Velocity profiles:* The velocity profiles are depicted in Figs 1-5. Figure-(1) shows the effect of the parameters  $k$  and  $Gr$  on velocity at any point of the fluid, when  $Pr=7$ ,  $Gm=2$ ,  $M=2$ ,  $K=2$ ,  $R=2$ ,  $h=0.2$  and  $Sc=0.22$ . It is noticed that velocity increases for the flow past a cooled plate ( $Gr > 0$ ) to flow past a heated plate ( $Gr < 0$ ) both for Newtonian fluid and visco-elastic fluid. The

visco-elasticity factor ( $k=0.01$  and  $0.02$ ) present in the various fluid flow mechanisms diminishes the speed of the fluid in comparison to the Newtonian fluid

Figure-(2) shows the effect of the parameters  $G_m$  and  $k$  on velocity at any point of the fluid, when  $Pr=7$ ,  $Gr=2$ ,  $M=2$ ,  $K=2$ ,  $h=0.2$ ,  $R=2$  and  $Sc=0.22$ . It is noticed that the velocity decreases with the increase of modified Grashof number ( $G_m$ ), where as visco-elasticity factor ( $k=0.01$  and  $0.02$ ) present in the various fluid flow mechanisms diminishes the speed of the fluid in comparison to the Newtonian fluid.

Figure-(3) shows the effect of the parameters  $Sc$ ,  $Pr$  and  $k$  on velocity at any point of the fluid, when  $G_m=2$ ,  $Gr=2$ ,  $M=2$ ,  $K=2$ ,  $h=0.2$  and  $R=2$ . It is noticed that the velocity decreases for Newtonian fluid and increases for visco-elastic fluid with the increase of Schmidt number ( $Sc$ ) and Prandtl number ( $Pr$ ).

Figure-(4) shows the effect of the parameters  $M$  and  $K$  on velocity at any point of the fluid, when  $G_m=2$ ,  $Gr=2$ ,  $Pr=7$ ,  $Sc=0.22$ ,  $h=0.2$  and  $R=2$ . It is noticed that the velocity decreases for Newtonian fluid and increases for visco-elastic fluid with the increase of magnetic parameter ( $M$ ) and decrease of permeability of porous medium ( $K$ ).

Figure-(5) shows the effect of the parameters  $R$  and  $h$  on velocity at any point of the fluid, when  $M=2$ ,  $K=2$ ,  $G_m=2$ ,  $Gr=2$ ,  $Pr=7$ ,  $Sc=0.22$  and  $R=2$ . It is noticed that the velocity increases with the increase of chemical parameter ( $R$ ) for both Newtonian fluid and visco-elastic fluid, whereas it increases for Newtonian fluid and decreases for visco-elastic fluid with the increase refraction parameter ( $h$ ).

*Heat Profile:* Figure-(6) shows the effect of the parameters  $Pr$  on Heat profile at any point of the fluid in the absence of other parameters. It is noticed that the temperature falls in the increase of Prandtl number ( $Pr$ ).

*Mass concentration profile:* Figure-(7) shows the effect of the parameters  $Sc$  and  $R$  on mass concentration profile at any point of the fluid in the absence of other parameters. It is noticed that the mass concentration decreases with the increase of Schmidt number ( $Sc$ ) and chemical reaction parameter ( $R$ ).

**Table :(1) Effect of different parameters on shearing stress**

Pr	Sc	Gr	$G_m$	h	R	M	K	Shearing stress for $k=0$	Shearing stress for $k=0.02$
7	0.22	2	2	0.02	2	2	2	4.8416	-458.96
1	0.22	2	2	0.02	2	2	2	-3.5403	-3.0327
0.25	0.22	2	2	0.02	2	2	2	-4.588	-4.3958
7	0.3	2	2	0.02	2	2	2	5.0604	-458.6493
7	0.78	2	2	0.02	2	2	2	6.1213	-457.2409
7	0.22	4	2	0.02	2	2	2	2.9194	-460.7041
7	0.22	6	2	0.02	2	2	2	0.9975	-462.4421

7	0.22	2	4	0.02	2	2	2	11.6033	-916.1990
7	0.22	2	6	0.02	2	2	2	18.369	-1373.4
7	0.22	2	2	0.4	2	2	2	5.2217	-458.4080
7	0.22	2	2	0.6	2	2	2	5.5147	-457.9724
7	0.22	2	2	0.02	4	2	2	3.72	-352.64
7	0.22	2	2	0.02	6	2	2	3.02	-286.3729
7	0.22	2	2	0.02	2	4	2	3.1558	-382.5594
7	0.22	2	2	0.02	2	6	2	1.9588	328.3051
7	0.22	2	2	0.02	2	2	4	5.1123	-471.32
7	0.22	2	2	0.02	2	2	6	5.2069	-475.52

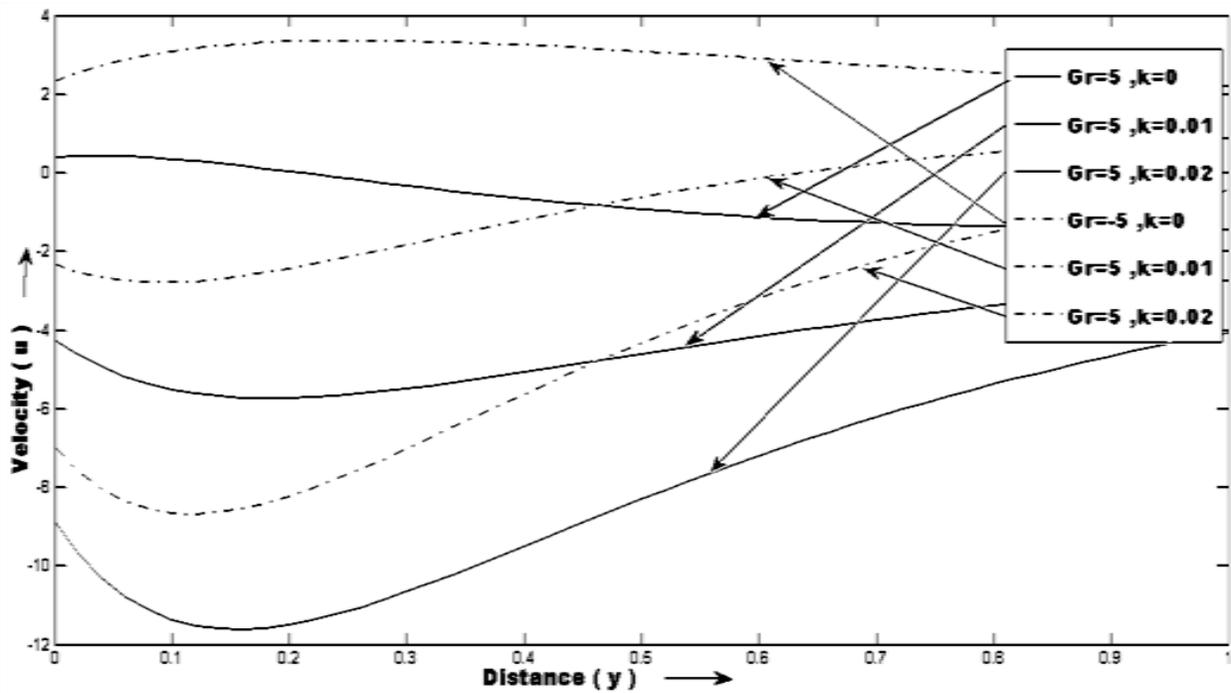


Fig-(1)-Effect of Gr and k on Velocity profile, when Pr=7, Gm=2, M=2, K=2, R=2,h=0.2 and Sc=0.22

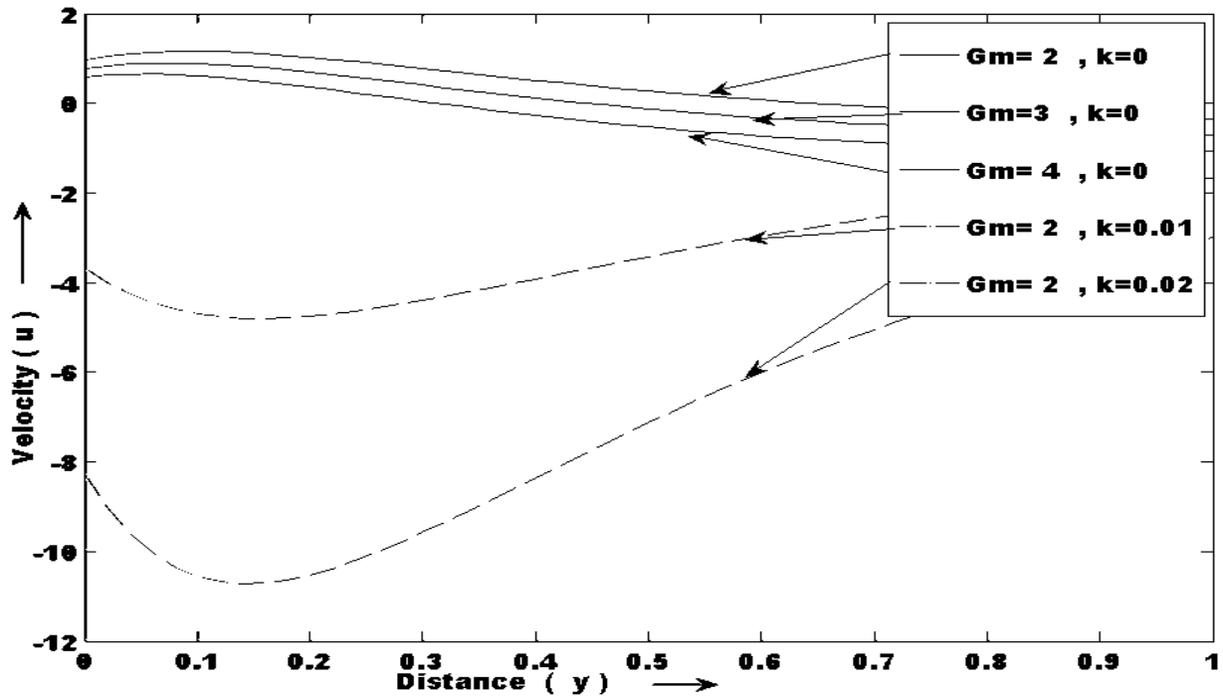
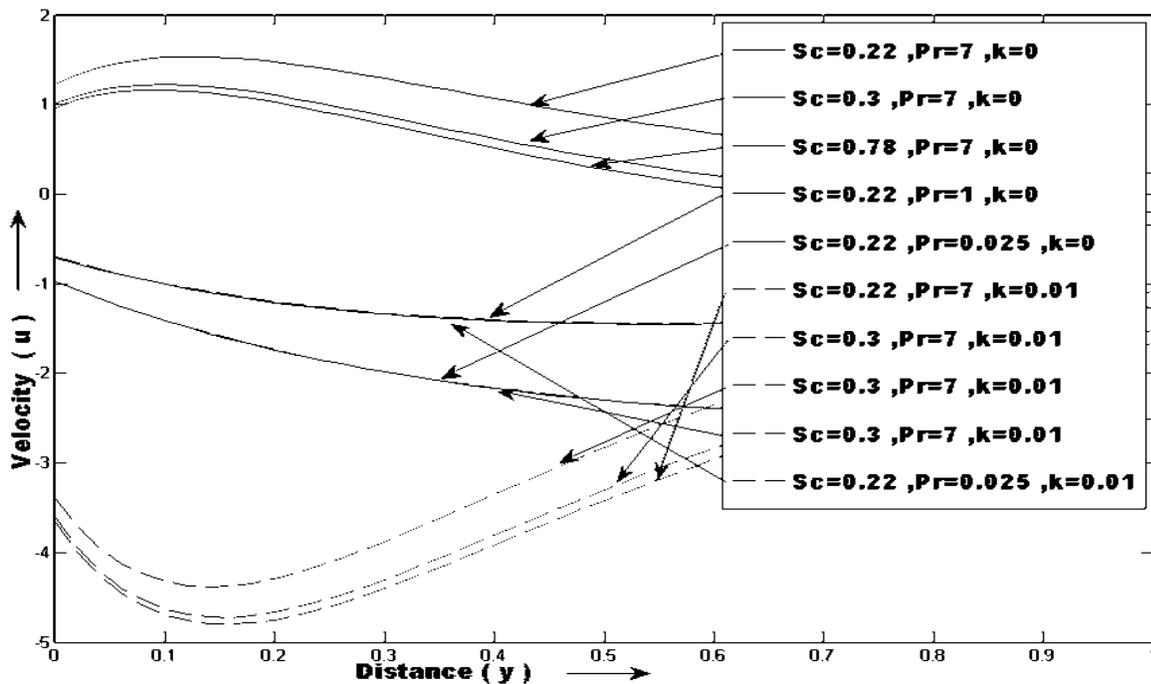
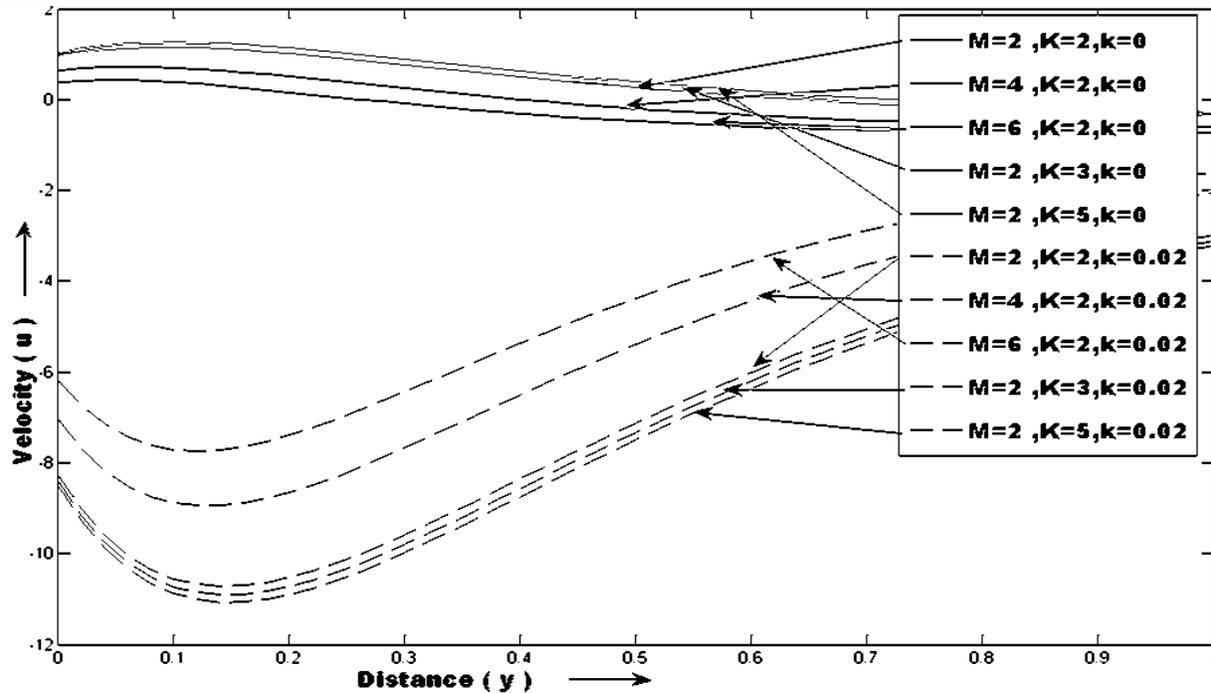


Fig-(2)-Effect of Gm and k on Velocity profile, when Pr=7, Gr=2, M=2, K=2, h=0.2,R= 2 and Sc=0.22



**Fig-(3)-Effect of  $Sc$ ,  $Pr$  and  $k$  on Velocity profile, when  $Gm=2$ ,  $Gr=2$ ,  $M=2$ ,  $K=2$ ,  $h=0.2$  and  $R=2$ .**



**Fig-(4)-Effect of  $M$ ,  $K$  and  $k$  on Velocity profile, when  $Gm=2$ ,  $Gr=2$ ,  $Pr=7$ ,  $Sc=0.22$ ,  $h=0.2$  and  $R=2$**

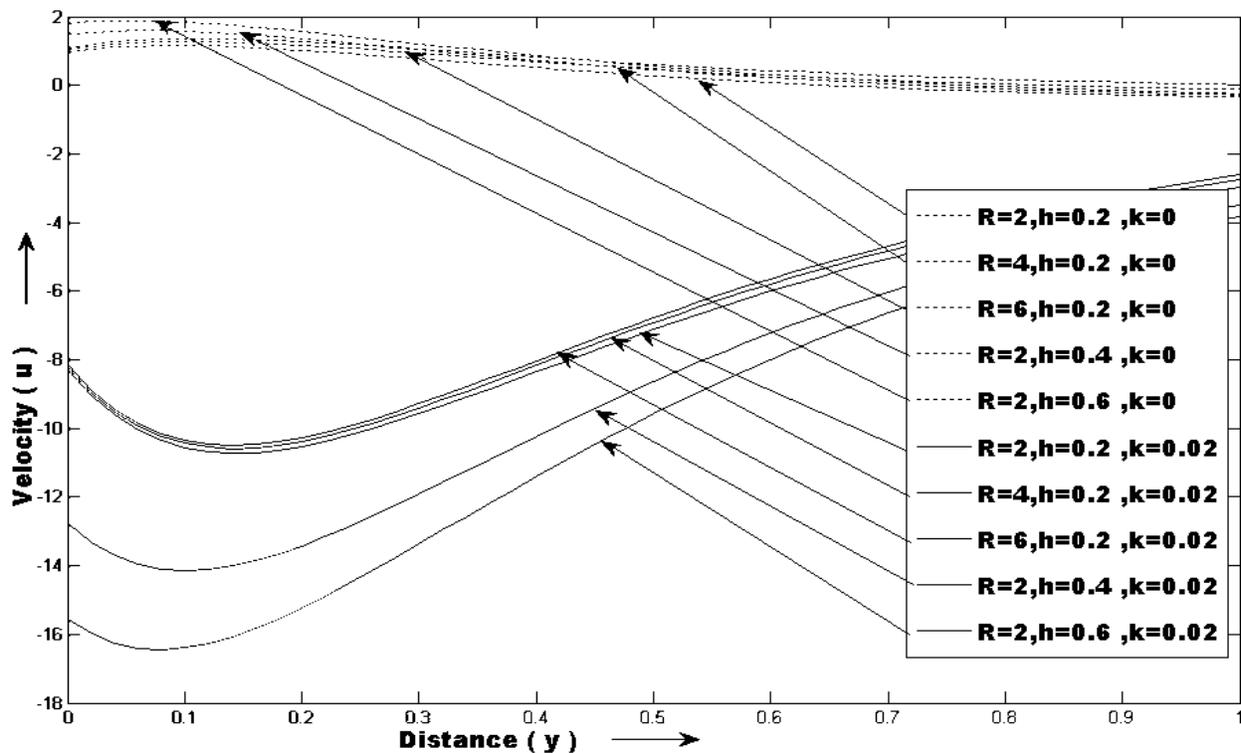


Fig-(5)-Effect of  $R$ ,  $h$  and  $k$  on Velocity profile, when  $M=2, K=2, Gm=2, Gr=2, Pr=7, Sc=0.22$  and  $R=2$

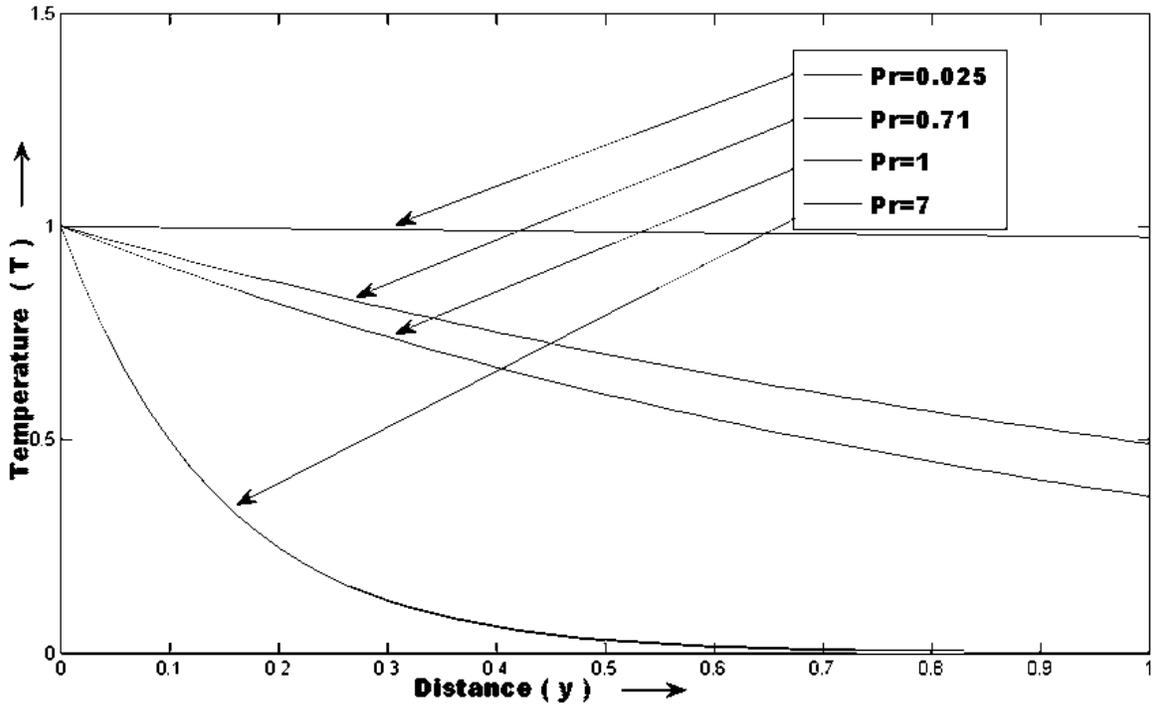


Fig-(6)-Effect of  $Pr$  on Temperature profile in the absence of other parameters

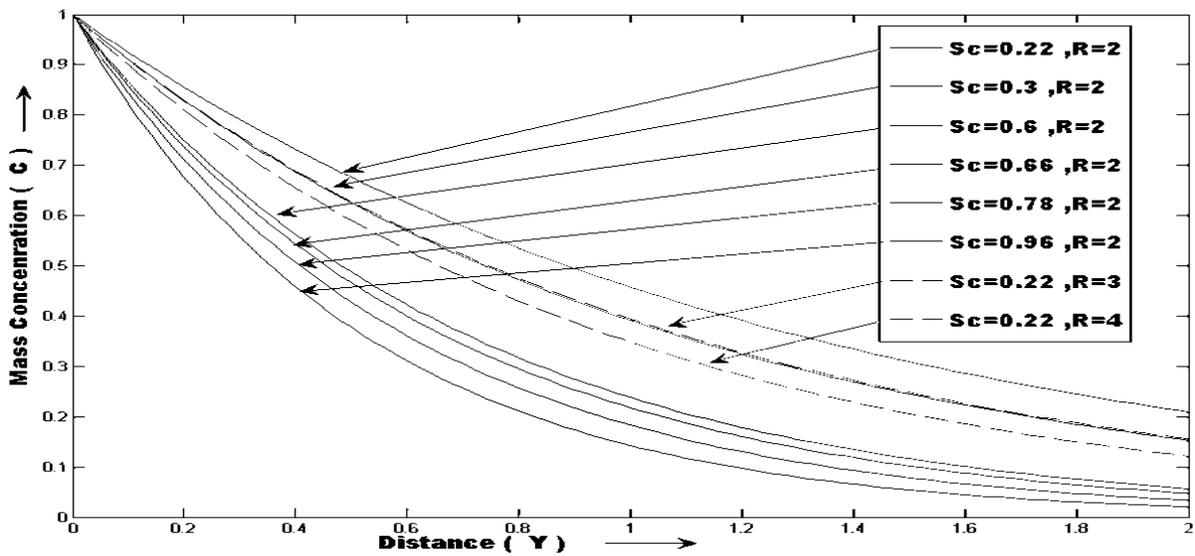


Fig-(7)-Effect of  $Sc$  and  $R$  on Mass concentration profile in the absence of other parameters

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