International Research Journal of Human Resources and Social Sciences
Vol. 4, Issue 6, June 2017 Impact Factor- 5.414
ISSN(O): (2349-4085) ISSN(P): (2394-4218)

Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

SUM SQUARE PRIME LABELING OF SOME SNAKE GRAPHS

Sunoj B S[1], Mathew Varkey T K[2]

[1]Assistant Professor, Department of Mathematics, Government Polytechnic College, Attingal, India.

[2]Assistant Professor, Department of Mathematics, T K M College of Engineering, Kollam, India.

ABSTRACT

Sum square prime labeling of a graph is the labeling of the vertices with {0,1,2------,p-1} and the edges with square of the sum of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits sum square prime labeling. Here we identify some snake graphs for sum square prime labeling.

KEYWORDS - Graph labeling, Greatest Common Incidence Number, Prime Labeling, Snake Graphs.

INTRODUCTION

All graphs in this paper are simple, connected, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)-graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated sum square prime labeling of some snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

MAIN RESULTS

Definition 2.1 Let G = (V,E) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{ssp}^* : E(G) → set of natural numbers N by $f_{ssp}^*(uv) = \{f(u) + f(v)\}^2$. The induced function f_{ssp}^* is said to be a sum square prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits sum square prime labeling is called a sum square prime graph.

Definition 2.3 Graph obtained by replacing the internal edges of a comb by triangles, we get a new graph called comb triangular snake and it is denoted by Com(T_n).

Theorem 2.1 Triangular snake T_n admits sum square prime labeling.

Proof: Let G = T_n and let v_1, v_2,----------------,v_{2n-1} are the vertices of G

Here |V(G)| = 2n-1 and |E(G)| = 3n-3

Define a function f : V → {0,1,2,3,----------------,2n-2 } by

$$f(v_i) = i-1 , \quad i = 1,2,------,2n-1$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i\, v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,2n-2

$f_{ssp}^*(v_{2i-1}\, v_{2i+1})$ = $(4i-2)^2$, i = 1,2,------------,n-1

Clearly f_{ssp}^* is an injection.

gcin of (v_{i+1}) = gcd of $\{f_{ssp}^*(v_i\, v_{i+1}) , f_{ssp}^*(v_{i+1}\, v_{i+2}) \}$

= gcd of $\{ (2i-1)^2, (2i+1)^2\}$

= gcd of {2i-1 , 2i+1} = 1, i = 1,2,------------,2n-3

gcin of (v_1) = gcd of $\{f_{ssp}^*(v_1\, v_2) , f_{ssp}^*(v_1\, v_3) \}$

= gcd of {1,4} = 1.

gcin of (v_{2n-1}) = gcd of $\{f_{ssp}^*(v_{2n-2}\, v_{2n-1}) , f_{ssp}^*(v_{2n-3}\, v_{2n-1}) \}$

= gcd of { $(4n-5)^2$, $(4n-6)^2$}

= gcd of { (4n-5), (4n-6)} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence T_n , admits sum square prime labeling. ■

Theorem 2.2 Alternate triangular snakes $A(T_n)$ admits sum square prime labeling, when n is even and triangle starts from the first vertex.

Proof: Let G = $A(T_n)$ and let v_1, v_2,----------------,$v_{\frac{3n}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,----------------,$\frac{3n-2}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------,$\frac{3n}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------, $\frac{3n-2}{2}$

$f_{ssp}^*(v_{3i-2}\ v_{3i})$ = $(6i-4)^2$, i = 1,2,------------,$\frac{n}{2}$

Clearly f_{ssp}^* is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------, $\frac{3n-4}{2}$

gcin of (v_1) = gcd of {$f_{ssp}^*(v_1\ v_2)$, $f_{ssp}^*(v_1\ v_3)$ }

= gcd of {1, 4}

gcin of $(v_{(\frac{3n}{2})})$ = gcd of {$f_{ssp}^*(v_{(\frac{3n}{2})}\ v_{(\frac{3n-2}{2})})$, $f_{ssp}^*(v_{(\frac{3n}{2})}\ v_{(\frac{3n-4}{2})})$ }

= gcd of {$(3n-3)^2$, $(3n-4)^2$}

= gcd of {(3n-3), (3n-4)} = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits sum square prime labeling. ■

Theorem 2.3 Alternate triangular snakes $A(T_n)$ admits sum square prime labeling, when n is even and triangle starts from the second vertex.

Proof: Let G = $A(T_n)$ and let v_1, v_2,----------------,$v_{(\frac{3n-2}{2})}$ are the vertices of G

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3

Define a function f : V → {0,1,2,3,----------------,$\frac{3n-4}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------,$\frac{3n-2}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f^*_{ssp}(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------, $\frac{3n-4}{2}$

$f^*_{ssp}(v_{3i-1}\ v_{3i+1})$ = $(6i-2)^2$, i = 1,2,------------,$\frac{n-2}{2}$

Clearly f^*_{ssp} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------, $\frac{3n-6}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits sum square prime labeling. ■

Theorem 2.4 Alternate triangular snakes $A(T_n)$ admits sum square prime labeling, when n is odd and triangle starts from the first vertex.

Proof: Let G = $A(T_n)$ and let v_1, v_2,----------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n-2$

Define a function $f : V \rightarrow \{0,1,2,3,$----------------,$\frac{3n-3}{2}\}$ by

$f(v_i) = i-1$, i = 1,2,------,$\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f^*_{ssp}(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------, $\frac{3n-3}{2}$

$f^*_{ssp}(v_{3i-2}\ v_{3i})$ = $(6i-4)^2$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f^*_{ssp} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------, $\frac{3n-5}{2}$

gcin of (v_1) = gcd of $\{f^*_{ssp}(v_1\ v_2)$, $f^*_{ssp}(v_1\ v_3)\}$

= gcd of {1, 4}

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits sum square prime labeling. ■

Theorem 2.5 Alternate triangular snakes $A(T_n)$ admits sum square prime labeling, when n is odd and triangle starts from the second vertex.

Proof: Let G = $A(T_n)$ and let v_1, v_2,----------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n-2$

Define a function $f : V \rightarrow \{0,1,2,3,$----------------,$\frac{3n-3}{2}\}$ by

$f(v_i) = i-1$, i = 1,2,------,$\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f_{ssp}^{*}(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------, $\frac{3n-3}{2}$

$f_{ssp}^{*}(v_{3i-1}\ v_{3i+1})$ = $(6i-2)^2$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f_{ssp}^{*} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------, $\frac{3n-5}{2}$

gcin of ($v_{(\frac{3n-1}{2})}$) = gcd of {$f_{ssp}^{*}(v_{(\frac{3n-1}{2})}\ v_{(\frac{3n-3}{2})})$, $f_{ssp}^{*}(v_{(\frac{3n-1}{2})}\ v_{(\frac{3n-5}{2})})$ }

= gcd of {$(3n-4)^2$, $(3n-5)^2$}

= gcd of {(3n-4), (3n-5)} = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n), admits sum square prime labeling. ■

Theorem 2.6 Double triangular snakes D(T_n) admits sum square prime labeling.

Proof: Let G = D(T_n) and let v_1,v_2,----------------,v_{3n-2} are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 5n-5

Define a function f : V → {0,1,2,3,----------------,3n-3 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^{*} is defined as follows

$f_{ssp}^{*}(v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,n-1

$f_{ssp}^{*}(v_{3i+1}\ v_{3i-1})$ = $(6i-2)^2$, i = 1,2,------------,n-1

$f_{ssp}^{*}(v_{3i-2}\ v_{3i})$ = $(6i-4)^2$, i = 1,2,------------,n-1

$f_{ssp}^{*}(v_{3i}\ v_{3i+1})$ = $(6i-1)^2$, i = 1,2,------------,n-1

$f_{ssp}^{*}(v_{3i-2}\ v_{3i+1})$ = $(6i-3)^2$, i = 1,2,------------,n-1

Clearly f_{ssp}^{*} is an injection.

gcin of (v_{3i-1}) = gcd of {$f_{ssp}^{*}(v_{3i-2}\ v_{3i-1})$, $f_{ssp}^{*}(v_{3i-1}\ v_{3i+1})$ }

= gcd of {$(6i-5)^2$, $(6i-2)^2$}

= gcd of {(6i-5), (6i-2)}

= gcd of {3,6i-5} = 1, i = 1,2,----------------,n-1.

gcin of (v_{3i}) = gcd of {$f_{ssp}^{*}(v_{3i-2}\ v_{3i})$, $f_{ssp}^{*}(v_{3i}\ v_{3i+1})$ }

= gcd of {$(6i-4)^2$, $(6i-1)^2$}

= gcd of {(6i-4), (6i-1)}

= gcd of {3,6i-4} = 1, i = 1,2,----------------,n-1.

gcin of (v_{3i+1}) = gcd of {$f_{ssp}^{*}(v_{3i+1}\ v_{3i})$, $f_{ssp}^{*}(v_{3i-1}\ v_{3i+1})$ }

= gcd of {$(6i-2)^2$, $(6i-1)^2$}

= gcd of {(6i-2), (6i-1)}

= 1, i = 1,2,----------------,n-1.

gcin of (v_1) = gcd of $\{f^*_{ssp}(v_1\ v_2)\ ,\ f^*_{ssp}(v_1\ v_3)\ \}$

= gcd of {1,4} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence D(T_n), admits sum square prime labeling. ■

Theorem 2.7 Comb snake graph Com(T_n) admits sum square prime labeling.

Proof: Let G = Com(T_n) and let v_1,v_2,----------------,v_{3n-1} are the vertices of G

Here |V(G)| = 3n-1 and |E(G)| = 4n-3.

Define a function f : V → {0,1,2,3,----------------,3n-2 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f^*_{ssp}\ (v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,n-1

$f^*_{ssp}\ (v_{3i+1}\ v_{3i-1})$ = $(6i-2)^2$, i = 1,2,------------,n-1

$f^*_{ssp}\ (v_{3i-2}\ v_{3i})$ = $(6i-4)^2$, i = 1,2,------------,n-1

$f^*_{ssp}\ (v_{3i-2}\ v_{3i+1})$ = $(6i-3)^2$, i = 1,2,------------,n-1

$f^*_{ssp}\ (v_{3n-2}\ v_{3n-1})$ = $(6n-5)^2$,

Clearly f^*_{ssp} is an injection.

gcin of (v_{3i-1}) = gcd of $\{f^*_{ssp}\ (v_{3i-2}\ v_{3i-1})\ ,\ f^*_{ssp}\ (v_{3i-1}\ v_{3i+1})\ \}$

= gcd of $\{(6i-5)^2, (6i-2)^2\}$

= gcd of {(6i-5), (6i-2)}

= gcd of {3,6i-5} = 1, i = 1,2,----------------,n-1.

gcin of (v_{3i+1}) = gcd of $\{f^*_{ssp}\ (v_{3i+1}\ v_{3i})\ ,\ f^*_{ssp}\ (v_{3i-1}\ v_{3i+1})\ \}$

= gcd of $\{(6i-2)^2, (6i-1)^2\}$

= gcd of {(6i-2), (6i-1)}

= 1, i = 1,2,----------------,n-1.

gcin of (v_1) = gcd of $\{f^*_{ssp}\ (v_1\ v_2)\ ,\ f^*_{ssp}\ (v_1\ v_3)\ \}$

= gcd of {1,4} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Com(T_n), admits sum square prime labeling. ■

CONCLUSIONS

Here we proved the results for some snake graphs. This topic is open for all other researchers to prove more snake graphs admit sum square prime labeling. Graph labeling as a whole has applications in various scientific and engineering problem. So surely sum square prime labeling is also useful to solve various application problems.

REFERENCES

1. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
2. F Harary, Graph Theory, Addison-Wesley,Reading, Mass, (1972)
3. Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics(2016), #DS6, pp 1 – 408.
4. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.